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# Effects of Hall Current and Thermal Radiation on an Unsteady Free Convection Slip Flow along a Vertical Plate Embedded in a Porous Medium with Constant Heat and Mass Flux

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**Abstract** The present investigation is made to study the effects of Hall current, thermal radiation and other parameters on an unsteady free convection flow of an incompressible, viscous fluid through a porous medium in a rotating system. The plate moves with a constant velocity $U_0$ in its own plane. A perturbation technique is used to obtain the solutions of velocity, temperature and concentration. Expressions for skin friction, Nusselt number and Sherwood number are also derived. It is observed that as we increase the hall parameter (m), $C_{fP}$ and $|C_{f_s}|$ rises. Also increase in radiation tends the Nusselt number to rise.

***Keywords:*** *hall current, heat flux, mass flux, porous medium, rotation, unsteady*

## 1. Introduction

Process involving coupled heat and mass transfer occurs frequently in nature. It occurs not only due to the temperature differences, but also, due to concentration differences or the combination of the two in different geophysical cases. Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Sivaiah et al. [1] and Das and Jana [2] have studied heat and mass transfer effects on MHD free convection flow past a vertical plate with different boundary conditions. Chaudhary and Jain [3] have also studied combined heat and mass transfer effects on MHD free convection flow past an oscillating plate.

The thermal radiation effects, on free convection flow are important in context of space technology and in certain applications involving, heat storage in aquifiers and gasification of oil. In the case of gasification large temperature gradients exists in the neighborhood of combustion, due to radiation. These type of problems are also extended in the case of magnetohydrodynamics and if the strength of magnetic field is strong, then the effect of Hall current cannot be neglected.

Recently, the effect of these currents have been studied by many research workers, on different flows and boundary conditions. Aboeldahab et al. [4] and Shit [5] studied the effects of Hall currents on MHD free convection flow with mass transfer. Also, Abuga et al. [6] investigated the effects of hall current and rotational parameters on dissipative fluid. The effects of thermal radiation, Hall current on MHD flow by mixed convection over a vertical surface in porous media have been studied by Shateyi et al. [7]. Saha et al. [8] studied the effects of Hall current on the MHD laminar natural convection flow and Mbeledogu and Ogulu [9] observed heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in presence of radiation. Further, Zhukovsky [10] studied modeling of the current limitations of PEFC.

The flows of fluid through porous media are quite prevalent in nature. Such flows have application in many branches of science and technology viz. to study seepage of water in river beds, to study movement of natural gas, oil and water through oil reservoirs etc. Acharya et al. [11] studied the magnetic effects on the free convection and mass transfer flow through porous medium, whereas, Rajesh and Varma [12] observed the heat source effects on MHD flow through a porous medium. Taking into account the Hall currents, Farhad et al. [13] studied a hydromagnetic rotating flow in a porous medium with slip condition and Hall currents. Moreover, Hall effect on MHD mixed convective flow past a vertical porous plate immersed in a porous medium have been studied by Sharma et al. [14].

The no slip boundary condition is valid only when particles close to the surface do not move with the flow i.e when adhesion is stronger than cohesion, however this is only true macroscopically. Effect of Hall currents, with slip conditions at the boundary have been studied by Abbas and Asghar [15] and Jain and Gupta [16]. Rescently Jain and Singh [17], have observed Hall and thermal radiative effects on an unsteady rotating free convection slip flow.

In general, Hall effect devices produce a very low signal level and thus require amplification. While suitable for laboratory instruments, the vacuum tube amplifiers available in the first half of the 20th century were too expensive, power consuming, and unreliable for everyday applications. It was only with the development of the low cost integrated circuit that the Hall effect sensor became suitable for mass applications. These are readily available from a number of different manufacturers, and may be used in various sensors such as rotating speed sensors (bicycle wheels, gear-teeth, automotive speedometers, and electronic ignition systems), fluid flow sensors, current sensors, and pressure sensors. Moreover, the most common application of thermal radiation is a microwave.

In the present paper, we study the effects of Hall currents, thermal radiation, Soret number etc., on an unsteady, rotating, free convection slip flow, past an infinite vertical porous plate, embedded in a porous medium, with constant heat and mass flux at the plate. The plate moves with a constant velocity $U_0$. A perturbation technique is used to obtain the expression for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number. It is noteworthy, that when we increase our hall parameter (m), primary velocity (u) increases at the plate but retards as we move away from the plate where as for the same secondary velocity (v) drops throughout the distance. Also, we notice that our Nusselt number rises when we increase the radiation.

## 2. Formulation of the Problem

We consider an unsteady free convection flow of an incompressible viscous fluid through a porous medium past an infinite vertical porous plate, with a constant heat flux and mass flux at z = 0. Let both the fluid and the plate be in a state of rigid rotation with uniform angular velocity Ω about the z-axis taken normal to the plate. A uniform magnetic field $H_0$ is imposed along the z-axis and the plate is considered to be electrically non-conducting. Moreover, the plate is assumed to coincide with the plane z = 0. As the plate is infinite in extent and the flow is unsteady, all the physical variables depend on z and t only. In addition, the permeability of the porous medium is assumed to be constant.

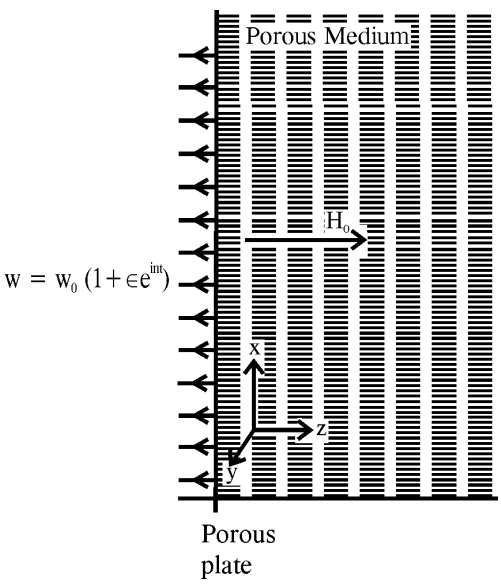



**Figure 1.** Schematic diagram

The equation of continuity $\nabla.q = 0$ gives on integration $w = -w_0\left(1+\in e^{-nt}\right)$, $(w_0 > 0)$, the negative sign indicates the suction velocity is towards the plate, where q = (u, v, w). The induced magnetic field has to be assumed negligible so that H = (0, 0, $H_0$). This assumption is justified when the magnetic Reynolds number is small (Shereliff, [18]). The equation of conservation of electrical charge $\nabla.J = 0$ gives $J_z$ = constant, where J = ($J_x$, $J_y$, $J_z$).

The constant is taken to be zero since $J_z$ = 0 at the plate which is electrically non-conducting. Thus $J_z$ = 0, is everywhere in the flow. Thus, for the present configuration and under Boussinesq's approximation, the governing equations are as follows:

Momentum equations:

$$\frac{\partial u}{\partial t} - 2\Omega v - w_0(1+\in e^{-nt})\frac{\partial u}{\partial z} = g\beta(T-T_\infty) + g\beta*(C-C_\infty) + \nu\frac{\partial u}{\partial z} - \frac{\nu}{K}u + \frac{\mu_e H_o}{\rho}J_y \quad (1)$$

$$\frac{\partial v}{\partial t} + 2\Omega u - w_0\left(1+\in e^{-nt}\right)\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\nu}{K}v - \frac{\mu_e H_0}{\rho}J_x \quad (2)$$

Energy Equation

$$\frac{\partial T}{\partial t} - w_0\left(1+\in e^{-nt}\right)\frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_P}\frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial z} + \frac{Q}{\rho C_P}\frac{\partial T}{\partial z}, \quad (3)$$

Concentration Equation

$$\frac{\partial C}{\partial t} - w_0\left(1+\in e^{-nt}\right)\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} + D_\ell\frac{\partial^2 T}{\partial z^2}, \quad (4)$$

where

$$J_x = \frac{\sigma\mu_e H_0}{1+m^2}(v+mu), \quad J_y = \frac{\sigma\mu_e H_0}{1+m^2}(-u+mv)$$

The local radiant is expressed by:

$$\frac{\partial q_r}{\partial z} = -4a*\sigma*\left(T_\infty^4 - T^4\right), \quad (5)$$

it is assumed that the temperature differences within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T^4$ in a Taylor series about $T_\infty$ and neglecting higher-order, thus

$$T^4 \cong 4T_d^3\,T - 3T_d^4 \quad (6)$$

by using (5) and (6), we obtain

$$\frac{\partial q_r}{\partial z} = -16a*\sigma*T_\infty^3\left(T_\infty - T\right) \quad (7)$$

where σ* is Stephen-Boltzmann constant and a* is absorption coefficient, England and Emery [19].

The boundary conditions are:

$$\left.\begin{array}{llllll} z=0: & u = U_0 + L\frac{\partial u}{\partial z}, & v = L\frac{\partial v}{\partial z}, & \frac{\partial T}{\partial z} = -\frac{q}{\kappa}, & \frac{\partial C}{\partial z} = -\frac{m_0}{D}, \\ z\to\infty: & u\to 0 \quad , & v\to 0 \quad , & T\to T_\infty \quad , & C\to C_\infty \quad . \end{array}\right] \quad (8)$$

Where q and $m_0$ are uniform heat and concentration flux at the plate respectively, m = $W_e\tau_e$ is the Hall parameter, $W_e$ is the cyclotron frequency and $\tau_e$ is the collision time of

electrons, σ is the fluid conductivity, $\mu_e$ is the magnetic permeability, u and v are the velocities is x and y directions respectively, T is the temperature of the fluid, $T_\infty$ is the temperature of the fluid far away from the plate, β is the coefficient of thermal expansion, κ is the thermal conductivity, $C_P$ is the specific heat at constant pressure, K is the permeability of the medium, ρ is the density, ν is the kinematic viscosity, g is acceleration due to gravity, β* is the coefficient of expansion due to concentration, μ is coefficient of viscosity, D is molecular diffusivity and $D_\ell$ is thermal diffusivity and other symbols have their usual meaning.

