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Hurst Exponent as a Part of Wavelet Decomposition Coefficients to Measure Long-term Memory Time Series Based on Multiresolution Analysis

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Abstract Processing and analysis of data sequences using wavelet-decomposition and subsequent analysis of the all relevant coefficients of such decomposition is one of strong methods to study various processes and phenomena. The key point of data sequence analysis lies in the concept of Hurst exponent. This is due to the fact that Hurst exponent gives an indication of the complexity and dynamics of the correlation structure of any given time series taking into consideration the importance of Hurst exponent estimation for such analysis. There are various methods and approaches to find the Hurst exponent estimation with varying degrees of accuracy and complexity. Therefore, in this paper we have made an attempt to prove the possibility of considering an estimation of Hurst exponent based on the properties of coefficients of wavelet decomposition of a given time series. The obtained results which mainly based on the properties of detailing coefficients of wavelet decomposition show that estimation is easy to calculate and comparable with classic estimation of Hurst exponent. Also ratios has been obtained, that allow to analyze the self-similarity of a given time series.

Keywords: *time series, self-similar, wavelet decomposition, Hurst exponent, wavelet-coefficients, detailing coefficient*

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1. Introduction

The analysis and processing of a sequence of the data presented in the form of time series is one of the prevalent methodologies in studying various processes and phenomena which are concerned to different fields of activity and researches. This is due to the fact that data about a phenomenon or a process under study can be represented as a time series. Banking sector [1,2], securities market [3,4], health, when studying the dynamics of the analyzed processes [5] and in medicine, when the initial data are represented using time series or when the source data is converted from one form of representation into a time series are all good examples of such domains [6]. One of the important research methods of time series is a Wavelet analysis [7,8,9,10]; because it allows us to highlight the characteristics of time series, where the more important role played in this analysis is by wavelet decomposition, taking into consideration that the tool for decomposition on a set of different wavelet coefficients is the multiresolution analysis [11].

A multiresolution wavelet-analysis transforms time series to hierarchical structure by means of the wavelet transformations which results into a set of wavelet coefficients. On each new level of wavelet- decomposition there is a division of an approximating signal of the previous level (presented by some time series) on its high-frequency component and on more smoothed approximating signal [12]. Thus, the multiresolution analysis splits the time series into two components: (1) approximating coefficients and (2) detailing coefficients.

One of the characteristics of coefficients of wavelet decomposition is the Hurst exponent [13] which connects coefficients of wavelet decomposition at different levels of decomposition, besides that, the Hurst exponent also plays as an indicator of the complexity of dynamics and correlation structure of time series. The pervious discussion makes it possible to talk about the properties of the coefficients of wavelet decomposition and the role of Hurst exponent in determining these properties, while Particular importance in such analysis of time series belongs to detailing coefficients of wavelet decomposition.

2. Materials and Methods

According to discrete wavelet-transformation the time series $X(t)$, $(t = t_1, t_2, ...)$ consists of a set of coefficients – detailing and approximating [10,11,13,14]:

$$X(t) = \sum_{k=1}^{N_a} apr(N-1,k)\phi_{N-1,k}(t) + \sum_{j=0}^{N-1}\sum_{k=1}^{N_j} \det(j,k)\psi_{j,k}(t) \quad (1)$$

Where $apr(N-1,k)$ – Approximating wavelet-coefficients of level N

$\det(j,k)$ – Detailing wavelet-coefficients of level j

N – Chosen maximum level of decomposition

N_j – Quantity of detailing coefficients at j level of decomposition

N_a – Quantity of approximating coefficients at level N

$\psi(t)$ – Mother wavelet-function

$\phi(t)$ – Corresponding scaling-function.

$$d_X(j,k) = \langle X, \psi_{j,k} \rangle \quad (2)$$

Where $d_X(j,k)$ – Detailing wavelet-coefficients $k = \overline{1, N_j}$ at level j, $\langle X, \psi_{j,k} \rangle$ – Scalar product of investigated sequence of data in the form of time series $X(t)$ and a mother wavelet ψ on corresponding level of decomposition j.

