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# The Mathematical Modelling of the Road Safety Equipment's Market Penetration

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**Abstract** The aim of this paper is to theoretically investigate the interaction between fatal road accidents and the road safety equipment's market penetration. The basis of analysis is the modernization of passenger cars and the analysis of market penetration. The hypothesis of the authors that need to be proven by tools of mathematical statistics is that the growing market penetration of safety equipment can lead to safer environment and less fatal accidents.

***Keywords:*** *passive and active road safety, mathematic modelling, statistical analysis, market penetration*

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## 1. Introduction

The object of this article is the mathematic modelling of the market penetration of the road safety equipment, especially front seat belts, rear seat belts, ABS and airbags. At first the relation between the passenger car's modernization and lethal accidents were examined, but only accidents were analysed, where the victim was in the car [1]. The analysis was created on the basis of the Hungarian vehicle fleet between 2002 and 2012. The vehicle fleet has been segregated on the basis of EURO categories. The categories EURO 0 to 5, were used to characterize the passenger cars. The mathematic model of the market penetration of the safety equipment was created after the statistical analysis, especially focusing on correlation analysis and analysis of variance. Finally, differentials were prepared to investigate the relations between active and passive safety equipment.

## 2. Methodology

The methodology is based on the data of the Hungarian Central Statistical Office. Statistical data has been analysed between 2002 and 2012 a in this paper. As the analysis required the manufacturer of passenger car involved in accident and some records were missing, therefore authors have investigated the data availability as well. This indicator can be created by the number of fatal accidents by each year (1).

$$\beta_t = 1 - \frac{\sum_{i=1}^{n} N_i - \sum_{i=1}^{n} (N_{h,i} + N_{n,i})}{\sum_{i=1}^{n} N_i} \qquad (1)$$

where:

$\beta_t$: uncertainty of the availability of year t.

$N_i$: the number of fatal accidents for EURO category.

$N_h$: not corrected, erroneously recorded year of production.

$N_n$: unknown year of production.

i: the current year of EURO category.

The second step was the statistical analysis, which was made of the passenger vehicle fleet and groups of EURO categories by production year. The fatal accident's percentage was valued from the relative frequency of EURO category (2).

$$N_{kt} = \sum_{i=1}^{n} \left( J_{i,t} \cdot N_{i,t} \right) \qquad (2)$$

where:

$N_k$: estimated number of fatal accidents per category

$J_i$: distribution of the specific vehicle for EURO category [%]

$N_i$: number of fatal accidents in a given year [-]

t: examined year

With the annual and category summary of the passenger cars, a new data set were given, which forms a the basis for the analysis of the market penetration of safety equipment. Three passive safety equipment were examined (front and rear seat belt, airbag) and one active safety equipment (ABS). The investigation of safety equipment was started with correlation analysis, to find the relation between them.

$R^2$ (coefficient of determination) (3):

$$R^2 = \frac{SSR}{SSTO}, 0 \le R^2 \le 1 \qquad (3)$$

where:

SSR *Squared Sum of Regression*,

SSTO *Squared Sum of Total errors,*
$R^2$ *coefficient of determination*

After the analysis of correlation, the analysis of variance was examined, to find the relation between fatal accidents and passenger car's safety equipment. The F test and the one-sample T test was made with the help of Windows Excel and SPSS programs. All passive safety equipment were weighted with 0.33.

The next step was the creation of the composite function. The number of road accident victims in the passenger car as the dependent variable (Y), willingness to wear a seat belt ($x_1$, $x_2$), presence of ABS ($x_3$) and the airbag ($x_4$) market penetration as explanatory variables were considered. The result was a five-dimensional dynamic space (the four variable plus time) and the annual breakdown of the statistic tools were examined further on (4).

$$\overline{Y} = \sum_{i=1}^{i=4} \left( \overline{w_i} \cdot \overline{R_i\left(x_i\right)} + \overline{\varepsilon} \right) \qquad (4)$$

where:

$w_i$: safety equipment weighting factor.
$R_i(x_i)$: *i*, safety device fitted to the spread regression R.
ε: error term.