On introducing the following non-dimensional quantities:

$$u^* = \frac{u}{U_0},\ v^* = \frac{v}{U_0},\ z^* = \frac{zw_0}{\nu},\ n^* = \frac{n\nu}{w_0^2},\ t^* = \frac{w_0^2 t}{\nu}$$

$$\theta = \frac{(T - T_\infty)\kappa w_0}{\nu q},\ \ C^* = \frac{(C - C_\infty)Dw_0}{\nu m_0}$$

$$K^* = \frac{Kw_0^2}{\nu^2}\ (\text{permeability parameter}).$$

in equations (1) –(4) and after dropping the asterisks over them yields:

$$\frac{\partial u}{\partial t} - 2Ev - \left(1 + \in e^{-nt}\right)\frac{\partial u}{\partial z} = Gr\,\theta + Gc\,C + \frac{\partial^2 u}{\partial z^2} - \frac{u}{K} + \frac{M^2}{\left(1+m^2\right)}(-u + mv), \tag{9}$$

$$\frac{\partial v}{\partial t} + 2Eu - \left(1 + \in e^{-nt}\right)\frac{\partial v}{\partial z} = \frac{\partial^2 v}{\partial z^2} - \frac{v}{K} - \frac{M^2}{\left(1+m^2\right)}(v + mu), \tag{10}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \in e^{-nt}\right)\frac{\partial \theta}{\partial z} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial z^2} - \frac{R}{\Pr}\theta + H\frac{\partial \theta}{\partial z}, \tag{11}$$

$$\frac{\partial C}{\partial t} - \left(1 + \in e^{-nt}\right)\frac{\partial C}{\partial z} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} + \frac{So}{Sc}\frac{\partial^2 \theta}{\partial z^2} \tag{12}$$

with boundary conditions as:

$$\left.\begin{array}{llllll} z = 0: & u = 1 + h\dfrac{\partial u}{\partial z}, & v = h\dfrac{\partial v}{\partial z}, & \dfrac{\partial \theta}{\partial z} = -1, & \dfrac{\partial C}{\partial z} = -1, \\ z \to \infty: & u \to 0, & v \to 0, & \theta \to 0, & C \to 0. \end{array}\right] \tag{13}$$

where

$$E = \frac{\Omega \nu}{w_0^2}\quad (\text{rotational parameter}),$$

$$\Pr = \frac{\mu C_P}{\kappa}\quad (\text{prandtl number}),$$

$$Sc = \frac{\nu}{D}\quad (\text{Schmidt number}),$$

$$Gr = \frac{g\beta\nu^2 q}{U_0 w_0^3 \kappa}\quad (\text{thermal Grashof number}),$$

$$Gc = \frac{g\beta^*\nu^2 m_0}{U_0 w_0^3\, D}\quad (\text{mass Grashof number}),$$

$$M = \left(\frac{\nu\mu_e^2 H_0^2 \sigma}{\rho\, w_0^3}\right)^{1/2}\quad (\text{Hartmann number}),$$

$$h = \frac{Lw_0}{\nu}\quad (\text{velocity slip parameter}),$$

$$So = \frac{D_\ell q}{m_0 \kappa}\quad (\text{soret number}),$$

$$R = \frac{16a^*\sigma^*\nu^2 T_\infty^3}{\kappa w_0^2}\quad (\text{radiation parameter}),$$

$$H = \frac{Q}{\rho\, C_P\, w_0}\quad (\text{Heat source parameter}).$$

We assume q = u + iv. Hence from equations (9) and (10), we obtain:

$$\frac{\partial^2 q}{\partial z^2} + \left(1 + \in e^{-nt}\right)\frac{\partial q}{\partial z} - \frac{\partial q}{\partial t} - \left[s + \frac{1}{K}\right]q = -Gr\,\theta - Gc\,C \tag{14}$$

where $s = \left[2Ei + \dfrac{M^2}{(1 - im)}\right]$

with boundary conditions as:

$$\left.\begin{array}{lll} z = 0: q = 1 + h\dfrac{\partial q}{\partial z}, & \dfrac{\partial \theta}{\partial z} = -1, & \dfrac{\partial C}{\partial z} = -1 \\ z \to \infty: q \to 0 \quad , & \theta \to 0 \quad , & C \to 0 \end{array}\right] \tag{15}$$

## 3. Solution of the Problem

To solve the equations (11), (12) and (14), which are coupled partial differential equations, we follow the perturbation technique of the form:

$$\left.\begin{array}{l} q = q_0(z) + \in e^{-nt} q_1(z), \\ \theta = \theta_0(z) + \in e^{-nt}\theta_1(z), \\ C = C_0(z) + \in e^{-nt} C_1(z) \end{array}\right\} \tag{16}$$

Where $\in (<<1)$ is very small.

Substituting equation (16) in equations (11), (12) and (14) and equating the coefficients of like powers of ∈ (neglecting $\in^2$ and higher orders), we obtain the following set of differential eauations:

$$\frac{d^2 q_0}{dz^2} + \frac{dq_0}{dz} - \left(s + \frac{1}{K}\right)q_0 = -Gr\theta_0 - Gc\,C_0, \tag{17}$$

$$\frac{d^2 q_1}{dz^2} + \frac{dq_1}{dz} - \left(s - n + \frac{1}{K}\right)q_1 = -Gr\theta_1 - Gc\,C_1 - \frac{dq_0}{dz}, \tag{18}$$

$$\frac{d^2\theta_0}{dz^2} + \Pr(1 + H)\frac{d\theta_0}{dz} - R\theta_0 = 0. \tag{19}$$

$$\frac{d^2\theta_1}{dz^2}+\Pr(1+H)\frac{d\theta_1}{dz}+\left(n\Pr-R\right)\theta_1=-\Pr\frac{d\theta_0}{dz}, \quad (20)$$

$$\frac{d^2C_0}{dz^2}+Sc\frac{dC_0}{dz}=-So\frac{d^2\theta_0}{dz^2}, \quad (21)$$

$$\frac{d^2C_1}{dz^2}+Sc\frac{dC_1}{dz}+n\,Sc\,C_1=-Sc\frac{dC_0}{dz}-So\frac{d^2\theta_1}{dz^2}, \quad (22)$$

with boundary conditions as:

$$\left.\begin{array}{lll} z=0: q_0=1+h\dfrac{dq_0}{dz}, & \dfrac{d\theta_0}{dz}=-1, & \dfrac{dC_0}{dz}=-1 \\ q_1=h\dfrac{dq_1}{dz}, & \dfrac{d\theta_1}{dz}=0, & \dfrac{dC_1}{dz}=0 \\ z\to\infty: q_0\to 0, & \theta_0\to 0, & C_0\to 0 \\ q_1\to 0, & \theta_1\to 0, & C_1\to 0 \end{array}\right] \quad (23)$$

The solution of equations (17) to (22) under the transformed boundary conditions (23) yields:

$$q_0(z)=m_9e^{-x_7z}+J_6e^{-x_1z}+J_7e^{-Sc\,z}, \quad (24)$$

$$\begin{aligned} q_1(z)=&\,m_{11}e^{-x_9z}+J_8e^{-x_3z}+J_9e^{-x_1z} \\ &+J_{10}e^{-x_5z}+J_{11}e^{-x_7z}+J_{12}e^{-Sc\,z}, \end{aligned} \quad (25)$$

$$\theta_0(z)=m_1e^{-x_1z}, \quad (26)$$

$$\theta_1(z)=m_3e^{-x_3z}+J_1e^{-x_1z}, \quad (27)$$

$$C_0(z)=m_6e^{-Sc\,z}+J_2e^{-x_1z}, \quad (28)$$

$$C_1(z)=m_7e^{-x_5z}+J_3e^{-Scz}+J_4e^{-x_3z}+J_5e^{-x_1z}. \quad (29)$$

Using $q=u+iv$, the expressions for the primary and secondary velocities are obtained as:

$$u(z,t)=d_{19}+\in e^{-nt}s_{31} \quad (30)$$

$$v(z,t)=d_{20}+\in e^{-nt}s_{32} \quad (31)$$

$$\theta(z,t)=\theta_0(z)+\in e^{-nt}\theta_1(z) \quad (32)$$

$$C(z,t)=C_0(z)+\in e^{-nt}C_1(z) \quad (33)$$

## 4. Skin Friction

The skin friction $C_{fP}$ due to primary velocity u and the skin friction $C_{fs}$ due to secondary velocity v at the plate are as follows:

$$\begin{aligned} C_{fP}=&\left(\frac{\partial u}{\partial z}\right)_{z=0} \\ =&\left\{-d_3d_{17}+d_4d_{18}-x_1d_6-Scd_9\right\} \\ &+\in e^{-nt}\left\{-s_1s_{29}+s_2s_{30}-s_4x_3\right. \\ &\left.-s_9x_1-s_{12}x_5-d_3s_{18}+d_4s_{19}-Scs_{21}\right\} \end{aligned} \quad (34)$$

and

$$\begin{aligned} C_{fs}=&\left(\frac{\partial v}{\partial z}\right)_{z=0} \\ =&\left\{-d_4d_{17}-d_3d_{18}-d_7x_1-d_{10}Sc\right\} \\ &+\in e^{-nt}\left\{-s_1s_{30}-s_{29}s_2-s_5x_3\right. \\ &\left.-s_{10}x_1-s_{13}x_5-d_3s_{19}-d_4s_{18}-s_{22}Sc\right\} \end{aligned} \quad (35)$$

## 5. Nusselt Number

An important physical parameter, Nusselt number can be obtained after θ is obtained, as follows:

$$\begin{aligned} Nu=&\frac{1}{\theta(0,t)} \\ =&\frac{1}{\left\{m_1+\in e^{-nt}\left(m_3+J_1\right)\right\}} \end{aligned}$$

