American Journal of Systems and Software, 2014, Vol. 2, No. 3, 65-71
Available online at http://pubs.sciepub.com/ajss/2/3/3

DOI:10.12691/ajss-2-3-3

Behaviours, Processes and Probabilistic Environmental Functions in H-Open Systems

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Received June 04, 2014; Revised June 17, 2014; Accepted June 24, 2014

Abstract The Patten's Theory of the Environment, supposes an impotent contribution to the Theoretical Ecology. The hypothesis of the duality of environments, the creaon and genon functions and the three developed propositions are so much of great importance in the field of the Applied Mathematical as Ecology. The authors have undertaken an amplification and revision of this theory, developing the following steps: 1) A theory of processes. 2) A definition of structural and behavioural functions. 3) A probabilistic definition of the environmental functions. In this paper the authors develop the theory of behavioural functions, begin the theory of environmental functions and give a complementary focus to the theory of processes that has been developed in precedent papers.

Keywords: *creaon, environment theory, genon, Holon, probabilistic functions, process, semiotic system*

Cite This Article: Josep-LluisUso-Domenech, Josue-Antonio Nescolarde-Selva, and Miguel Lloret-Climent, "Behaviours, Processes and Probabilistic Environmental Functions in H-Open Systems." *American Journal of Systems and Software*, vol. 2, no. 3 (2014): 65-71. doi: 10.12691/ajss-2-3-3.

1. Introduction

A system is a partially interconnected set of components. Interconnections mean interactions, and components mean processes or objects, the later being slow processes in reference to time frame norms of system. Systems can be considered classically in two ways: (1) closed or (2) open, in interaction with their environments. A characteristic of all closed systems is that they have an inherent tendency to move toward a static equilibrium and entropy. A system is closed if it does not interact with other system and open if it receives causes from or generates effects to another system. A system boundary provides the interface with other systems and is defined by specifying its component set. Input is any movement of energy-matter or information, and output is any similar movement across the system boundary in the opposite direction (Patten, 1978).

The open-system view recognizes that the ecological system is a dynamic relationship with is environment and receives various stimuli, transform these stimulus in some way, and export responses. The receipt of stimuli in the form of material, energy, and information allows the open system to offset the process of entropy. These systems are open not only in relation to their environment but also in relation to themselves (internally related) in that interactions between components affect the systems as a whole. Thus, a three level hierarchy is usually implied in definition of a system: suprasystem or environment, system, and subsystem or components (Patten, 1980).

The open system adapts to its environment by changing the structure and processes of its internal components. The structure's concept can be considered in terms of a generic open system, because the open system is in a continual interaction with its environments (stimulus environment H' and response environment H") and achieve a steady state or dynamic equilibrium while still retaining the capacity for energy transformation. The survival of system, in effect, would not be possible without continuos inflow, transformation, and outflow. In a complex structural system this is a continuous recycling process. The system must receive sufficient stimuli of resources to maintain its operations and also to export the transformed resources to the environment in sufficient quantity to continue the cycle.

The behaviour of a system is the time course of systems's state variables. It is therefore, its performance before certain situations in a temporary interval, for what interests to consider the dependence of the time of the different magnitudes x_i associated to the system. This dependence is expressed by means of functions x(t) that represent what is denominated trajectory of each variable. The set of trajectories describes the evolution of the system during a certain period of time, that is to say, it constitutes the history of the system during that period.

Patten (1978, 1980) introduces the Koestler's (1967) dialectic term holon as systems that are simultaneously part of a greater whole, and whole made up of lesser parts. Holon faces two directions at once, inward and downward toward its own parts, and outward and upward toward the system of which it is a part.

An H-open system (Lloret-Climent et al., 2002; Uso-Domenech et al., 2002; Villacampa and Uso-Domenech, 1999.), is one whose component set M is ontic but whose relational set R_I is formed by "inferential interactions". *Inferential interactions* $R_I \subset \{R_r, R_R\}$ are transactions or relations whose significance is *informational*, transcending the matter and physical forces underneath. Inferential interactions involves ontic signs and signal flows which take semantic meaning within habituated espistemological frames established between interactive pairs from M. H-open systems are thus oriented, causal, functional systems defined by an external observer, as before, whose object set A responds to information. Definition of H-open systems enlarges Holon's definition, because the two directions of Holon are the flow of matter, energy and information. Holon is also a semiotic or conceptual model of reality in the mind of observer.