In this case, the main tool to analyze time series is by processing the detailing coefficients which have been obtained on different levels. As a result the obtained series of the detailing coefficients will have the following properties [10,14,15]:

1. If time series $X(t)$ is a self-similar process, then detailing coefficients $d_X(j,k)$, $k = \overline{1, N_j}$ at each level of decomposition j are all self-similar, which is mean that equal distribution of small series of wavelet coefficients at each level of decomposition with some scale will take the form:

$$(d_X(j,1), d_X(j,2), ..., d_X(j,N_j)) \cong 2^{j(H+\frac{1}{2})}(d_X(0,1), d_X(0,2), ..., d_X(0,N_j)) \quad (3)$$

Where $j = \overline{1, N-1}$.

2. Wavelet coefficients resulting from the decomposition process with fixed increments will be fixed for each level 2^j ($j = \overline{1, N-1}$).
3. If there are moments of order P then the coefficients of small waves that were obtained as a result of the decomposition process must satisfy the following equation:

$$M\left[|d_X(j,k)|^p\right] = M\left[|d_X(0,k)|^p\right] 2^{jp(H+\frac{1}{2})} \quad (4)$$

$$M\left[(d_X(j,k))^2\right] = M\left[(d_X(0,k))^2\right] 2^{j(2H+1)} \quad (4a)$$

Where $M[...]$ $j = \overline{1, N-1}$ – estimation value of the process.

4. If time series $X(t)$ is self-similar, then the correlation function of wavelet-coefficients of j level will decrease according to:

$$M\left[d_X(j,k)d_X(j,k+n)\right] \cong n^{2(H-n_\psi)}, n \to \infty \quad (5)$$

Where n_ψ – number of zero moments of a mother wavelet ψ.

5. For all different levels of decomposition $j_1 \neq j_2$ and for all n correlations of detailing coefficients of these levels, $d_X(j_1,k)$ and $d_X(j_2,k+n)$ must equal to 0.
6. Detailing coefficients of DWT at each level of decomposition j have normal distribution with a zero average $N(0,\sigma)$.

It is clear that the detailing Hurst exponent (H) has been used when considering the properties of wavelet decomposition coefficients, which represents a measure of self-similarity. Hurst's exponent lies within the range $0 < H < 1$ and represents a key measure for the analysis of long-term dependence duration. In the case when Hurst's exponent lies in range $0.5 < H < 1$, this mean that the time series is persistent and has a trend-stable behavior, but when Hurst's exponent lies in range $0 < H < 0.5$, this refers to anti-persistent process (growth in the past means reduction in the future, and the tendency to reduction in the future makes probable increase in the future). At $H = 0.5$ the deviations of the process are really casual and don't depend on the previous values. Therefore, the estimation of Hurst exponent values is an important task. There are various methods to estimate Hurst exponent [16,17,18,19,20], but all of them provide only approximate values, while some of them have a high computational complexity. In this work we propose a new method to estimate the Hurst exponent to expand the existing approaches to analyze the time series, using individual properties of detailing coefficients of wavelet decomposition

3. Data for Analysis

To test the obtained results, we will use the self-similar time series, which are presented in Figure 1, Figure 2 and Figure 3.

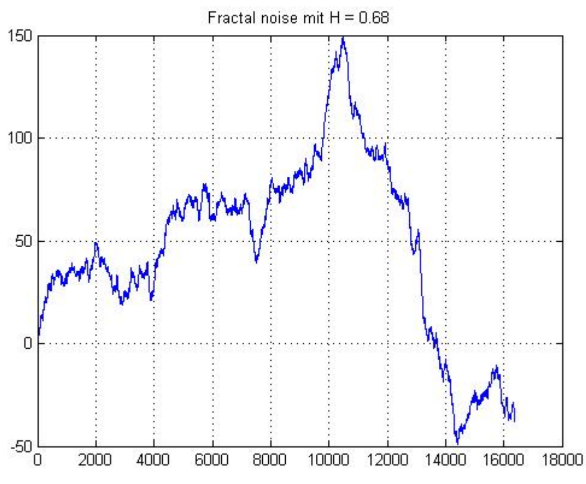

Figure 1. A modeling time series - Brownian motion ($H = 0.68$)