Through the verification of a mathematical model a new composite function was defined, that has been made by idealized exponential regression curves of market penetrations. This idealized model created four two-dimensional space and the goodness of approximation was defined too. With the composition of four two-dimensional space functions, the result was given (5).

$$\overline{Y} = \sum_{i=1}^{i=4} \left( \overline{Y_i} + \overline{\varepsilon} \right) \qquad (5)$$

During the creation of mathematic modelling a limit test was made to realize the maximum utilization of the safety equipment, in addition to define the remaining number of fatal accidents when the market penetration is 100%. With the help of the limit test a mathematically correct solution was found, however, it's leaden with a statistically significant error (6).

$$F_i = Sbe * 5777,2 * e^{-2,792*be} + Sbj * 1643,6 * e^{-1,84*bh} + SABS * 1138,5 * e^{-2,48*ABS} + SL * 1208 * e^{-0,989*L} \qquad .(6)$$

where:

$F_i$: idealized composite functions.
$S_{be}$: weighted front seat belt.
be: front seat belt.
$S_{bh}$: weighted rear seat belt.
bh: rear seat belt.
$S_{ABS}$: weighted ABS.
ABS: ABS.
$S_L$: weighted airbag.
L: Airbag.

Finally, differences of the market penetration were created and the correlations between the first and second differences were examined [2].

## 3. Results

The availability of uncertainty indicators were determined (Figure 1) [3].

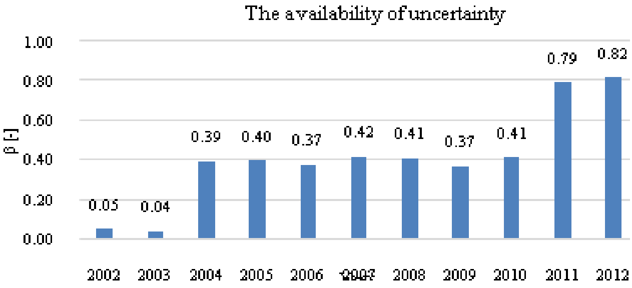


**Figure 1.** The availability of uncertainty (source: own research)

It could be said that the database was worse year after year and the uncertainty grew. The fatal accidents were allocated each year - the appropriate EURO category - considering the year of production. Knowing the vehicle fleet representation annually, the data of the percentage of fatal accidents for each categories were summarized. Statistically (from Accident database) and weighted (calculated from vehicle fleet) databases were made and were compared with each other. The resulted figure is time independent, and the EURO categories can be seen. Those bubbles that are under the identification line (blue dot line) has more fatal accidents statistical as they represented in the fleet. Those bubbles that are above the identical line has less accident statistically as they appear on the road. This phenomena strongly connected to the mileage of vehicle fleet. The size of circles are related to the uncertainty of the data(Figure 2) [4].

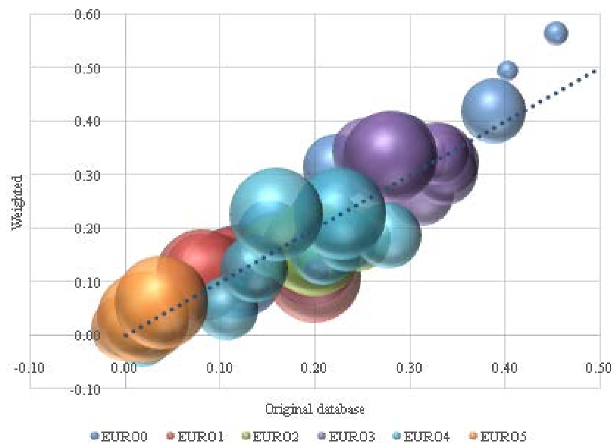


**Figure 2.** Representation of data on a graph (source: own research)

During the analysis of variance, the time period between 2006 and 2011 as been examined, because in this period the market penetration of security devices shows continuous growth. It was assumed that this trend will continue in the future, namely that newer vehicles will have more safety equipment on board. The following figure shows the table of coefficient of determination ($R^2$), which was generated by using MS Excel. Mostly the factors are independent from each other. (Table 1) [5].