Beings do not have an intrinsic meaning and they only transform themselves into signs when we have invested them with meaning. The signs are significant units that take the form from words, images, sounds, gestures and objects, studied within a system of semiotic signs, like means or code. In any process, we can distinguish between having a significant like inherent property, and having significance when it is related to other processes of Reality that the Subject considers like system. The existence of information is independent of the fact that there is a Subject able to decode the message, which it is wished to communicate. This objective information is termed significant. The information in a message acquires meaning if a Subject decodes the message. This subjective information is termed significance. (Sastre-Vazquez, P.. Uso-Domenech, J.L., Y. Villacampa, J. Mateu and P. Salvador. 1999; Uso-Domenech, J.L., G. Stubing, J. Lopez-Vila, and P. Sastre Vazquez, 2002; Uso-Domenech, J.L., J. Mateu. 2004; Villacampa, Y., et. al., 1999[a]; Villacampa-Esteve, Y., et. al., 1999; Uso-Domenech, J.L. and Villacampa, Y., 2001; Nescolarde-Selva, J. and Uso-Domenech, J.L., 2013; Gash, H., 2014).

To understand the behaviour of the system means to figure out the causal relationships that allow explaining this system to the Observer, so that it allows giving a mechanism with the one that to build a mathematical model of the ontologic system. We will consider therefore the studied systems as H-open systems, since the behaviour can only be determined by the Observer's presence. In this paper the authors develop the theory of behavioural functions, begin the theory of environmental functions and give a complementary focus to the theory of processes that has been developed in precedent papers.

2. Open H-Systems and Behaviours

Let $H[t_0, t_f] = (M[t_0, t_f], R[t_0, t_f])$ be an open semiotic system and $x \in M[t_0, t_f]$, being x a behaviour. $x[t_0, t_f]$ is the behaviour in any instant of the interval $[t_0, t_f]$ and it is the value of the behaviour in any instant of the interval $[t_0, t_f]$ (time of duration of the behaviour). $x(t)$ it is the behaviour and/or value of the behaviour in time t (presents). x^t is the behaviour and/or value of the behaviour in any previous instant to t, that is to say, in $[t_0, t]$ (past). x_t is the behaviour and/or value of the behaviour in any later instant to t, that is to say $[t_0, t_f]$ (future). Similarly, $\forall r \in R[t_0, t_f]$ being r a relationship. $r[t_0, t_f]$ is the relationship in any instant of the interval $[t_0, t_f]$. $r(t)$ it is the relationship in time t (presents). r^t is the relationship in any previous instant to t (past) and r_t is the relationship in any later instant to t (future).

Definition 1: *In a semiotic open system* $H[t_0, t_f] = (M[t_0, t_f], R[t_0, t_f])$, *behaviour* $x(t) \in M[t_0, t_f]$ *is a stimulus with response and it is not a response of any stimulus iff:*

$$\left\{\exists y[t_0,t_f] \in M[t_0,t_f]\ \exists r[t_0,t_f] \in R[t_0,t_f] / \left(x(t), y_t\right) \in r_t\right\}$$
$$\wedge \left\{\forall z[t_0,t_f] \in M[t_0,t_f]\ \forall[t_0,t_f] \in R[t_0,t_f] \left(z', x(t)\right) \notin r'\right\}$$

If $(x(t), y_t) \in r_t$ and being able to be expressed as $r_t(x(t)) = y_t$, it indicates that the behaviour and/or value of the behaviour x in t when being influenced by the relationship r in any later instant to t, produces a behaviour and/or value of the behaviour y_t. This won't be the only behaviour and/or value of the behaviour y_t, since there are other relationships and other behaviours that will be able to affect to the behaviour of y.