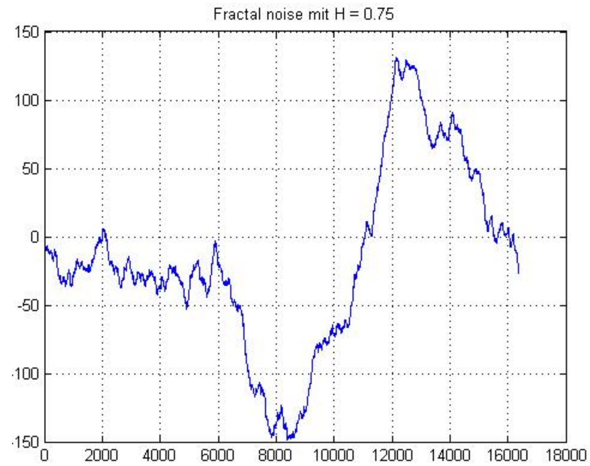

Figure 2. A modeling time series - Brownian motion ($H = 0.75$)

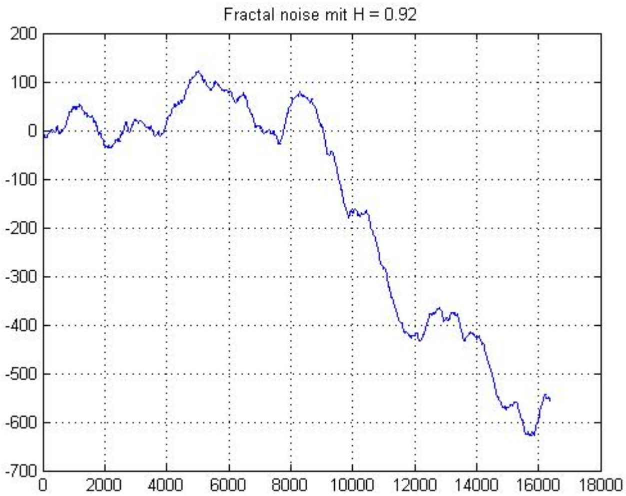

Figure 3. A modeling time series - Brownian motion ($H = 0.92$)

4. Results and Discussion

Let us consider the property in point 3 which is presented in equation (4). This property takes into consideration the value of the Hurst exponent and can be represented as follows:

$$2^{jp(H+\frac{1}{2})} = \frac{M\left[\left|d_x(j,k)\right|^p\right]}{M\left[\left|d_x(0,k)\right|^p\right]} \qquad (6)$$

By admitting logarithm into equation (6):

$$\log_2(2^{jp(H+\frac{1}{2})}) = \log_2(\frac{M\left[\left|d_x(j,k)\right|^p\right]}{M\left[\left|d_x(0,k)\right|^p\right]}) \qquad (7)$$

Which can be transformed into:

$$jp(H+\frac{1}{2}) = \log_2(\frac{M\left[\left|d_x(j,k)\right|^p\right]}{M\left[\left|d_x(0,k)\right|^p\right]}) \qquad (8)$$

Or

$$H = \frac{\log_2(M\left[\left|d_x(j,k)\right|^p\right]) - \log_2(M\left[\left|d_x(0,k)\right|^p\right])}{jp} - \frac{1}{2}. \quad (9)$$

So equation (9) can be used to calculate the values of the Hurst exponent. But we have to make one comment: When we consider the property in point 3 which is presented in equation (4) for the detailing coefficients of the wavelet decomposition, we, in fact, operate with the module provided by the coefficient values $(d_X(j,k)$ to calculate the mathematical expectation ($M[...]$). At the same time in the traditional case it is the mathematical expectation of the aggregate coefficients $(d_X(j,k)$, which are presented without a module (see equation. 4a). The assumption that the mathematical expectation $M[...]$ can be negative makes equation (9) meaningless. So, it should be borne in mind that:

$$M[|...|] \geq M[...]. \qquad (10)$$

More clearly

$$M\left[\left|d_x(j,k)\right|^p\right] \geq M\left[(d_x(j,k))^p\right] \qquad (11)$$

Thus, when calculating the Hurst exponent in accordance with equation (9) an error may occur. Table 1 shows the calculation of the Hurst exponent for time series in accordance with the classical approaches using equation (9), where all calculations were performed in MATLAB. Hereinafter, for the wavelet decomposition of a specified time series, we use wavelet db1, knowing that experiments have shown that the use of other wavelets gives similar results.