## 6. Sherwood Number

Once we obtain C, another important physical parameter, Sherwood number can be obtained as

$$\begin{aligned} Sh=&\frac{1}{C\left(0,t\right)} \\ =&\frac{1}{\left\{\left(m_6+J_2\right)+\in e^{-nt}\left(m_7+J_3+J_4+J_5\right)\right\}} \end{aligned}$$

where

$$x_1=\frac{\Pr(1+H)+\sqrt{\Pr^2(1+H)^2+4R}}{2},$$

$$x_2=\frac{-\Pr(1+H)+\sqrt{\Pr^2(1+H)^2+4R}}{2},$$

$$x_3=\frac{\Pr(1+H)+\sqrt{\Pr^2(1+H)^2-4(n\Pr-R)}}{2},$$

$$x_4=\frac{-\Pr(1+H)+\sqrt{\Pr^2(1+H)^2-4(n\Pr-R)}}{2},$$

$$x_5=\frac{Sc+\sqrt{Sc^2-4nSc}}{2},\quad x_6=\frac{-Sc+\sqrt{Sc^2-4nSc}}{2}$$

$$x_7=\frac{1+\sqrt{1+4\left(s+\frac{1}{K}\right)}}{2}=d_3+id_4,$$

$$x_8=\frac{-1+\sqrt{1+4\left(s+\frac{1}{K}\right)}}{2},$$

$$x_9=\frac{1+\sqrt{1+4\left(s-n+\frac{1}{K}\right)}}{2}=s_1+is_2,$$

$$x_{10}=\frac{-1+\sqrt{1+4\left(s-n+\frac{1}{K}\right)}}{2},$$

$$m_1 = \frac{1}{x_1}, \quad m_3 = -\frac{J_1 x_1}{x_3},$$

$$J_1 = \frac{\Pr}{x_1^2 - \Pr x_1 (1+H) + (n\Pr - R)}, \quad J_2 = \frac{-So\, x_1}{x_1^2 - Scx_1}$$

$$m_6 = \frac{-(J_2 x_1 - 1)}{Sc}, \quad J_3 = \frac{m_6 Sc}{n}, \quad J_4 = \frac{-Som_3 x_3^2}{x_3^2 - Scx_3 + nSc}$$

$$J_5 = \frac{\left(J_2 Scx_1 - J_1 S_0 x_1^2\right)}{x_1^2 - Scx_1 + nSc}, \quad m_7 = \frac{-\left(J_3 Sc + J_4 x_3 + J_5 x_1\right)}{x_5}$$

$$J_6 = \frac{\left(-Grm_1 - Gc\, J_2\right)}{x_1^2 - x_1 - \left(s + \frac{1}{K}\right)} = d_6 + id_7$$

$$J_7 = \frac{-Gc\, m_6}{Sc^2 - Sc - \left(s + \frac{1}{K}\right)} = d_9 + id_{10}$$

$$m_9 = \frac{1 - J_6\left(1 + hx_1\right) - J_7(1 + hSc)}{\left(1 + hx_7\right)} = d_{17} + id_{18}$$

$$J_8 = \frac{-\left(Grm_3 + GcJ_4\right)}{x_3^2 - x_3 - \left(s - n + \frac{1}{K}\right)} = s_4 + is_5$$

$$J_9 = \frac{\left(-GrJ_1 - GcJ_5 + J_6 x_1\right)}{x_1^2 - x_1 - \left(s - n + \frac{1}{K}\right)} = s_9 + is_{10}$$

$$J_{10} = \frac{-Gcm_7}{x_5^2 - x_5 - \left(s - n + \frac{1}{K}\right)} = s_{12} + is_{13}$$

$$J_{11} = \frac{m_9 x_7}{x_7^2 - x_7 - \left(s - n + \frac{1}{K}\right)} = s_{18} + is_{19}$$

$$J_{12} = \frac{\left(-GcJ_3 + J_7 Sc\right)}{Sc^2 - Sc - \left(s - n + \frac{1}{K}\right)} = s_{21} + is_{22}$$

$$m_{11} = \frac{-\left\{\begin{array}{c} J_8\left(1 + hx_3\right) + J_9\left(1 + hx_1\right) + J_{10}\left(1 + hx_5\right) \\ + J_{11}\left(1 + hx_7\right) + J_{12}\left(1 + hSc\right) \end{array}\right\}}{\left(1 + hx_9\right)} = s_{29} + is_{30}$$

$$s = d_1 + id_2 \;, \quad d_1 = \frac{M^2}{(1 + m^2)} \;, \quad d_2 = \left\{2E + \frac{M^2 m}{\left(1 + m^2\right)}\right\}$$

$$d_3 = \left\{\frac{1}{2} + \frac{A_1}{2}\right\} \;, \quad d_4 = \frac{B_1}{2} \;, \quad d_5 = \left(x_1^2 - x_1 - d_1 - \frac{1}{K}\right)$$

$$d_6 = \frac{\left(-Gr\, d_5 m_1 - Gc\, d_5\, J_2\right)}{\left(d_5^2 + d_2^2\right)},$$

$$d_7 = \frac{\left(-Gr\, d_2\, m_1 - Gc\, d_2\, J_2\right)}{\left(d_5^2 + d_2^2\right)}$$

$$d_8 = \left(Sc^2 - Sc - d_1 - \frac{1}{K}\right),$$

$$d_9 = \frac{-Gc\, m_6\, d_8}{\left(d_8^2 + d_2^2\right)},$$

$$d_{10} = \frac{-Gc\, m_6\, d_2}{\left(d_8^2 + d_2^2\right)}$$

$$d_{11} = \left\{1 - d_6 - d_6 hx_1 - d_9 - h\,Sc\, d_9\right\},$$
$$d_{12} = \left\{-d_7 - hx_1 d_7 - d_{10} - h\,Sc\, d_{10}\right\},$$

$$d_{13} = \left(1 + hd_3\right), \; d_{14} = hd_4, \; d_{15} = \left(d_{11} d_{13} + d_{12} d_{14}\right)$$

$$d_{16} = \left(d_{12}\, d_{13} - d_{11}\, d_{14}\right), d_{17} = \frac{d_{15}}{\left(d_{13}^2 + d_{14}^2\right)}, \; d_{18} = \frac{d_{16}}{\left(d_{13}^2 + d_{14}^2\right)}$$

$$d_{19} = \left\{e^{-d_3 z}\left(d_{17} C_1 - d_{18} C_2\right) + d_6 e^{-x_1 z} + d_9 e^{-Sc\, z}\right\}$$

$$d_{20} = \left\{e^{-d_3 z}\left(d_{17} C_2 + C_1 d_{18}\right) + d_7 e^{-x_1 z} + d_{10} e^{-Scz}\right\}$$

$$C_1 = C\mathrm{os}\left(-d_4 z\right) \quad , \quad C_2 = S\mathrm{in}\left(-d_4 z\right)$$

$$s_1 = \frac{1}{2} + \frac{A_2}{A}, \quad s_2 = \frac{B_2}{2}$$

$$A_1 = \sqrt{\frac{1}{2}\left\{\sqrt{\left(1 + 4d_1 + \frac{4}{K}\right)^2 + 16d_2^2} + \left(1 + 4d_1 + \frac{4}{K}\right)\right\}}$$

$$B_1 = \sqrt{\frac{1}{2}\left\{\sqrt{\left(1 + 4d_1 + \frac{4}{K}\right)^2 + 16d_2^2} - \left(1 + 4d_1 + \frac{4}{K}\right)\right\}}$$

$$A_2 = \sqrt{\frac{1}{2}\left\{\sqrt{\left(1 + 4d_1 - 4n + \frac{4}{K}\right)^2 + 16d_2^2} + \left(1 + 4d_1 + \frac{4}{K} - 4n\right)\right\}}$$

$$B_2 = \sqrt{\frac{1}{2}\left\{\sqrt{\left(1 + 4d_1 - 4n + \frac{4}{K}\right)^2 + 16d_2^2} - \left(1 + 4d_1 - 4n + \frac{4}{K}\right)\right\}}$$

$$s_4 = \frac{\left(-Gr\, m_3\, s_3 - Gc\, J_4\, s_3\right)}{\left(s_3^2 + d_2^2\right)},$$

$$s_5 = \frac{\left(-Gr\, m_3\, d_2 - Gc\, J_4\, d_2\right)}{\left(s_3^2 + d_2^2\right)}$$

$$s_6 = \left(x_1^2 - x_1 - d_1 + n - \frac{1}{K}\right)$$

$$s_7 = \left(-GrJ_1 s_6 - Gc\, J_5\, s_6 + d_6\, x_1\, s_6 - d_7 x_1 d_2\right)$$

$$s_8 = \left(-GrJ_1 d_2 - Gc\, J_5\, d_2 + d_6\, x_1\, d_2 + d_7 x_1 s_6\right)$$

$$s_9 = \frac{s_7}{\left(s_6^2 + d_2^2\right)}, \quad s_{10} = \frac{s_8}{\left(s_6^2 + d_2^2\right)}$$

$$s_{11} = \left(x_5^2 - x_5 - d_1 + n - \frac{1}{K}\right), \quad s_{12} = \frac{-Gc\, m_7\, s_{11}}{\left(s_{11}^2 + d_2^2\right)}$$

$$s_{13} = \frac{-Gc\, m_7\, d_2}{\left(s_{11}^2 + d_2^2\right)}, \quad s_{14} = \left(d_{17}\, d_3 - d_{18}\, d_4\right)$$

$$s_{15} = \left(d_3 d_{18} + d_4 d_{17}\right),$$

$$s_{16} = \left(d_3^2 - d_4^2 - d_3 - d_1 + n - \frac{1}{K}\right)$$

$$s_{17} = \left(2d_3 d_4 - d_4 - d_2\right)$$

$$s_{18} = \frac{\left(s_{14}s_{16} + s_{15}s_{17}\right)}{\left(s_{16}^2 + s_{17}^2\right)}, \quad s_{19} = \frac{\left(s_{15}s_{16} - s_{14}s_{17}\right)}{\left(s_{16}^2 + s_{17}^2\right)}$$

$$s_{20} = \left(Sc^2 - Sc - d_1 + n - \frac{1}{K}\right),$$

$$s_{21} = \frac{\left\{\left(-Gc\, J_3 + d_9 Sc\right)s_{20} - d_{10} Scd_2\right\}}{\left(s_{20}^2 + d_2^2\right)}$$

$$s_{22} = \frac{\left\{\left(-Gc\, J_3 + d_9\, Sc\right)d_2 + d_{10}\, Sc\, s_{20}\right\}}{\left(s_{20}^2 + d_2^2\right)}$$

$$s_{23} = \left\{\begin{array}{r} s_4\left(1 + hx_3\right) + s_9\left(1 + hx_1\right) + s_{12}\left(1 + hx_5\right) + \\ s_{18}\left(1 + hd_3\right) - s_{19}d_4 + s_{21}\left(1 + hSc\right) \end{array}\right\}$$

$$s_{24} = \left\{\begin{array}{r} s_5\left(1 + hx_3\right) + s_{10}\left(1 + hx_1\right) + s_{13}\left(1 + hx_5\right) \\ + s_{18}d_4 + s_{19}\left(1 + hd_3\right) + s_{22}(1 + hSc \end{array}\right\}$$

$$s_{25} = \left(1 + hs_1\right), \quad s_{26} = hs_2$$

$$s_{27} = -\left(s_{23}s_{25} + s_{24}s_{26}\right), \quad s_{28} = \left(s_{23}s_{26} - s_{25}s_{24}\right)$$

$$s_{29} = \frac{s_{27}}{\left(s_{25}^2 + s_{26}^2\right)}, \; s_{30} = \frac{s_{28}}{\left(s_{25}^2 + s_{26}^2\right)}$$

$$s_{31} = \left\{e^{-s_1 z}\left(s_{29}C_3 - s_{30}C_4\right) + s_4 e^{-x_3 z} + s_9 e^{-x_1 z} + s_{12} e^{-x_5 z} + e^{-d_3 z}\left(s_{18}C_1 - s_{19}C_2\right) + s_{21} e^{-Sc\, z}\right\}$$

$$s_{32} = \left\{\begin{array}{r} e^{-s_1 z}\left(C_3 s_{30} + s_{29}C_4\right) + s_5 e^{-x_3 z} + s_{10} e^{-x_1 z} + \\ s_{13} e^{-x_5 z} + e^{-d_3 z}\left(s_{19}C_1 + s_{18}C_2\right) + s_{22} e^{-Sc\, z} \end{array}\right\}$$

$$C_3 = C\mathrm{os}\left(-s_2 z\right), \quad C_4 = S\mathrm{in}\left(-s_2 z\right)$$