The behaviours $x(t) \in M[t_0, t_f]$ stimuli with response and not response of any stimulus, they act as behaviours stimulus, are known a priori. When in a system all the behaviours are known, they can also be calculated to posteriori by means of the stimulus-response behavioural function f_0: $M[t_0, t_f] \rightarrow P(M)[t_0, t_f]$ in the following way:

$$f_D\left(x(t)\right) = \begin{Bmatrix} y_t \in M\left[t_0,t_f\right] / \exists r_t \in R\left[t_0,t_f\right] \\ \wedge \left(x(t), y_t\right) \in r_t \end{Bmatrix}$$
$$= \left\{y_t \in M\left[t_0,t_f\right] / \exists r_t \in R\left[t_0,t_f\right], r_t\left(x(t)\right) = y_t\right\}$$
$$= \left\{y_t^1, y_t^2, \ldots, y_t^q\right\}$$

The behaviour of x in t will come determined by one transformed function of order q of the previous values of the behaviours: $x(t) = y_t^1 \otimes y_t^2 \otimes \ldots \otimes y_t^q$ being $\otimes$ a mathematical operation.

Similarly $(z^t, x(t)) \notin r^t$ and being able to be expressed as $r^t(z^t) \neq x(t)$, means that the behaviour and/or value of the behaviour of z in any previous instant to t, when is influenced by the relationship r in any previous instant to t, doesn't affect to the behaviour and/or value of the behaviour x in t.

Definition 2: *In a semiotic open system* $H\left[t_0,t_f\right] = \left(M\left[t_0,t_f\right], R\left[t_0,t_f\right]\right)$, *a behavior* $x(t) \in M\left[t_0,t_f\right]$ *is stimulus and response simultaneously iff:*

$$\begin{Bmatrix} \exists y\left[t_0,t_f\right] \in M\left[t_0,t_f\right] \exists r\left[t_0,t_f\right] \\ \in R\left[t_0,t_f\right] / \left(x(t), y_t\right) \in r_t \end{Bmatrix}$$
$$\wedge \begin{Bmatrix} \exists z\left[t_0,t_f\right] \in M\left[t_0,t_f\right] \exists r\left[t_0,t_f\right] \\ \in R\left[t_0,t_f\right] / \left(z^t, x(t)\right) \in r^t \end{Bmatrix}$$

If $(z^t, x(t)) \in r^t$ and being able to be expressed as $r^t(z^t) = x(t)$, means that the behavior and/or value of the behaviour z in any previous instant to t, when is influenced by the relationship r in any previous instant to t, produces a behaviour and/or value of the behaviour x in t.

The value of the behaviour $x(t) \in M\left[t_0,t_f\right]$ that is stimulus and response simultaneously, will come determined by the stimulus-response behavioural function and response-stimulus behavioural function in the following way:

$$f_D(x(t)) = \begin{Bmatrix} y_t \in M[t_0,t_f] / \exists r_t \in R[t_0,t_f] \\ \wedge (x(t), y_t) \in r_t \end{Bmatrix}$$

$$= \begin{Bmatrix} y_t \in M[t_0,t_f] / \exists r^t \in R[t_0,t_f], \\ r_t(x(t)) = y_t \end{Bmatrix}$$

$$= \{y_t^1, y_t^2, ..., y_t^q\}$$

and

$$g_D(x(t)) = \begin{Bmatrix} y^t \in M[t_0,t_f] / \exists r^t \in R[t_0,t_f] \\ \wedge (x(t), y^t) \in r^t \end{Bmatrix}$$

$$= \begin{Bmatrix} y^t \in M[t_0,t_f] / \exists r_t \in R[t_0,t_f], \\ r^t(x(t)) = y^t \end{Bmatrix}$$

$$= \{y_1^t, y_2^t, ..., y_q^t\}$$

The behaviour of x in t will come determined by one transformed function of order p+q of the values previous of the behaviours:

$$x(t) = y_1^t \otimes y_2^t \otimes ... \otimes y_q^t \otimes y_t^1 \otimes y_t^2 \otimes ... \otimes y_t^q$$

where each $\otimes$ represents a mathematical operation.