Table 1. Values of the Hurst exponent for a specified time series

Hurst Parameter Estimation	Time series		
	Figure 1	Figure 2	Figure 3
Discrete second derivative estimator (DSOD)	0.6672	0.7622	0.9206
Wavelet version of DSOD	0.6620	0.7538	0.9233
Wavelet details regression estimator	0.6531	0.7332	0.8900
Equation (9), $j = 7$:			
$p = 1$	0.6368	0.7585	0.8713
$p = 2$	0.6490	0.7516	0.8714
$p = 3$	0.6598	0.7445	0.8682
$p = 4$	0.6688	0.7381	0.8636

The data in Table 1 shows that the value of Hurst exponent calculated by equation (9) are comparable with the values of the Hurst exponent which are designed in accordance with conventional approaches. Therefore, equation (9) can be used to estimate the value of the Hurst exponent. Consider the two levels of decomposition (j_1 *and* j_2) for the same series $X(t)$. Then, using equation (9) we get:

$$H_1 = \frac{\left\{ \begin{array}{l} \log_2(M\left[|d_x(j_1,k)|^p\right]) \\ -\log_2(M\left[|d_x(0,k)|^p\right]) \end{array} \right\}}{j_1 p} - \frac{1}{2} \quad (12)$$

Or

$$H_2 = \frac{\left\{ \begin{array}{l} \log_2(M\left[|d_x(j_2,k)|^p\right]) \\ -\log_2(M\left[|d_x(0,k)|^p\right]) \end{array} \right\}}{j_2 p} - \frac{1}{2} \quad (13)$$

But $H_1 \cong H_2$ (on the basis of self-similarity of the time series $X(t)$ [14]), therefore:

$$\begin{array}{l} \frac{\log_2(M\left[|d_x(j_1,k)|^p\right]) - \log_2(M\left[|d_x(0,k)|^p\right])}{j_1 p} - \frac{1}{2} \\ \cong \frac{\log_2(M\left[|d_x(j_2,k)|^p\right]) - \log_2(M\left[|d_x(0,k)|^p\right])}{j_2 p} - \frac{1}{2} \end{array} \quad (14)$$

$$\begin{array}{l} \frac{\log_2(M\left[|d_x(j_1,k)|^p\right]) - \log_2(M\left[|d_x(0,k)|^p\right])}{j_1 p} \\ \cong \frac{\log_2(M\left[|d_x(j_2,k)|^p\right]) - \log_2(M\left[|d_x(0,k)|^p\right])}{j_2 p} \end{array} \quad (15)$$

Then,

$$\begin{array}{l} pj_2(\log_2(M\left[|d_x(j_1,k)|^p\right]) - \log_2(M\left[|d_x(0,k)|^p\right])) \\ \cong pj_1(\log_2(M\left[|d_x(j_2,k)|^p\right]) - \log_2(M\left[|d_x(0,k)|^p\right])) \end{array} \quad (16)$$

$$\begin{array}{l} \log_2\left(\frac{M\left[|d_x(j_1,k)|^p\right]}{M\left[|d_x(0,k)|^p\right]}\right)^{pj_2} \\ \cong \log_2\left(\frac{M\left[|d_x(j_2,k)|^p\right]}{M\left[|d_x(0,k)|^p\right]}\right)^{pj_1} \end{array} \quad (17)$$

Or

$$\left(\frac{M\left[|d_x(j_1,k)|^p\right]}{M\left[|d_x(0,k)|^p\right]}\right)^{pj_2} \cong \left(\frac{M\left[|d_x(j_2,k)|^p\right]}{M\left[|d_x(0,k)|^p\right]}\right)^{pj_1} \quad (18)$$

The equations (16-18) can be used as an indicators of the self-similar time series. Table 2 shows the calculations for different ratios according to equation (17) for a specific time series.

Table 2 shows that the values obtained in accordance with equation (17) are comparable within the specified terms of comparison. However, it should be noted that the closer levels of decomposition of time series the more comparable calculated values in accordance with equation (17). By rewriting equation (16) we get:

$$\begin{array}{l} \left\{ \begin{array}{l} pj_2 \log_2(M\left[|d_x(j_1,k)|^p\right]) \\ -pj_2 \log_2(M\left[|d_x(0,k)|^p\right]) \end{array} \right\} \\ \cong \left\{ \begin{array}{l} pj_1 \log_2(M\left[|d_x(j_2,k)|^p\right]) \\ -pj_1 \log_2(M\left[|d_x(0,k)|^p\right]) \end{array} \right\} \end{array} \quad (19)$$

$$\begin{array}{l} \left\{ \begin{array}{l} pj_2 \log_2(M\left[|d_x(j_1,k)|^p\right]) \\ -pj_1 \log_2(M\left[|d_x(j_2,k)|^p\right]) \end{array} \right\} \\ \cong \left\{ \begin{array}{l} pj_2 \log_2(M\left[|d_x(0,k)|^p\right]) \\ -pj_1 \log_2(M\left[|d_x(0,k)|^p\right]) \end{array} \right\} \end{array} \quad (20)$$