On the next step, the weighted passive safety equipment and the one active safety equipment were used (Table 2).

| Analysis of variance | | | | | |
|---|---|---|---|---|---|
| | *df* | *SS* | *MS* | *F* | *significance of F* |
| Regression | 2 | 0,237578867 | 0,118789434 | 18,27470206 | 0,020891613 |
| Residual error | 3 | 0,019500635 | 0,006500212 | | |
| Total | 5 | 0,257079502 | | | |

**Figure 3.** Analysis of variance, F test (source: own research)

The calculation and estimation of variance were based on the mathematical fact, that the entire MS (mean square), is equal to ratio of the total SS (deviation sum of squares); and the df (degrees of freedom) *(Figure 3).*

In the analysis of variance the T test, the coefficients, the standard error, the t value, the p value and the lower and top 95% confidence intervals (Table 3) can be seen.

**Table 1. Coefficient of determination (source: own research)**

| $R^2$ | Year | Fatality [person] | Safety belt wearing (front) [0,1] | Safety belt wearing (rear) [0,1] | ABS penetration [0,1] | Airbag penetration [0,1] |
|---|---|---|---|---|---|---|
| Year | 1 | 0 | 0 | 0 | 0 | 0 |
| Fatality [person] | 0.850932557 | 1 | 0 | 0 | 0 | 0 |
| Safety belt wearing (front) [0,1] | 0.955315615 | 0.750590546 | 1 | 0 | 0 | 0 |
| Safety belt wearing (rear) [0,1] | 0.964395649 | 0.749001442 | 0.95808881 | 1 | 0 | 0 |
| ABS penetration [0,1] | 0.831923282 | 0.78220641 | 0.840148189 | 0.815673816 | 1 | 0 |
| Airbag penetration [0,1] | 0.701996902 | 0.418119889 | 0.776567059 | 0.843181873 | 0.669596802 | 1 |

**Table 2. Regression statistics from 2006 to 2011 (source: own research)**

| Regression statistics | |
|---|---|
| R | 0,961324871 |
| R square | 0,924145507 |

**Table 3. Analysis of variance, T test (source: own research)**

| | Coefficients | Standard error | t value | p-value | Lower 95% | Top 95% |
|---|---|---|---|---|---|---|
| Deadly victims [%] | 2,523694046 | 0,899222928 | 2,80652769 | 0,067486606 | -0,338034637 | 5,385422729 |
| Passive safety equipment's market penetration [%] | -2,753487291 | 2,128127065 | -1,29385474 | 0,286322177 | -9,526137405 | 4,019163823 |
| Active safety equipment's market penetration [%] | -1,185468725 | 1,65721252 | -0,715338983 | 0,526046543 | -6,45948617 | 4,088521167 |

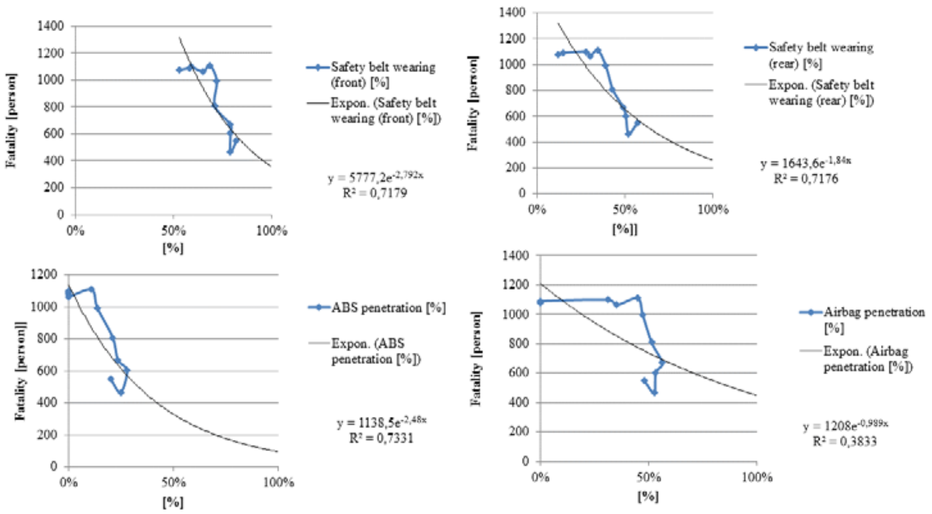


**Figure 4.** Functions of safety equipment market penetration (source: own research)

On the figures below the market penetration of front and rear seat belt, the penetration of ABS and the penetration of airbag in the function of fatalities can be seen. Exponential curves were superposed on the statistical data (Figure 4).