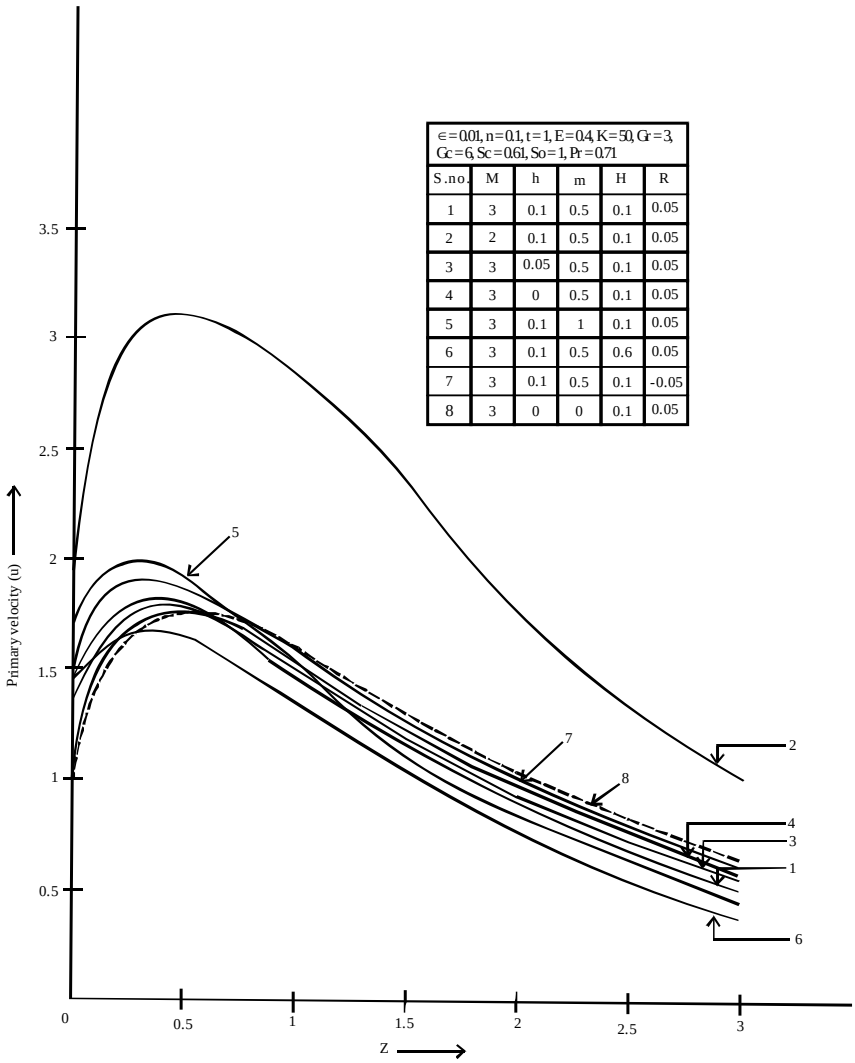



**Figure 2.** Primary velocity profiles plotted against z for different values of M, h, m, H and R

## 7. Results and Discussion

To study the results, numerical values of the primary and secondary velocities (u, v), temperature (θ), primary and secondary skin frictions $\left(C_{f_p}, C_{f_s}\right)$ are computed for different parameters viz. Hartmann number (M), velocity slip parameter (h), Hall parameter (m), Grashof number (Gr), mass Grashof number (Gc), radiation parameter (R), Soret number (So) etc. we fix Pr=0.71, $\in = 0.01$, $E = 0.4$, $n = 0.1$.

In Figure 2 and Figure 3, primary velocity (u) profiles are plotted against z. We fix t = 1 and Sc = 0.61. It is observed that on increasing K, Gr, Gc and So, our primary velocity increases where as primary velocity decreases on increasing M, R and H. We note that on increasing K (permeability parameter), primary velocity increases since medium becomes more porous. Also, it is interesting to note that when we increase h and m, primary velocity increases near the plate but decreases as we move away from the plate. It is not worthy that when we consider (h = 0, m = 0), the case of no slip and no Hall current, primary velocity decreases near the plate but then rises as compared to the case of $h \neq 0, m \neq 0$.

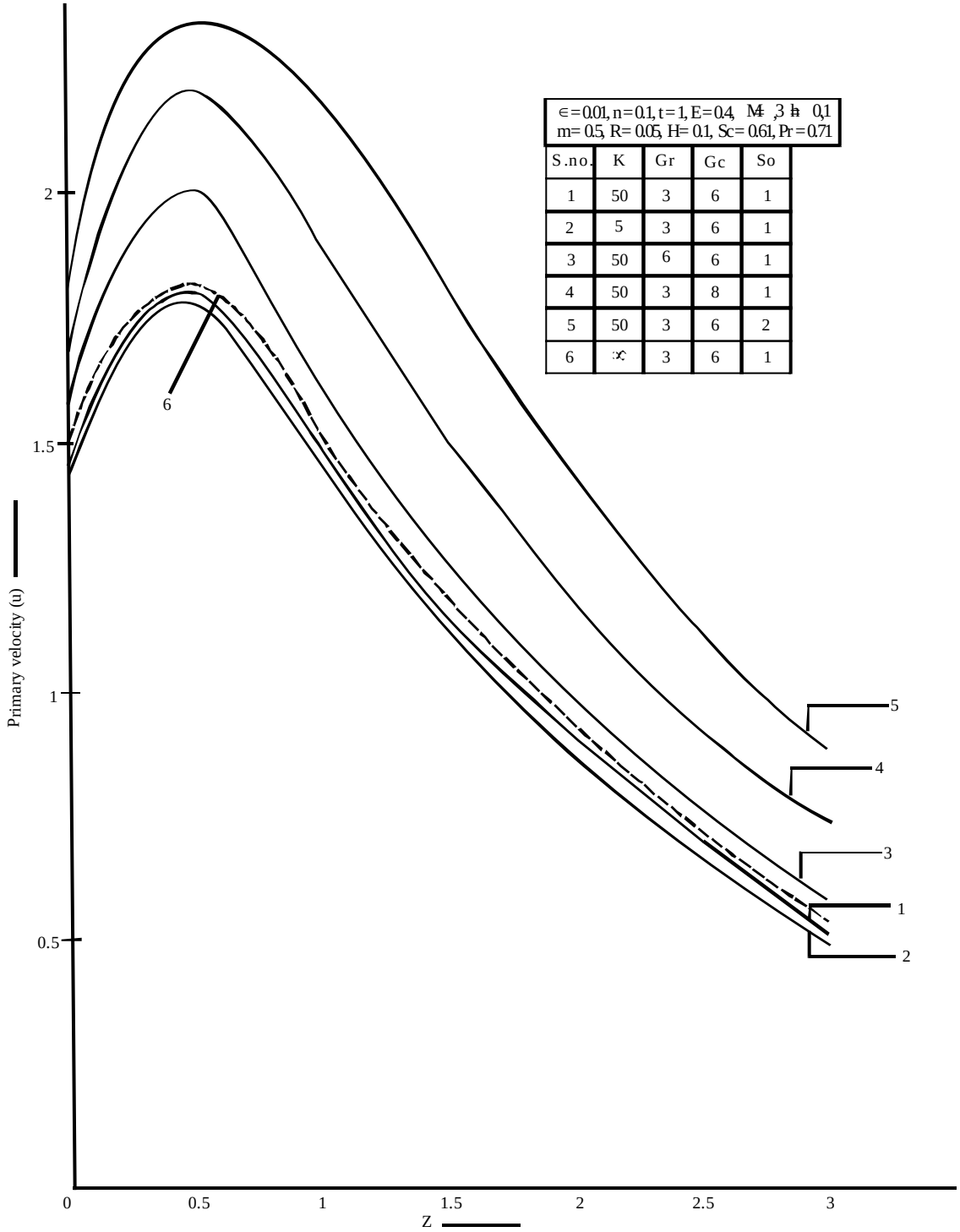
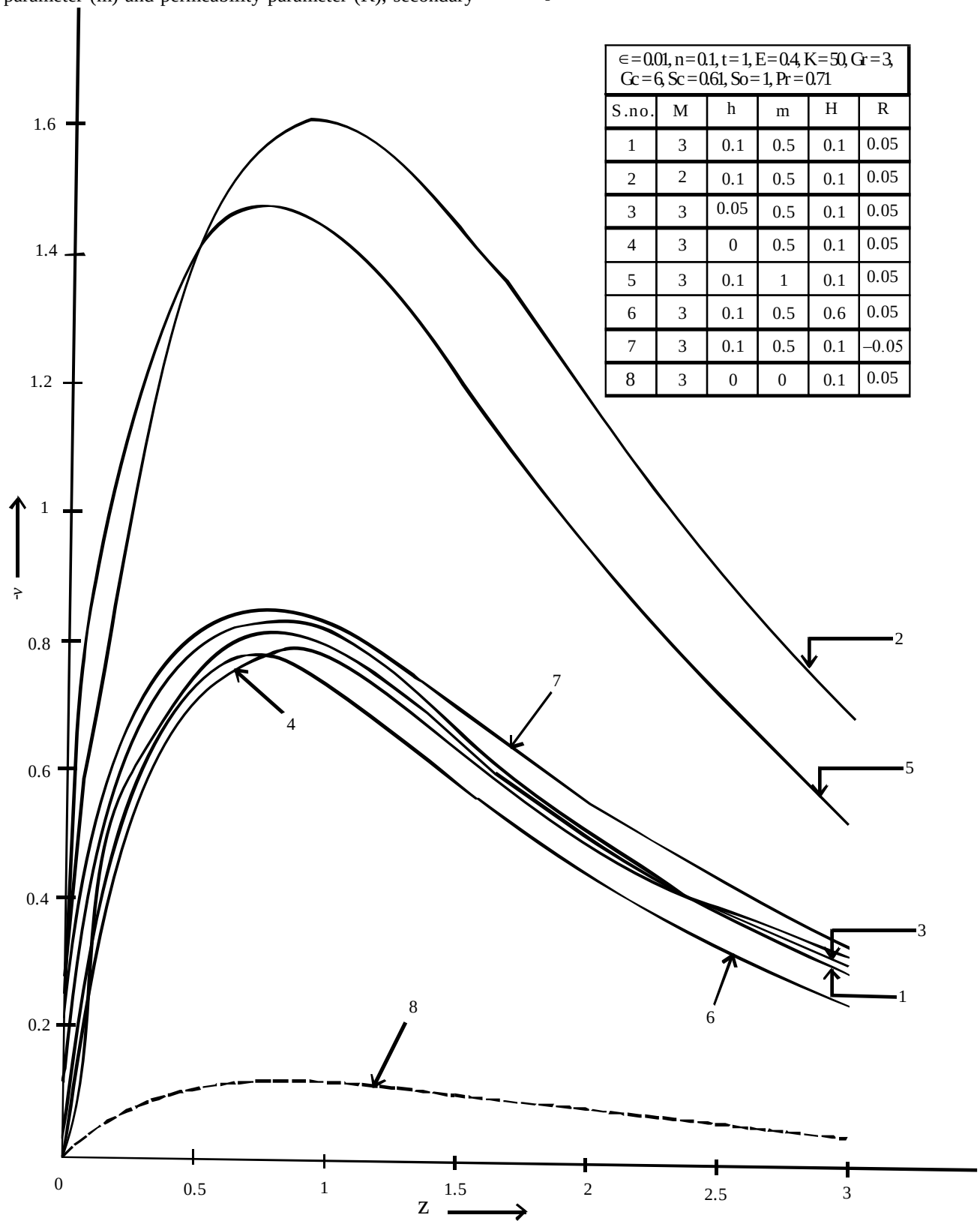