Definition 3: *In a semiotic open system* $H[t_0,t_f] = (M[t_0,t_f], R[t_0,t_f])$, *a behavior* $x(t) \in M[t_0,t_f]$ *is a response of some stimulus but it is not stimulus of any response iff*:

$$\begin{Bmatrix} \exists y[t_0,t_f] \in M[t_0,t_f] \exists r[t_0,t_f] \\ \in R[t_0,t_f] / (y^t, x(t)) \in r^t \end{Bmatrix}$$
$$\wedge \begin{Bmatrix} \forall z[t_0,t_f] \in M[t_0,t_f] \exists r[t_0,t_f] \\ \in R[t_0,t_f] / (x(t), z_t) \notin r_t \end{Bmatrix}$$

If $(x(t), z_t) \notin r_t$ and being able to be expressed as $r_t(x(t)) \neq z_t$ means that the behavior and/or value of the behaviour x in t, when is influenced by the relationship r in any later instant to t, doesn't affect to the behaviour and/or value of the behaviour after t.

The value of the behaviour $x(t) \in M[t_0,t_f]$ that is response of some stimulus but it is not stimulus of any response, will come determined by the response-stimulus behavioural function from the following way:

$$g_D(x(t)) = \begin{Bmatrix} y^t \in M[t_0,t_f] / \exists r^t \in R[t_0,t_f] \\ \wedge (x(t), y^t) \in r^t \end{Bmatrix}$$

$$= \begin{Bmatrix} y^t \in M[t_0,t_f] / \exists r_t \in R[t_0,t_f], \\ r^t(x(t)) = y^t \end{Bmatrix}$$

$$= \{y_1^t, y_2^t, ..., y_q^t\}$$

The behaviour of x in t will come determined by one transformed function of order p of the values previous of the behaviours: $x(t) = y_1^t \otimes y_2^t \otimes ... \otimes y_q^t$ where each $\otimes$ represents a mathematical operation.

This way partition of the system $H[t_0,t_f]$ is made in three groups:

a) Behaviours that are stimuli with response and that are not response of any stimulus $H_S[t_0,t_f]$.

b) Behaviours that are simultaneously stimuli and response $H_{SR}[t_0,t_f]$.

c) Behaviours that are response of some stimulus but are not stimuli of any response $H_R[t_0,t_f]$.

That it to say:

$$H[t_0,t_f] = H_S[t_0,t_f] \cup H_{SR}[t_0,t_f] \cup H_R[t_0,t_f]$$
$$H_S[t_0,t_f] \cap H_{SR}[t_0,t_f] = \varnothing$$
$$H_S[t_0,t_f] \cap H_R[t_0,t_f] = \varnothing$$
$$H_{SR}[t_0,t_f] \cap H_R[t_0,t_f] = \varnothing$$

Definition 4: *In a semiotic open system* $H[t_0,t_f] = (M[t_0,t_f], R[t_0,t_f])$, *the stimulusresponse behavioural function* $f_D : M[t_0,t_f] \to P(M)[t_0,t_f]$ *is defined as*

$$\forall x[t_0,t_f] \in M[t_0,t_f], t \in]t_0,t_f[,$$
$$f_D(x(t)) = \begin{Bmatrix} y_t \in M[t_0,t_f] / \exists r_t \in R[t_0,t_f] \\ \wedge (x(t), y_t) \in r_t \end{Bmatrix}$$

and being able to be expressed as

$$f_D(x(t)) = \begin{Bmatrix} y_t \in M[t_0,t_f] / \exists r_t \in R[t_0,t_f], \\ r_t(x(t)) = y_t \end{Bmatrix}$$

For a given behaviour x in an instant t, in the behavioural function, appear all the future behaviours able to be observed after t.