It can be seen, that the $R^2$ value shows a moderately good correlation. The weights of market penetrations of vehicle related safety equipments were determined for each year. The error term is not zero (Table 4) [6].

**Table 4. Dynamic testing of the weight of the security equipment (source: own research)**

| Year | Fatality [person] | Weight for Safety belt wearing | Weight for Safety belt wearing | Weight for ABS penetration | Weight for airbag penetration | Eps [person] | Theoretician |
|---|---|---|---|---|---|---|---|
| 2002 | 1075 | 60% | 40% | 0% | 0% | -245,37 | 1075 |
| 2003 | 1090 | 60% | 40% | 0% | 0% | -99,11 | 1090 |
| 2004 | 1099 | 54% | 36% | 0% | 10% | 52,41 | 1099 |
| 2005 | 1063 | 48% | 32% | 0% | 20% | 134,93 | 1063 |
| 2006 | 1109 | 44% | 34% | 10% | 12% | 254,02 | 1109 |
| 2007 | 993 | 42% | 32% | 9% | 17% | 206,87 | 993 |
| 2008 | 808 | 40% | 29% | 9% | 22% | 49,98 | 808 |
| 2009 | 669 | 38% | 27% | 9% | 26% | 7,07 | 669 |
| 2010 | 603 | 36% | 25% | 8% | 31% | -57,75 | 603 |
| 2011 | 465 | 33% | 24% | 8% | 35% | -199,60 | 465 |
| 2012 | 548 | 31% | 23% | 8% | 38% | -109,12 | 548 |

**Table 5. The free database after calibration (source: own research)**

| Year | Fatality [person] | Weight for Safety belt wearing | Weight for Safety belt wearing | Weight for ABS penetration | Weight for airbag penetration | Theoretician |
|---|---|---|---|---|---|---|
| 2002 | 1075 | 50% | 50% | 0% | 0% | 1075 |
| 2003 | 1090 | 51% | 49% | 0% | 0% | 1090 |
| 2004 | 1099 | 34% | 31% | 0% | 34% | 1099 |
| 2005 | 1063 | 33% | 33% | 0% | 33% | 1063 |
| 2006 | 1109 | 26% | 24% | 24% | 26% | 1109 |
| 2007 | 993 | 27% | 24% | 23% | 26% | 993 |
| 2008 | 808 | 26% | 24% | 24% | 26% | 808 |
| 2009 | 669 | 25% | 25% | 25% | 25% | 669 |
| 2010 | 603 | 23% | 27% | 27% | 23% | 603 |
| 2011 | 465 | 26% | 24% | 24% | 26% | 465 |
| 2012 | 548 | 25% | 25% | 25% | 25% | 548 |

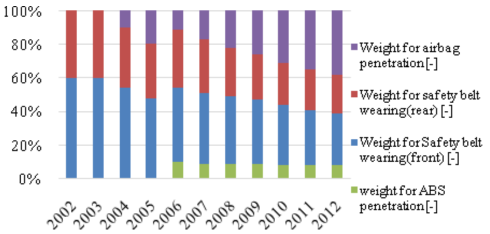


**Figure 5.** Mathematical solution (source: own research)

To the verification of the mathematical model an optimum was created, with the maximum dominance principle, to find the ideal weight of the penetration. It was assumed, that each passive equipment is responsible to the fatalities in the full year, so in this case only the weight of penetration was important. Thanks to the optimization, the idealized composite function was mathematically correct, because of a linear weight factor. The model was error free and the error value ε was always zero (Table 5).

The following diagram (Figure 5) represents mathematical solution in each year. The system is an equilibrium system, passive safety devices spread or entry of new items - according to international literature-are responding to a reduction of the weight factors. So in the real situation according to a mathematical model, the weight of the different equipment is not equal. During a fatal accident they don't have the same percent age, the weights are not changing either. In case of ideal situation we gave the same proportion of weights [7].