Table 2. The comparability of values for the equation (17)

Comparable conditions	Time series		
	Figure 1	Figure 2	Figure 3
$j1=7 \quad j2=5 \quad p=1$	27.2841	30.2050	32.9117
$j1=5 \quad j2=7 \quad p=1$	27.5619	29.6030	33.1274
$j1=7 \quad j2=5 \quad p=2$	110.3080	120.1508	131.6537
$j1=5 \quad j2=7 \quad p=2$	110.8266	118.2124	132.1597
$j1=7 \quad j2=5 \quad p=3$	250.5134	268.8135	295.5375
$j1=5 \quad j2=7 \quad p=3$	250.2070	265.8349	296.7108
$j1=7 \quad j2=5 \quad p=4$	448.8262	475.4117	523.6084
$j1=5 \quad j2=7 \quad p=4$	445.9148	473.0703	526.6400
$j1=9 \quad j2=6 \quad p=1$	45.1889	49.4181	54.0979
$j1=6 \quad j2=9 \quad p=1$	45.7336	49.2112	54.9194
$j1=9 \quad j2=6 \quad p=2$	181.3358	199.1977	216.9567
$j1=6 \quad j2=9 \quad p=2$	183.8070	197.4284	219.5123
$j1=9 \quad j2=6 \quad p=3$	408.2816	449.8565	487.8387
$j1=6 \quad j2=9 \quad p=3$	414.4472	443.9703	492.6907
$j1=9 \quad j2=6 \quad p=4$	724.8931	800.5587	865.2227
$j1=6 \quad j2=9 \quad p=4$	737.7660	787.7358	873.0272
$j1=6 \quad j2=5 \quad p=1$	22.8668	24.6056	27.4597
$j1=5 \quad j2=6 \quad p=1$	22.9683	24.6692	27.6062
$j1=6 \quad j2=5 \quad p=2$	91.9035	98.7142	109.7562
$j1=5 \quad j2=6 \quad p=2$	92.3555	98.5104	110.1331
$j1=6 \quad j2=5 \quad p=3$	207.2236	221.9852	246.3453
$j1=5 \quad j2=6 \quad p=3$	208.5058	221.5291	247.2590
$j1=6 \quad j2=5 \quad p=4$	368.8830	393.8679	436.5136
$j1=5 \quad j2=6 \quad p=4$	371.5957	394.2253	438.8667

$$\left\{\begin{array}{l} pj_2\log_2(M\left[\left|d_x(j_1,k)\right|^p\right]) \\ -pj_1\log_2(M\left[\left|d_x(j_2,k)\right|^p\right]) \end{array}\right\} \cong p(j_2-j_1)\log_2(M\left[\left|d_x(0,k)\right|^p\right]) \tag{21}$$

$$\log_2(M\left[\left|d_x(0,k)\right|^p\right]) \cong \frac{\left\{\begin{array}{l} pj_2\log_2(M\left[\left|d_x(j_1,k)\right|^p\right]) \\ -pj_1\log_2(M\left[\left|d_x(j_2,k)\right|^p\right]) \end{array}\right\}}{p(j_2-j_1)} \tag{22}$$

Or

$$M\left[\left|d_x(0,k)\right|^p\right] \cong \sqrt[p(j_2-j_1)]{\frac{(M\left[\left|d_x(j_1,k)\right|^p\right])^{pj_2}}{(M\left[\left|d_x(j_2,k)\right|^p\right])^{pj_1}}} \tag{23}$$

Based on the above, equation (23) can be used to find the mathematical expectation of detailing coefficients for various levels of wavelet decomposition of the original time series, besides, it is also possible to consider an expression for different values of p (p_1 *and* p_2) with same values on a specific level of decomposition j, such that:

$$\frac{\log_2(M\left[\left|d_x(j,k)\right|^{p_1}\right])-\log_2(M\left[\left|d_x(0,k)\right|^{p_1}\right])}{p_1} \cong \frac{\log_2(M\left[\left|d_x(j,k)\right|^{p_2}\right])-\log_2(M\left[\left|d_x(0,k)\right|^{p_2}\right])}{p_2} \tag{24}$$