Definition 5: *In a semiotic open system* $H[t_0,t_f] = (M[t_0,t_f], R[t_0,t_f])$, *the responsestimulus behavioural function* $g_D : M[t_0,t_f] \to P(M)[t_0,t_f]$ *is defined as*

$$\forall x[t_0,t_f] \in M[t_0,t_f], t \in]t_0,t_f[,$$
$$f_D(x(t)) = \begin{Bmatrix} y^t \in M[t_0,t_f] / \exists r^t \in R[t_0,t_f] \\ \wedge (y^t, x(t)) \in r^t \end{Bmatrix}$$

and being able to be expressed as

$$g_D(x(t)) = \begin{Bmatrix} y^t \in M[t_0,t_f] / \exists r_t \in R[t_0,t_f], \\ r^t(y^t) = x(t) \end{Bmatrix}$$

For behaviour x given in an instant t, in the behavioural function appear all the stimuli behaviours last observed before t.

The function g_D will be able to be observed in multitude of systems not happening since the same thing with the function f_D. It supposes to use (and know) future performances.

Definition 6: *In a semiotic open system* $H\left[t_0,t_f\right]=\left(M\left[t_0,t_f\right],R\left[t_0,t_f\right]\right)$, *the stimulusresponse behavioural function associated to relationship r* $f_D^r : M\left[t_0,t_f\right] \to P(M)\left[t_0,t_f\right]$
is defined as

$$\forall x\left[t_0,t_f\right] \in M\left[t_0,t_f\right], t \in \left]t_0,t_f\right[,$$
$$f_D^r\left(x(t)\right)=\left\{y_t \in M\left[t_0,t_f\right] / \wedge \left(x(t),y_t\right) \in r_t\right\}$$

and being able to be expressed as

$$f_D^r\left(x(t)\right)=\left\{y_t \in M\left[t_0,t_f\right] / r_t\left(x(t)\right)=y_t\right\}.$$

For a given behaviour x in an instant t and a relationship r, in the behavioural function appear all the future behaviours response that will be able to be observed after t. This function will be a stochastic behavioural function, that is to say, there will be a probability that the observed behaviour be in and or another way.

Definition 7: *In a semiotic open system* $H\left[t_0,t_f\right]=\left(M\left[t_0,t_f\right],R\left[t_0,t_f\right]\right)$, *the responsestimulus behavioural function associated to relationship r* $g_D^r : M\left[t_0,t_f\right] \to P(M)\left[t_0,t_f\right]$
is defined as

$$\forall x\left[t_0,t_f\right] \in M\left[t_0,t_f\right], t \in \left]t_0,t_f\right[,$$
$$g_D^r\left(x(t)\right)=\left\{y^t \in M\left[t_0,t_f\right] / \wedge \left(y^t,x(t)\right) \in r^t\right\}$$

and being able to be expressed as

$$g_D^r\left(x(t)\right)=\left\{y^t \in M\left[t_0,t_f\right] / r^t\left(y^t\right)=x(t)\right\}.$$

For a given behaviour x in an instant t and a relationship r, in the behavioural function appear all the stimuli behaviours last observed before t and that they have given place to the behaviour x in t.

Theorem 1: *In a semiotic open system* $H\left[t_0,t_f\right]=\left(M\left[t_0,t_f\right],R\left[t_0,t_f\right]\right)$, *the stimulusresponse behavioural function is the union of the stimulus-response behavioural functions associated to relationship r that exists in this H-system, that it to say* $f_D=\bigcup_{r\in R} f_D^r$.

Proof

If
$$y_t \in f_D\left(x(t)\right) \Rightarrow \exists r_t \in R\left[t_0,t_f\right],$$
$$r_t\left(x(t)\right)=y_t \in f_D\left(x(t)\right) \subseteq \bigcup_{r\in R} f_D^r\left(x(t)\right)$$

On other hand

$$y_t \in \bigcup_{r\in R} f_D^r\left(x(t)\right) \Rightarrow \exists r_0 \in R\left[t_0,t_f\right]$$
$$/ y_t \in f_D^{r_0}\left(x(t)\right) \Rightarrow y_t \in f_D\left(x(t)\right)$$

Theorem 2: *In a semiotic open system* $H\left[t_0,t_f\right]=\left(M\left[t_0,t_f\right],R\left[t_0,t_f\right]\right)$, *the responsestimulus behavioural function is the union of the response-stimulus behavioural functions associated to relationship r that exists in this H-system, that it to say* $g_D=\bigcup_{r\in R} g_D^r$.

