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# Identities of Common Factors of Generalized Fibonacci, Jacobsthal and Jacobsthal-Lucas Numbers

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**Abstract** The Fibonacci sequence is famous for possessing wonderful and amazing properties. In this paper, we present generalized identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers and related identities consisting even and odd terms. Binet's formula will employ to obtain the identities.

***Keywords:*** *generalized Fibonacci numbers, Jacobsthal and jacobsthal-Lucas numbers, Binet's formula*

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## 1. Introduction

It is well-known that the Fibonacci sequence is most prominent examples of recursive sequence. The Fibonacci sequence is famous for possessing wonderful and amazing properties. Fibonacci numbers are a popular topic for mathematical enrichment and popularization. The Fibonacci appear in numerous mathematical problems. The Fibonacci numbers $F_n$ are terms of the sequence $\{0,1,1,2,3,5,...\}$ wherein each term is the sum of the two previous terms, beginning with the values $F_0 = 0$ and $F_1 = 1$.

There are a lot of identities of Fibonacci and Lucas numbers described in [6]. Thongmoon [10], defined various identities of Fibonacci and Lucas numbers. Singh, Bhadouria and Sikhwal [9], present some generalized identities involving common factors of Fibonacci and Lucas numbers. Gupta and Panwar [1], present identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers. Panwar, Singh and Gupta [8], present Generalized Identities Involving Common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers. In this paper, we present generalized identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers.

## 2. Preliminaries

Before presenting our main theorems, we will need to introduce some known results and notations.

Generalized Fibonacci sequence ([2,7]) is defined as

$$F_k = pF_{k-1} + qF_{k-2},\ k \geq 2 \tag{2.1}$$
$$with\ F_0 = a,\ F_1 = b$$

where $p, q, a$ & $b$ are positive integers.

For different values of $p, q, a$ & $b$ many sequences can be determined.

If $p = 1,\ q = a = 2, b = 0$, we get

$$U_k = U_{k-1} + 2U_{k-2},\ k \geq 2 \tag{2.2}$$
$$with\ U_0 = 2,\ U_1 = 0.$$

The first few terms of $\{U_k\}_{k\geq 0}$ are 2, 0, 4, 4, 12, 20 and so on.

Its Binet forms is defined by

$$U_k = 4\frac{\Re_1^{k-1} - \Re_2^{k-1}}{\Re_1 - \Re_2} \tag{2.3}$$

The Jacobsthal sequence [3], is defined by the recurrence relation

$$J_k = J_{k-1} + 2J_{k-2},\ k \geq 2\ with\ J_0 = 0,\ J_1 = 1 \tag{2.4}$$

Its Binet's formula is defined by

$$J_k = \frac{\Re_1^k - \Re_2^k}{\Re_1 - \Re_2} \tag{2.5}$$

The Jacobsthal-Lucas sequence [3], is defined by the recurrence relation

$$j_k = j_{k-1} + 2j_{k-2},\ k \geq 2\ with\ j_0 = 2,\ j_1 = 1 \tag{2.6}$$

Its Binet's formula is defined by

$$j_k = \Re_1^k + \Re_2^k \tag{2.7}$$

where $\Re_1$ & $\Re_2$ are the roots of the characteristic equation $x^2 - x - 2 = 0$.

## 3. Main Results

Generalized Fibonacci sequence ([2,7]), similar to the other second order classical sequences. In this section we present generalized identities involving common factors of generalized Fibonacci, Jacobsthal and Jacobsthal-Lucas numbers. We shall use the Binet's formula for derivation.

**Theorem 1:** *If $U_k$ is the generalized Fibonacci numbers and $j_k$ is Jacobsthal-Lucas numbers, then,*

$$U_{2k+p}\, j_{2k+1} = U_{4k+p+1} - 2^{2k+1} U_{p-1} \qquad (3.1)$$

*where* $k \geq 0$ & $p \geq 0$.