**Figure 3**. Primary velocity profiles plotted against z for different values of K, Gr, Gc and So

Secondary velocity ($v$) profiles are plotted against z in Figure 4 and Figure 5, fixing t = 1, Sc = 0.61. We observe that on increasing M, R, h and H, secondary velocity decreases near the plate but rises immediately as we move away from the plate, whereas on increasing the hall parameter (m) and permeability parameter (K), secondary velocity drops throughout the distance. Also on increasing Gr, Gc and So we observe an increase in secondary velocity near the plate but it decreases as we move away from the plate. It is noteworthy from the figure that for the case of (h = 0, m = 0), secondary velocity rises as compared to the case of $h \neq 0, m \neq 0$.




| ∈ = 0.01, n = 0.1, t = 1, E = 0.4, K = 50, Gr = 3, Gc = 6, Sc = 0.61, So = 1, Pr = 0.71 | | | | | |
|---|---|---|---|---|---|
| S.no. | M | h | m | H | R |
| 1 | 3 | 0.1 | 0.5 | 0.1 | 0.05 |
| 2 | 2 | 0.1 | 0.5 | 0.1 | 0.05 |
| 3 | 3 | 0.05 | 0.5 | 0.1 | 0.05 |
| 4 | 3 | 0 | 0.5 | 0.1 | 0.05 |
| 5 | 3 | 0.1 | 1 | 0.1 | 0.05 |
| 6 | 3 | 0.1 | 0.5 | 0.6 | 0.05 |
| 7 | 3 | 0.1 | 0.5 | 0.1 | –0.05 |
| 8 | 3 | 0 | 0 | 0.1 | 0.05 |

**Figure 4**. Secondary velocity profiles against z for different values of M, h, m, H and R

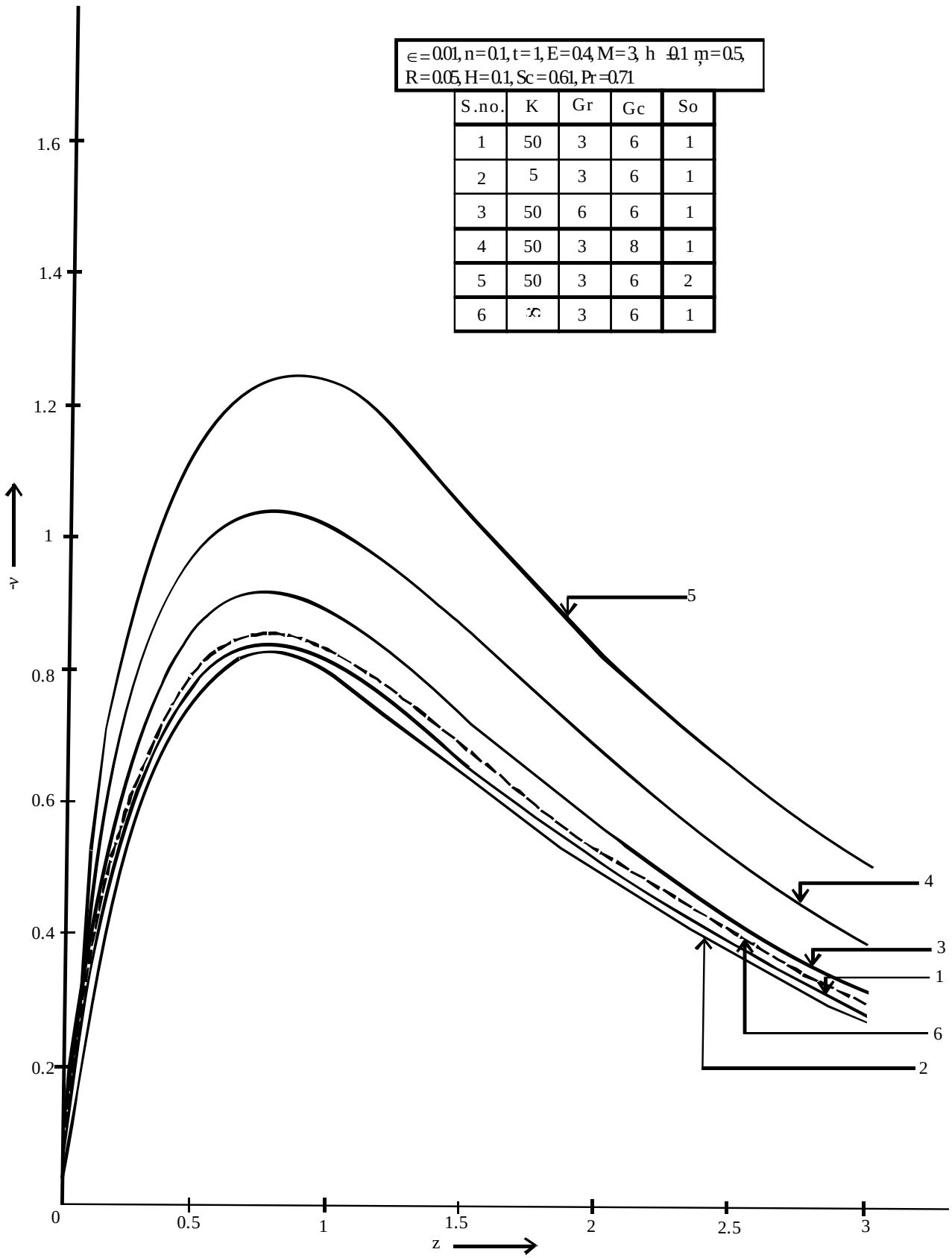
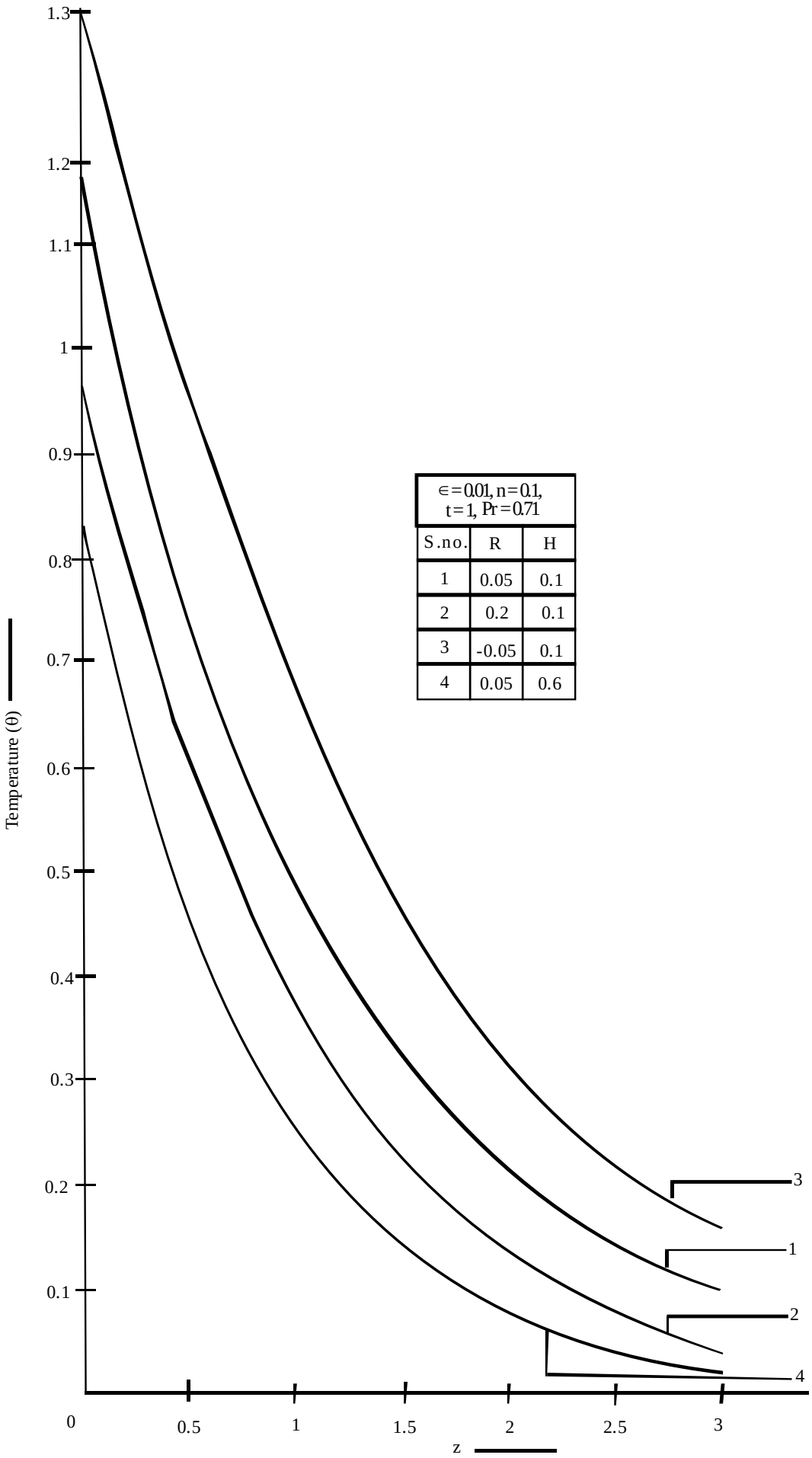