Or

$$p_2(\log_2(M\left[\left|d_x(j,k)\right|^{p_1}\right])-\log_2(M\left[\left|d_x(0,k)\right|^{p_1}\right])) \cong p_1(\log_2(M\left[\left|d_x(j,k)\right|^{p_2}\right])-\log_2(M\left[\left|d_x(0,k)\right|^{p_2}\right])) \tag{25}$$

$$\log_2\left(\frac{M\left[\left|d_x(j,k)\right|^{p_1}\right]}{M\left[\left|d_x(0,k)\right|^{p_1}\right]}\right)^{p_2} \cong \log_2\left(\frac{M\left[\left|d_x(j,k)\right|^{p_2}\right]}{M\left[\left|d_x(0,k)\right|^{p_2}\right]}\right)^{p_1} \tag{26}$$

$$\left(\frac{M\left[\left|d_x(j,k)\right|^{p_1}\right]}{M\left[\left|d_x(0,k)\right|^{p_1}\right]}\right)^{p_2} \cong \left(\frac{M\left[\left|d_x(j,k)\right|^{p_2}\right]}{M\left[\left|d_x(0,k)\right|^{p_2}\right]}\right)^{p_1} \tag{27}$$

The equations (25) – (27) can be used as an indicators of the self-similar of the time series. Table 3 shows calculations for different ratios in accordance with equation (26) for a specified time series.

Table 3 shows that the values obtained in accordance with formula (26) are comparable within the specified terms of comparison. Thus, the data confirm the theoretical calculations, allowing to use equations (25) – (27) in the study and comparison of dynamics of different time series.

Table 3. The comparability of values for equation (26)

Comparable conditions	Time series		
	Figure 1	Figure 2	Figure 3
$j=7$ $p1=1$ $p2=4$	27.2841	30.2050	32.9117
$j=7$ $p1=4$ $p2=1$	28.0516	29.7132	32.7255
$j=7$ $p1=2$ $p2=4$	55.1540	60.0754	65.8268
$j=7$ $p1=4$ $p2=2$	56.1033	59.4265	65.4511
$j=7$ $p1=3$ $p2=4$	83.5045	89.6045	98.5125
$j=7$ $p1=4$ $p2=3$	84.1549	89.1397	98.1766
$j=3$ $p1=1$ $p2=4$	8.8516	9.5777	10.8561
$j=3$ $p1=4$ $p2=1$	8.7740	9.4755	10.8636
$j=3$ $p1=2$ $p2=4$	17.6967	19.0549	21.7555
$j=3$ $p1=4$ $p2=2$	17.5479	18.9511	21.7272
$j=3$ $p1=3$ $p2=4$	26.4530	28.4788	32.6209
$j=3$ $p1=4$ $p2=3$	26.3219	28.4266	32.5908
$j=10$ $p1=1$ $p2=4$	41.2881	44.2505	48.5203
$j=10$ $p1=4$ $p2=1$	41.0870	45.0553	48.0098
$j=10$ $p1=2$ $p2=4$	82.7961	89.2490	96.6220
$j=10$ $p1=4$ $p2=2$	82.1741	90.1106	96.0197
$j=10$ $p1=3$ $p2=4$	123.7996	134.7129	144.4206
$j=10$ $p1=4$ $p2=3$	123.2611	135.1659	144.0295

5. Conclusions

In this paper, have been highlighted the main points regarding the use of multiresolution wavelet analysis as a method of analysis for data sequence. We also looked at the relationship and the value of the Hurst exponent in accordance with the properties of the decomposition of wavelet coefficients of time series at different levels. Based on the properties of decomposition of detailing wavelet coefficients, we have shown the possibility of settlement to obtain an estimate the Hurst exponent for self-similar time series. For example, we have demonstrated the feasibility to estimate Hurst exponent for time series presented by Brownian motion, and at the same time we have discussed and shown the main causes of errors in the calculated ratios to assess the Hurst exponent values, where the validity of all obtained theoretical results had been confirmed by a number of examples.

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