**Proof:**

$$U_{2k+p}\, j_{2k+1} = 4\left(\frac{\Re_1^{2k+p-1} - \Re_2^{2k+p-1}}{\Re_1 - \Re_2}\right)\left(\Re_1^{2k+1} + \Re_2^{2k+1}\right)$$

$$= 4\left(\frac{\Re_1^{4k+p} - \Re_2^{4k+p}}{\Re_1 - \Re_2}\right) + \frac{4}{(\Re_1 - \Re_2)}(\Re_1\Re_2)^{2k}\left(\Re_1^{p-1}\Re_2 - \Re_2^{p-1}\Re_1\right)$$

$$= 4\left(\frac{\Re_1^{4k+p} - \Re_2^{4k+p}}{\Re_1 - \Re_2}\right) - \frac{4}{(\Re_1 - \Re_2)} 2^{2k+1}\left(\Re_1^{p-2} - \Re_2^{p-2}\right)$$

$$= U_{4k+p+1} - 2^{2k+1} U_{p-1}$$

This completes the proof.

**Corollary 1.1:** For different values of $p$, (3.1) can be expressed for even and odd numbers:

(*i*) *If* $p = 0$, *then*: $U_{2k}\, j_{2k+1} = U_{4k+1} + 2^{2k+1}$

(*ii*) *If* $p = 1$, *then*: $U_{2k+1}\, j_{2k+1} = U_{4k+2} - 4^{k+1}$

(*iii*) *If* $p = 2$, *then*: $U_{2k+2}\, j_{2k+1} = U_{4k+3}$

**Corollary 1.2:**

$$U_{2k+p}\, j_{2k+1} = 4\left\{J_{4k+p} - 2^{2k+1} J_{p-2}\right\} \qquad (3.2)$$

Following theorems can be solved by Binet's formulae (2.3), (2.5) and (2.7).

**Theorem 2:**

$$U_{2k+p}\, j_{2k+2} = U_{4k+p+2} - 4^{k+1} U_{p-2} \qquad (3.3)$$

*where* $k \geq 0$ & $p \geq 0$.

**Corollary 2.1:** For different values of $p$, (3.3) can be expressed for even and odd numbers:

(*i*) *If* $p = 0$, *then*: $U_{2k}\, j_{2k+2} = U_{4k+2} - 3\left(2^{2k+1}\right)$

(*ii*) *If* $p = 1$, *then*: $U_{2k+1}\, j_{2k+2} = U_{4k+3} + 4^{k+1}$

(*iii*) *If* $p = 2$, *then*: $U_{2k+2}\, j_{2k+2} = U_{4k+4} - 2^{2k+3}$

**Corollary 2.2:**

$$U_{2k+p}\, j_{2k+2} = 4\left\{J_{4k+p+1} - 4^{k+1} J_{p-3}\right\} \qquad (3.4)$$

**Theorem 3:**

$$U_{2k+p}\, j_{2k} = U_{4k+p} + 4^k U_p \qquad (3.5)$$

*where* $k \geq 0$ & $p \geq 0$.

**Corollary 3.1:** For different values of $p$, (3.5) can be expressed for even and odd numbers:

(*i*) *If* $p = 0$, *then*: $U_{2k}\, j_{2k} = U_{4k} + 2^{2k+1}$

(*ii*) *If* $p = 1$, *then*: $U_{2k+1}\, j_{2k} = U_{4k+1}$

(*iii*) *If* $p = 2$, *then*: $U_{2k+2}\, j_{2k} = U_{4k+2} + 4^{k+1}$

**Corollary 3.2:**

$$U_{2k+p}\, j_{2k} = 4\left\{J_{4k+p-1} + 4^k J_{p-1}\right\} \qquad (3.6)$$

*where* $k \geq 0$ & $p \geq 0$.

**Theorem 4:**

$$U_{2k-p}\, j_{2k+1} = U_{4k+1-p} - 2^{2k+1} U_{-p-1} \qquad (3.7)$$

*where* $k \geq 0$ & $p \geq 0$.