**Figure 5**. Secondary velocity profiles plotted against z for different values of K, Gr, Gc and So

Temperature profiles are plotted against z in Figure 6, fixing t = 1. From the figure we observe that as we increase the radiation our temperature drops as heat is being radiated while the effect of absorption is obvious, the temperature rises. Also, increase is H, which is source parameter, decreases the temperature.

In Figure 7, effects of different parameters are studied on concentration (C), profiles are plotted against z fixing t = 1. We observe that on increasing the Schmidt number (Sc) our concentration drops while concentration rises on increasing the Soret number (So). It's interesting to note that when we increase R and H, concentration increases at the plate but decreases as soon as we move away from the plate.




| $\in = 0.01, n = 0.1, t = 1, Pr = 0.71$ | | |
|---|---|---|
| S.no. | R | H |
| 1 | 0.05 | 0.1 |
| 2 | 0.2 | 0.1 |
| 3 | -0.05 | 0.1 |
| 4 | 0.05 | 0.6 |

**Figure 6**. Temperature profiles plotted against zfor different values of R and H

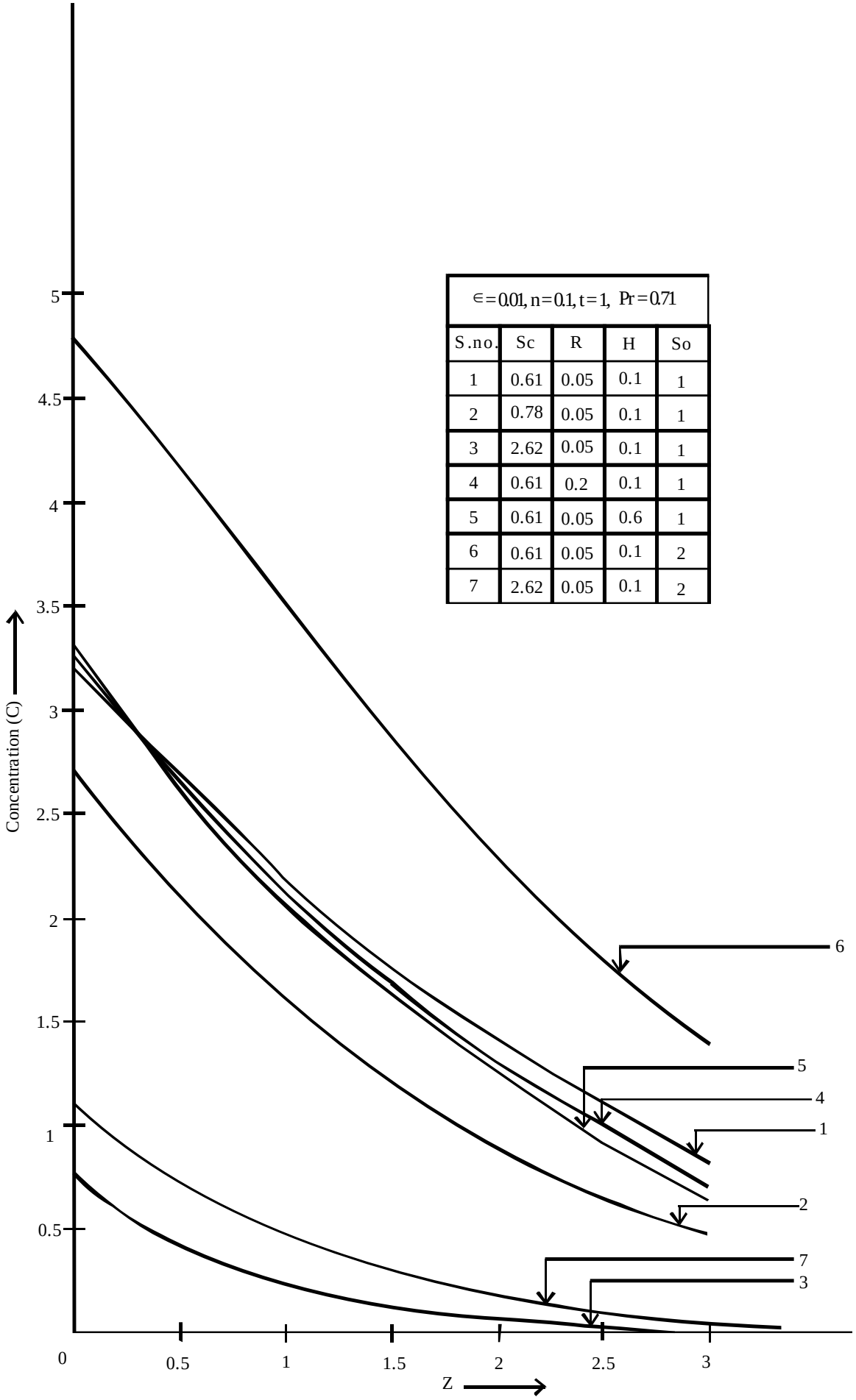



**Figure 7**. Concentration profiles plotted against z fordifferent values of Sc, R, H, and So

Skin friction due to primary velocity and secondary velocity $C_{fP}$ and $C_{f_s}$ are plotted in Figure 8 and Figure 9 against t. We fix Sc = 0.61 and So = 1. We observe that as we increase K, Gr, Gc and m, both $C_{fP}$ and $|C_{f_s}|$ rises. When we consider the case of free flow i.e., $K = \infty$ with h = 0 (no slip), we notice that $C_{fP}$ and $|C_{f_s}|$ rises as compared to the case of $K \neq \infty, h \neq 0$.

| ∈ = 0.01, n = 0.1, E = 0.4, Sc = 0.61, So = 1, Pr = 0.71 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| S.no. | K | M | Gr | Gc | m | h | R | H |
| 1 | 50 | 3 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 2 | 5 | 3 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 3 | 50 | 2 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 4 | 50 | 3 | 6 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 5 | 50 | 3 | 3 | 8 | 0.5 | 0.1 | 0.05 | 0.1 |
| 6 | 50 | 3 | 3 | 6 | 1 | 0.1 | 0.05 | 0.1 |
| 7 | 50 | 3 | 3 | 6 | 0.5 | 0.05 | 0.05 | 0.1 |
| 8 | 50 | 3 | 3 | 6 | 0.5 | 0.1 | 0.2 | 0.1 |
| 9 | 50 | 3 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.6 |
| 10 | ∞ | 3 | 3 | 6 | 0.5 | 0 | 0.05 | 0.1 |