Based on the data of the analysis of variance the examination of the data has been set for the period 2006-2011, as can be seen in the monotonous increase penetration and monotonic decrease in the number of deaths. Year 2006 was chosen as the base year (Table 6).

**Table 6. Active and passive safety equipment's market penetration from 2006 to 2011**

| Year | Fatality [person] | Fatality [%] | Safety belt wearing (front) | Safety belt wearing (rear) | ABS penetration [%] | Airbag penetration [%] |
|---|---|---|---|---|---|---|
| 2006 | 1109 | 100% | 68,5% | 34,5% | 10,8% | 44,8% |
| 2007 | 993 | 89,5% | 72,0% | 39,0% | 13,6% | 47,3% |
| 2008 | 808 | 72,9% | 71,0% | 43,0% | 21,0% | 51,4% |
| 2009 | 669 | 60,3% | 79,0% | 49,0% | 23,4% | 56,3% |
| 2010 | 603 | 54,4% | 79,0% | 50,5% | 27,8% | 53,3% |
| 2011 | 465 | 41,9% | 79,0% | 52,0% | 24,7% | 52,6% |

It can be seen in the data set, that these are discrete points, in line with it a discrete analysis was created. With the given data the first and the second difference were made as the analogue to road, speed and acceleration. With the help of the first and second differentials of market penetration and number of fatal accidents pairs were created to compare them. Main idea behind that was if not the penetration itself drives the decrease number of fatal accidents but the speed or acceleration of penetration. For easier understanding the next notations were used: fatalities Y, the front and rear seat belt $x_1$, $x_2$, ABS $x_3$ and the airbag $x_4$ The following figure conclude the first-order and second-order differential values. (Table 7) [8].

**Table 7. General and differential values (source: own research)**

| Y | X1 | X2 | X3 | X4 | Y' | X1' | X2' | X3' | X4' | Y'' | X1'' | X2'' | X3'' | X4'' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | 0.69 | 0.35 | 0.11 | 0.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.90 | 0.72 | 0.39 | 0.14 | 0.47 | -0.10 | 0.03 | 0.05 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.73 | 0.71 | 0.43 | 0.21 | 0.51 | -0.17 | -0.01 | 0.04 | 0.07 | 0.07 | -0.06 | -0.04 | -0.01 | 0.04 | 0.02 |
| 0.60 | 0.79 | 0.49 | 0.23 | 0.56 | -0.13 | 0.05 | 0.06 | 0.02 | 0.02 | 0.04 | 0.09 | 0.02 | -0.05 | 0.01 |
| 0.54 | 0.79 | 0.51 | 0.28 | 0.53 | -0.06 | 0.00 | 0.02 | 0.04 | 0.04 | 0.07 | -0.08 | -0.05 | 0.02 | -0.08 |
| 0.42 | 0.79 | 0.52 | 0.25 | 0.53 | -0.12 | 0.00 | 0.02 | -0.03 | -0.01 | -0.06 | 0.00 | 0.00 | -0.07 | 0.02 |

Totally 36 pairs were made for the analysis, but only the general data had shown correlation; the following four pairs were analysed (Figure 6), with the next conditions:

>0.7 strong correlation.

0.7-0.3 medium correlation, we need more analysis.

<0.3 low correlation, no significant interrelation.

Fatality - Front seat belt, Y = -4.1679x + 3.8139.

Fatality –Rear seat belt, Y = -3.1018x + 2.0838.

Fatality - ABS, Y = -3.1125x + 1.3278.

Fatality - Airbag, Y = -4.4374x + 2.9585.

For example in the case of fatality and front seat belt: If the market penetration or usage of front seat belts increase by 5% then the number of fatal accidents decrease by 1% (Table 8).