**Corollary 4.1:** For different values of $p$, (3.7) can be expressed for even and odd numbers:

(*i*) *If* $p = 0$, *then*: $U_{2k}\, j_{2k+1} = U_{4k+1} + 2^{2k+1}$

(*ii*) *If* $p = 1$, *then*: $U_{2k-1}\, j_{2k+1} = U_{4k} - 3\left(4^k\right)$

(*iii*) *If* $p = 2$, *then*: $U_{2k-2}\, j_{2k+1} = U_{4k-1} + 5\left(2^{2k-1}\right)$

**Corollary 4.2:**

$$U_{2k-p}\, j_{2k+1} = 4\left\{J_{4k-p} - 2^{2k+1} J_{-p-2}\right\} \qquad (3.8)$$

*where* $k \geq 0$ & $p \geq 0$.

**Theorem 5:**

$$U_{2k-p}\, j_{2k-1} = U_{4k-1-p} - 2^{2k-1} U_{-p-1} \qquad (3.9)$$

*where* $k \geq 0$ & $p \geq 0$.

**Corollary 5.1:** For different values of $p$, (3.9) can be expressed for even and odd numbers:

(*i*) *If* $p = 0$, *then*: $U_{2k}\, j_{2k-1} = U_{4k-1} + 2^{2k-1}$

(*ii*) *If* $p = 1$, *then*: $U_{2k-1}\, j_{2k-1} = U_{4k-2} - 3\left(4^{k-1}\right)$

(*iii*) *If* $p = 2$, *then*: $V_{2k-2}\, j_{2k-1} = U_{4k-3} + 5\left(2^{2k-3}\right)$

**Corollary 5.2:**

$$U_{2k-p}\, j_{2k-1} = 4\left\{J_{4k-p-2} - 2^{2k-1} J_{-p}\right\} \qquad (3.10)$$

*where* $k \geq 0$ & $p \geq 0$.

**Theorem 6:**

$$U_{2k-p}\, j_{2k} = U_{4k-p} + 4^k U_{-p} \qquad (3.11)$$

*where* $k \geq 0$ & $p \geq 0$.

**Corollary 6.1:** For different values of $p$, (3.11) can be expressed for even and odd numbers:

(*i*) *If* $p = 0$, *then*: $U_{2k}\, j_{2k} = U_{4k} + 2^{2k+1}$

(ii) If $p=1$, then: $U_{2k-1}j_{2k}=U_{4k-1}-4^k$

(iii) If $p=2$, then: $U_{2k-2}j_{2k}=U_{4k-2}+3\left(2^{2k-1}\right)$

**Corollary 6.2:**

$$V_{2k-p}j_{2k}=4\left\{J_{4k-p-1}+4^kJ_{-p-1}\right\} \tag{3.12}$$

where $k\geq 0$ & $p\geq 0$.

**Theorem 7**

$$U_{2k}j_{2k+p}=U_{4k+p}+2^{2k-1}U_{p+2} \tag{3.13}$$

where $k\geq 0$ & $p\geq 0$.

**Corollary 7.1:** For different values of $p$, (3.13) can be expressed for even and odd numbers:

(i) If $p=0$, then: $U_{2k}j_{2k}=U_{4k}+2^{2k+1}$

(ii) If $p=1$, then: $U_{2k}j_{2k+1}=U_{4k+1}+2^{2k+1}$

(iii) If $p=2$, then: $U_{2k}j_{2k+2}=U_{4k+2}+3\left(2^{2k+1}\right)$

**Corollary 7.2:**

$$U_{2k}j_{2k+p}=4\left\{J_{4k+p-1}+2^{2k-1}J_{p+1}\right\} \tag{3.14}$$

where $k\geq 0$ & $p\geq 0$.

# 4. Conclusion

In this paper we present generalized identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers and related identities consisting even and odd terms. Mainly Binet's formula employ for the identities. The concept can be executed for generalized second order recursive sequences as well as polynomials..

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