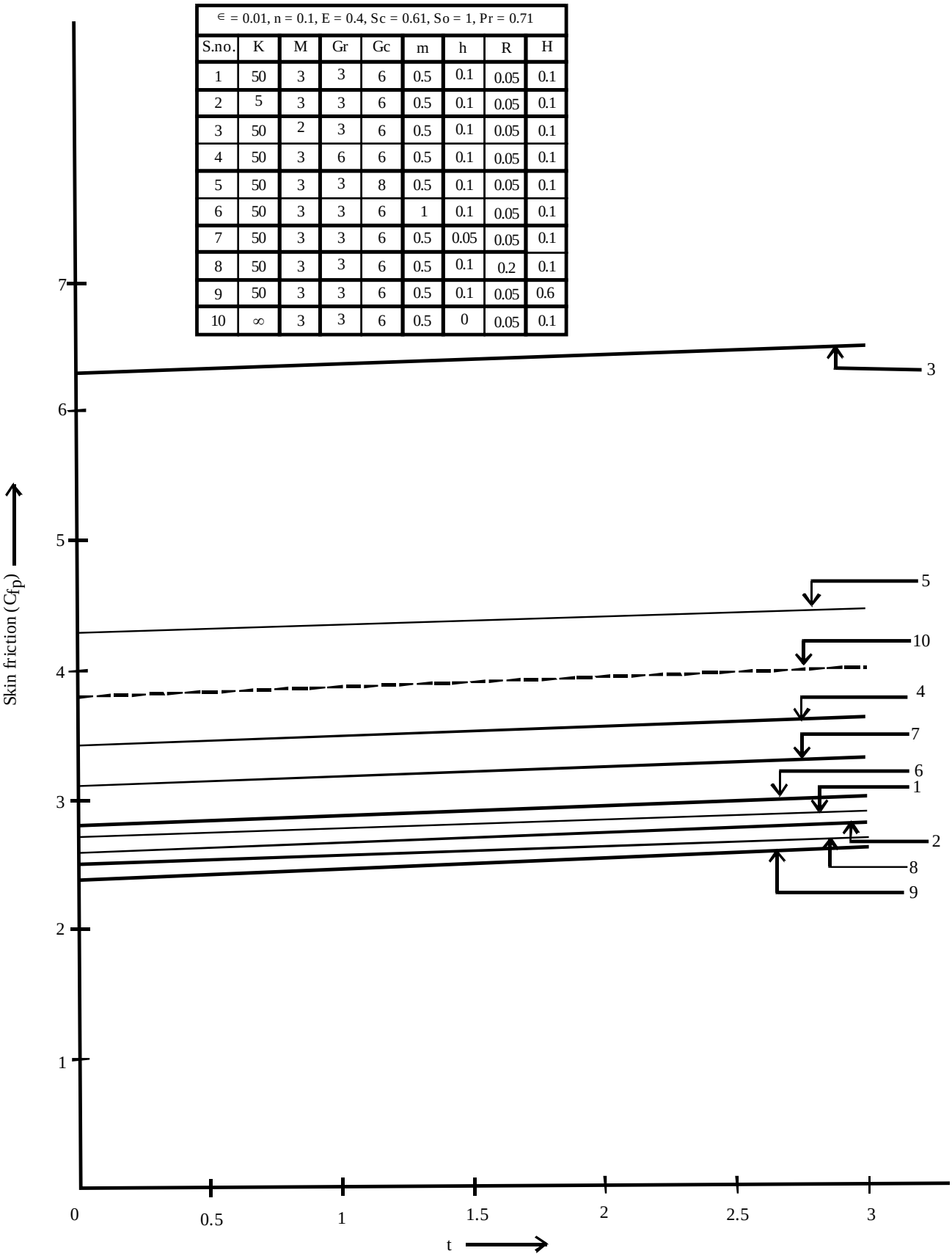
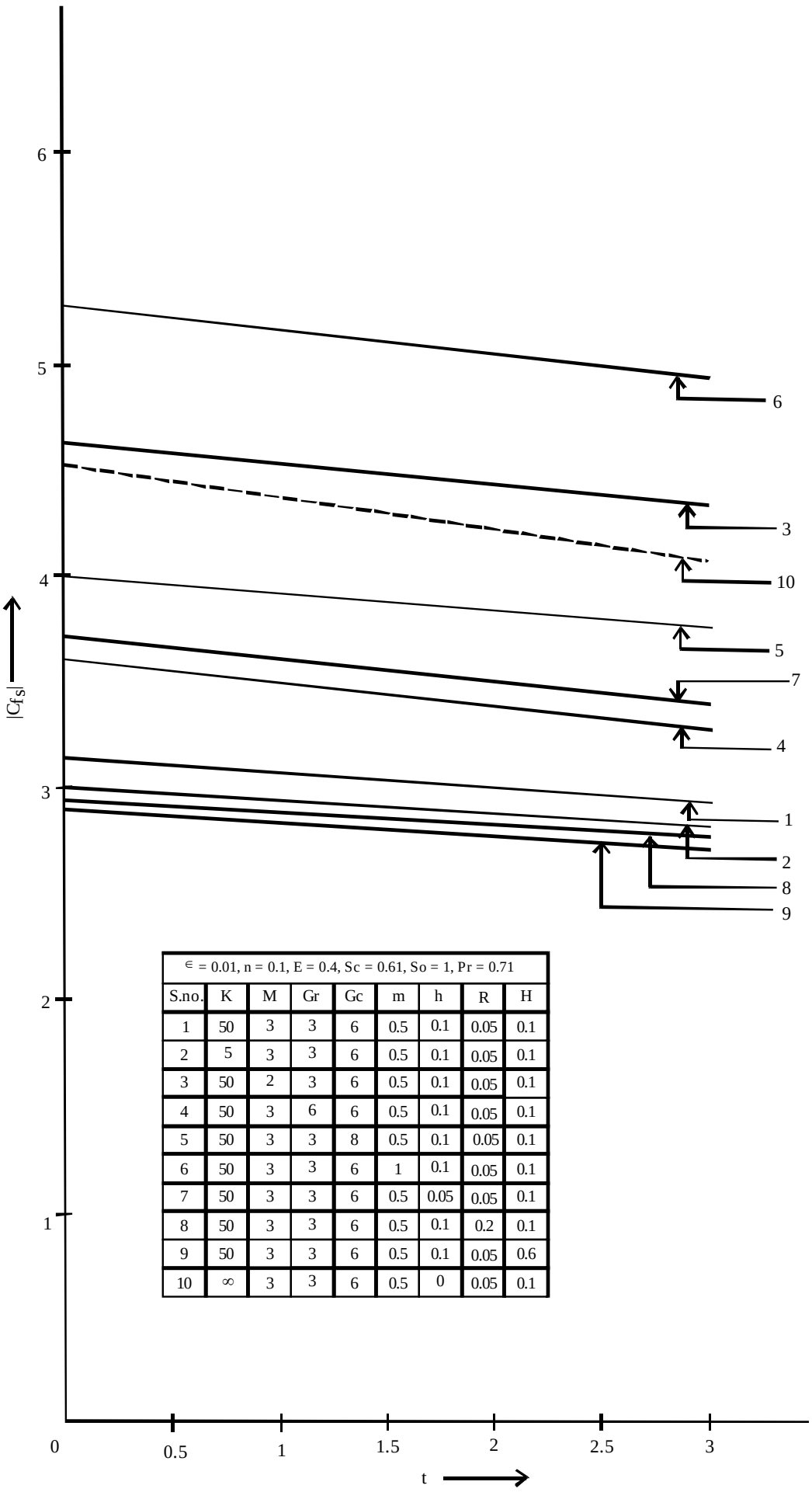



**Figure 8**. Skin friction due to primary velocity plotted against t for different values of K, M, Gr, Gc, m, h, R and H.




| ∈ = 0.01, n = 0.1, E = 0.4, Sc = 0.61, So = 1, Pr = 0.71 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| S.no. | K | M | Gr | Gc | m | h | R | H |
| 1 | 50 | 3 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 2 | 5 | 3 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 3 | 50 | 2 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 4 | 50 | 3 | 6 | 6 | 0.5 | 0.1 | 0.05 | 0.1 |
| 5 | 50 | 3 | 3 | 8 | 0.5 | 0.1 | 0.05 | 0.1 |
| 6 | 50 | 3 | 3 | 6 | 1 | 0.1 | 0.05 | 0.1 |
| 7 | 50 | 3 | 3 | 6 | 0.5 | 0.05 | 0.05 | 0.1 |
| 8 | 50 | 3 | 3 | 6 | 0.5 | 0.1 | 0.2 | 0.1 |
| 9 | 50 | 3 | 3 | 6 | 0.5 | 0.1 | 0.05 | 0.6 |
| 10 | ∞ | 3 | 3 | 6 | 0.5 | 0 | 0.05 | 0.1 |

**Figure 9**. Skin friction due to secondary velocity plotted against t for different values of K, M, Gr, Gc, m, h, R and H

Nusselt number is plotted against t in Figure 10. From the figure it is observed that when we increase the radiation parameter R and source parameter H, the rate of heat transfer rises. In Figure 11, effects of different parameters are plotted for Sherwood number against t. We notice that on increasing Sc, rate of mass transfer rises whereas, rate of mass transfer drops on increasing R, H and So.

## 8. Conclusions

1. On increasing the permeability parameter (K), primary velocity accelerates.

2. For the case of no slip and no Hall current i.e. h=0 and m=0, primary velocity decreases near the plate but then rises as we move away from the plate as compared to the case for h≠0 and m≠0.

3. Secondary velocity decreases on increasing the Hall parameter (m) and permeability parameter (K).

4. We observe that temperature drops for high radiation parameter (R).

5. Concentration of the fluid drops on increasing the Schmidt number where as it rises for high Soret number.

6. In the case of free flow with no slip at the boundary we observe a rise in the values of $C_{fP}$ and $\left|C_{f_s}\right|$.