**Table 8. Reduction rate of fatalities (source: own research)**

| Safety belt wearing (front) [%] | 0,7 | 0,75 | 0,8 | 0,85 | 0,9 |
|---|---|---|---|---|---|
| Fatality [%] | 0,89617 | 0,687775 | 0,47938 | 0,27098 | 0,06259 |
| Reduction rate of fatalities [%] | 0,188667 | 0,2325396 | 0,302999 | 0,43472 | 0,76903 |

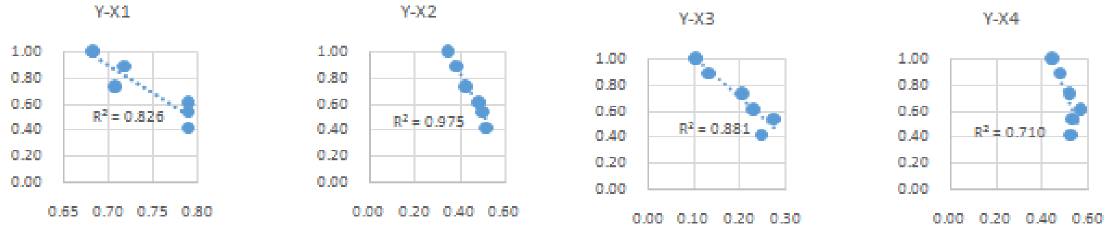


**Figure 6.** Fatality-Safety equipment penetration **(source: own research)**

**Table 9. General and differential values (source: own research)**

| $R^2$ | Y | X1 | X2 | X3 | X4 | Y' | X1' | X2' | X3' | X4' | Y'' | X1'' | X2'' | X3'' | X4'' |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y | X | 0,83 | 0,98 | 0,98 | 0,71 | X | 0,00 | 0,01 | 0,03 | 0,06 | X | 0,00 | 0,04 | 0,25 | 0,02 |
| X1 | 0,83 | | | | | 0,08 | | | | | 0,11 | | | | |
| X2 | 0,98 | | | | | 0,21 | | | | | 0,01 | | | | |
| X3 | 0,98 | | | | | 0,19 | | | | | 0,03 | | | | |
| X4 | 0,71 | | | | | 0,33 | | | | | 0,05 | | | | |
| Y' | X | 0,08 | 0,21 | 0,19 | 0,33 | X | 0,03 | 0,50 | 0,12 | 0,36 | X | 0,02 | 0,09 | 0,01 | 0,20 |
| X1' | 0,00 | | | | | 0,03 | | | | | 0,22 | | | | |
| X2' | 0,01 | | | | | 0,50 | | | | | 0,01 | | | | |
| X3' | 0,03 | | | | | 0,12 | | | | | 0,03 | | | | |
| X4' | 0,06 | | | | | 0,36 | | | | | 0,05 | | | | |
| Y'' | X | 0,11 | 0,01 | 0,03 | 0,05 | X | 0,22 | 0,01 | 0,03 | 0,05 | X | 0,01 | 0,11 | 0,01 | 0,56 |
| X1'' | 0,00 | | | | | 0,02 | | | | | 0,01 | | | | |
| X2'' | 0,04 | | | | | 0,09 | | | | | 0,11 | | | | |
| X3'' | 0,25 | | | | | 0,01 | | | | | 0,01 | | | | |
| X4'' | 0,02 | | | | | 0,20 | | | | | 0,56 | | | | |

A symmetrical matrix was made, where all of the 36 correlation value were shown (Table 9).

## 4. Conclusion

During the analysis of the passenger vehicle fleet we experienced, that in Hungary the representation of the car age is very bad, usually this information was not written on the statistical records or these were fixed wrongly in the official statistical database. This is a really big problem because the road fatality rate is still very high, not only in our country but also in most of the European Union countries. The historical dataset is also important not only for analysis past trends, but in order to prepare future forecasts. Unfortunately, there were not enough mileage data, but it is clear that the EURO 0 and EURO 1 category cars' mileage is below the average, the EURO 2, EURO 3 and EURO 4 cars' mileage is above the average, while the EURO 5 cars mileage is average.

After the testing of the safety devices, there sult showed that the active and passive safety features had very large impact on the evolution of the accidents. The introduced mathematical model proves, how large is the role of these equipments in the preservation of human life. All around the world the market penetration of road safety equipment is growing and the number of accidents are reducing. So after all the conclusion is that there are connections between road safety equipment and fatal accidents.

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