7. Nusselt number rises when we increase the radiation parameter (R).

8. Sherwood number defined as the rate of mass transfer rises on increasing the Schmidt number.

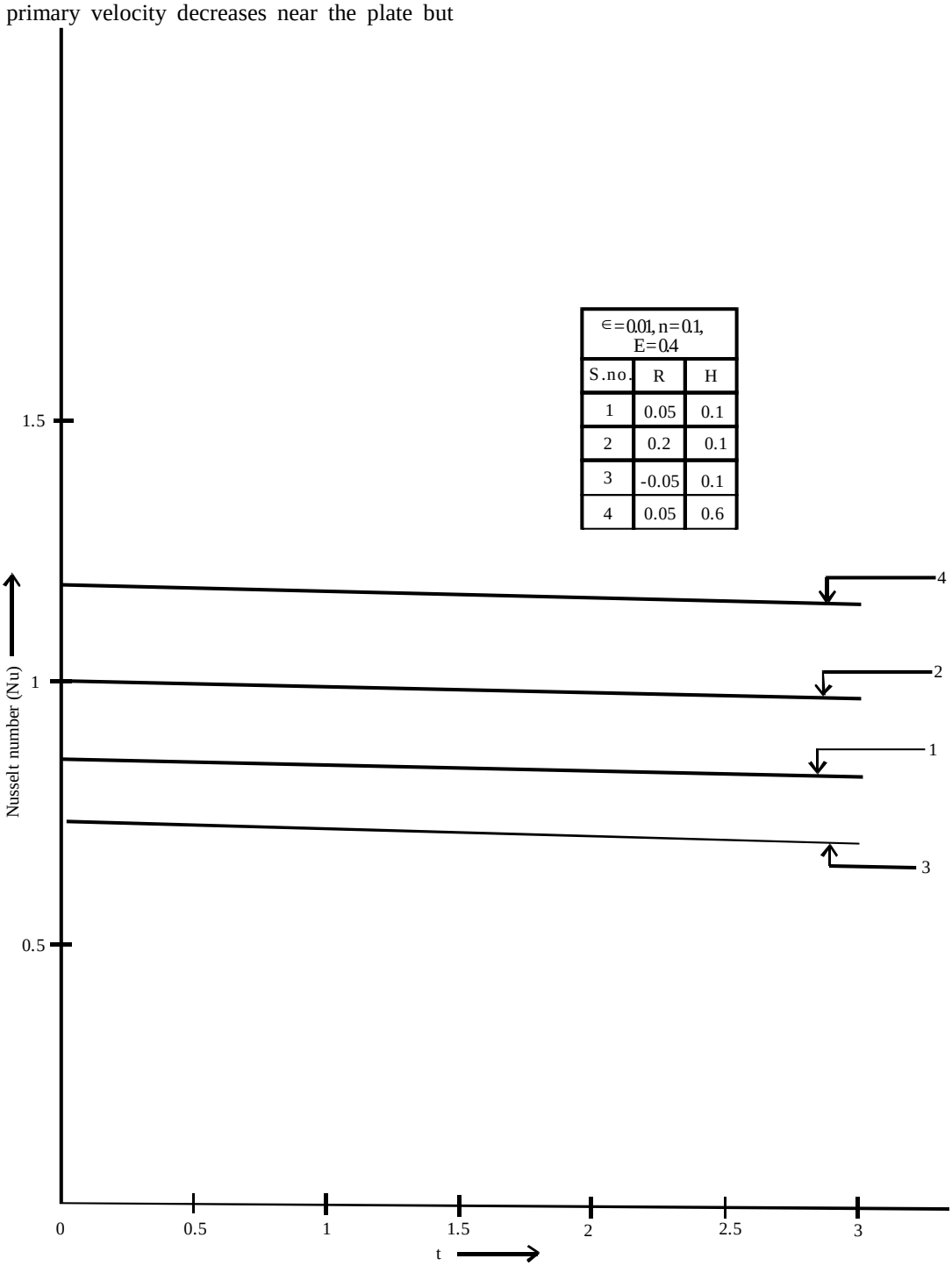
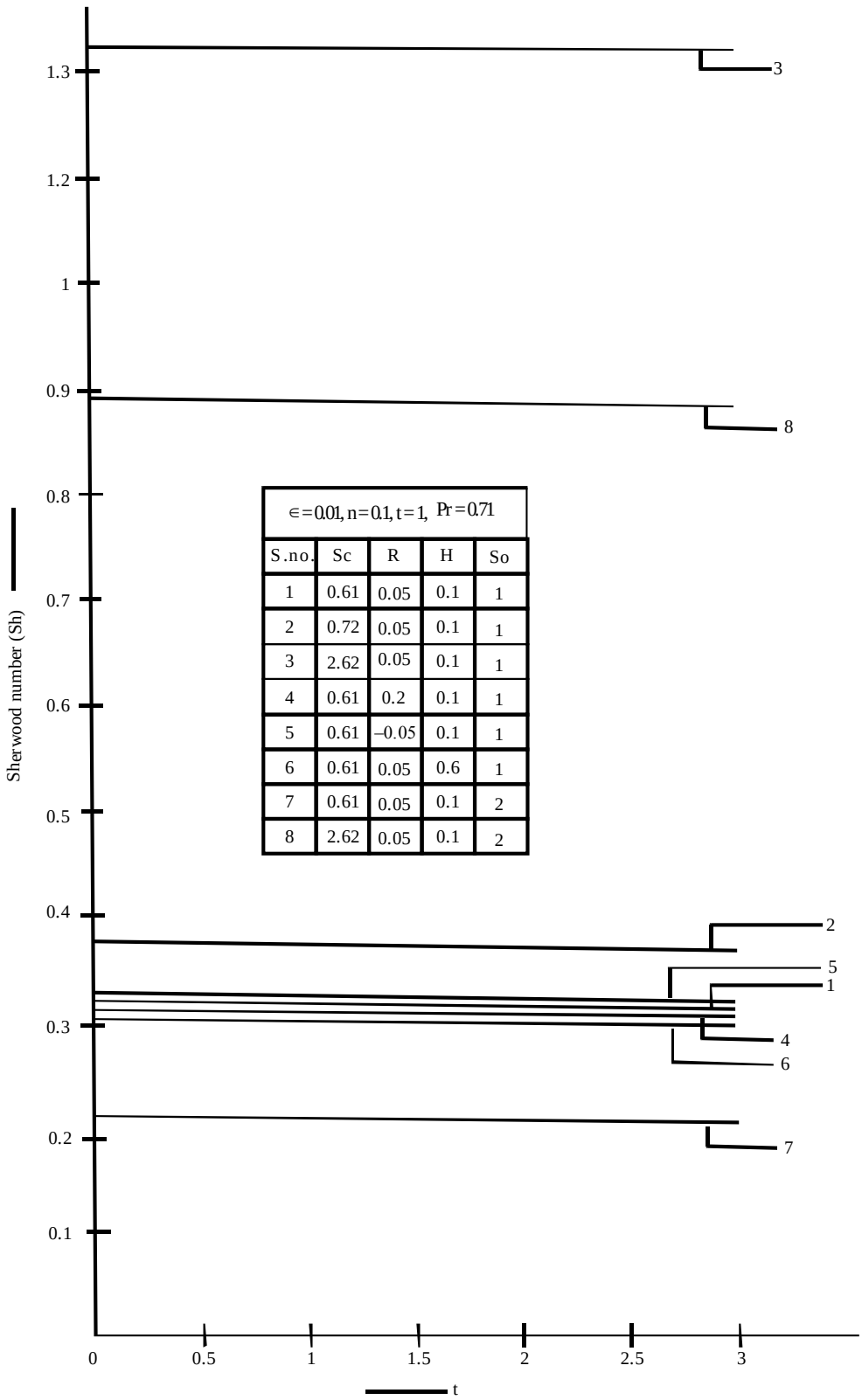



**Figure 10**. Nusselt number plotted against t for different values of R and H




| $\in=0.01, n=0.1, t=1, Pr=0.71$ | | | | |
|---|---|---|---|---|
| S.no. | Sc | R | H | So |
| 1 | 0.61 | 0.05 | 0.1 | 1 |
| 2 | 0.72 | 0.05 | 0.1 | 1 |
| 3 | 2.62 | 0.05 | 0.1 | 1 |
| 4 | 0.61 | 0.2 | 0.1 | 1 |
| 5 | 0.61 | –0.05 | 0.1 | 1 |
| 6 | 0.61 | 0.05 | 0.6 | 1 |
| 7 | 0.61 | 0.05 | 0.1 | 2 |
| 8 | 2.62 | 0.05 | 0.1 | 2 |

**Figure 11**. Sherwood number plotted against t for differentvalues of Sc, R, H and So.

# References

[1] Sivaiah, M., Nagarajan, A.S. and Reddy, P.S., Heat and mass transfer effects on MHD free convective flow past a vertical porous plate. The ICFAI University *J. of computational Mathematics* II (2), 14-21, 2009.

[2] Das, K. and Jana, S., Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in a porous medium. *Bull. Sco. Math.* 17, 15-32, 2010.

[3] Chaudhary, R.C. and Jain, A., combined heat and mass transfer effects on MHD free convection flow past oscillating plate embedded in aporous medium. *Rom. J. Phys.* 52 (5-7), 505-524, 2007.

[4] Aboeldahabn, E. M and Ebarbary, Elsayed, M. E., Hall current effect on Magnetohydrodynamic free convection flow past a semi infinite vertical plate with mass transfer. *Int. J. of Eng. Sci.* 39 (14), 1641-1652, 2001.

[5] Shit, G.C., Halll effects on MHD free convection flow and mass transfer over a stretching sheet. *Int. J. of Appl. Math. And Mech.* 5 (8), 22-38, 2009.

[6] Abuga, J.G., Kinyanjui, M. and Sigey, J.K., An investigation of the effect of Hall currents and rotational parameter on dissipative fluid flow past a vertical semi-infinite plate. *J. of Engi. And Tech. Research*, 3 (11), 314-320, 2011.

[7] Shateyi, S., Motsa, S.S. and Sibands, P., The effects of thermal radiation, Hall currents, secret and Dufour on MHD flow by mixed convection over a vertical surface in porous media. Math. Prob. Engi. 20.

[8] Saha, L.K., Hossain, M.A. and Gorla, Rama Subba Reddy, Effect of Hall current on MHD laminar natural convectin flow from a vertical permeable flat plate with uniform surface temperature. *Int. J. of Thermal Sci.*, 46 (8), 790 – 801, 2007.

[9] Mbeledogu, I.U. and Ogulu, A., Heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. *Int. J. Heat and mass transfer*, 50, 1902-1908, 2007.

[10] Zhukovsky, K., Modeling of the current limitations of PEFC. AIChE Journal, 52(7), 2356-2366, 2006.

[11] Acharya, M., Dash, G.C. and Singh, L.P., Magnetic field effects on the free convection and mass transfer flow through a porous medium with constant suction and constant heat flux. *Ind. J. Pure, Appl. Math*. 31, 1-18, 2000.

[12] Rajesh, V. and Varma, S.V.K., Heat source effects on MHD flow past an exponentially accelerated vertical plate with variable temperature through a porous medium. *Int. J. of Appl. Math & Mech*., 6 (12), 68-78, 2010.

[13] Farhad, A., Norzeiha, M., Sharidan, S., Khan, I. and Samiulhaq, Hydromagnetic rotating flow in a porous medium with slip conditions and Hall current. *Int. J. of Physical Sci.*, 7 (10), 1540-1548, 2012.

[14] Sharma, B.K., Jha, A.K. and Chaudhary, R.C., Hall Effect on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed is a porous medium with heat source/sink. *Rom. J. Phys*. 52 (5-7), 487-503, 2007.

[15] Abbas, Z. and Asghar, S., Effect of Hall current and heat transfer on the flow in a porous medium with slip conditions. *J. of porous media*, 10 (1), 35-50, 2007.

[16] Jain, N.C. and Gupta; P., Hall effects on unsteady free convection slip flow in a rotating fluid along a porous vertical surface bounded by a porous medium. *Int. J. of Appl. Mech. And Eng*. 12 (3), 609-627, 2007.

[17] Jain, N.C., and Singh H., Hall and thermal radiation effects on an unsteady rotating free convection slip flow along a porous vertical moving plate. *Int. J. of Appl. Mech. And Eng.* 17 (1), 53-70, 2012.

[18] Shereliff, J.A., A text book of magnetohydrodynamic-$^{1st}$ ed. London -: Pergamon Press, 1965.

[19] England, W.G. and Emery, A.F., Thermal radiation Effects on the laminar free convection boundary layer of an absorbing on the laminar free convection boundary layer of an absorbing gas. *J. Heat Transf.* 91, 37-44, 1969.