Proof

If
$$y^t \in g_D\left(x(t)\right) \Rightarrow \exists r^t \in R\left[t_0,t_f\right],$$
$$r^t\left(x(t)\right)=y^t \in g_D\left(x(t)\right) \subseteq \bigcup_{r\in R} g_D^r\left(x(t)\right)$$

On other hand

$$y_t \in \bigcup_{r\in R} g_D^r\left(x(t)\right) \Rightarrow \exists r_0 \in R\left[t_0,t_f\right]$$
$$/ y^t \in g_D^{r_0}\left(x(t)\right) \Rightarrow y^t \in g_D\left(x(t)\right)$$

Definition 8: *In a semiotic open system* $H\left[t_0,t_f\right]=\left(M\left[t_0,t_f\right],R\left[t_0,t_f\right]\right)$, *a behavior* $x(t) \in M\left[t_0,t_f\right]$ *is invariable, unalterable or independent for the relationship* $r\left[t_0,t_f\right] \in R\left[t_0,t_f\right]$ *if* $\left(x',x(t)\right) \notin r^t$, $\left(x(t),x(t)\right) \notin r(t), \left(x(t),x_t\right) \notin r_t$. *or expressing it functionally if* $r^t\left(x^t\right) \neq x(t), r_t\left(x(t)\right) \neq x(t)\ r_t\left(x(t)\right) \neq x_t$; *that is to say when the value of the behaviour remains invariable for the relationship r*.

Definition 9: *In a semiotic open system* $H\left[t_0,t_f\right]=\left(M\left[t_0,t_f\right],R\left[t_0,t_f\right]\right)$. , *a behavior* $x(t) \in M\left[t_0,t_f\right]$ *is constant for the relationship r t* $r\left[t_0,t_f\right] \in R\left[t_0,t_f\right]$

a) $\left(x^t,x(t)\right) \in r^t, \left(x(t),x(t)\right) \in r(t), \left(x(t),x_t\right) \in r_t$

. *or expressing it functionally* $r^t\left(x^t\right)=x(t), r_t\left(x(t)\right)=x(t)\ r_t\left(x(t)\right)=x_t$

b) $x(t)=x_t=x^t$

Definition 10: *In a semiotic open system* $H\left[t_0,t_f\right]=\left(M\left[t_0,t_f\right],R\left[t_0,t_f\right]\right)$. , *a behavior* $x(t) \in M\left[t_0,t_f\right]$ *is steady for the relationships* $\forall r\left[t_0,t_f\right] \in R\left[t_0,t_f\right]$ *iff*

a) $\left(x^t,x(t)\right) \in r^t, \left(x(t),x(t)\right) \in r(t), \left(x(t),x_t\right) \in r_t$ *or expressing it functionally* $r^t\left(x^t\right)=x(t), r_t\left(x(t)\right)=x(t), r_t\left(x(t)\right)=x_t$

b) $x(t)=x_t=x^t$

Consequence 1: *If behaviour is constant for any relationship, then is steady*.

Consequence 2: *If behaviour is steady for any relationship, then is constant.*

3. Processes and Behaviours

A process is a mechanism that implies a group of successive operations between stimuli and responses. Explicitly, a process is a succession of behaviours and relationships in different instants of time: $P=\left\{r_i\left(x_i\left(t_i\right)\right)\right\}_{i=1}^{n}$ where $r_i\left(x_i\left(t_i\right)\right)$ is the value of the behaviour x_i in the instant t_i for the relationship r_i (n can be infinite, what would indicate that the process doesn't have end) and $t_0< t_1<t_2<....<t_n<t_f$. accomplishing that for any two serial behaviours $x_j(t_j)$ and $x_{j+1}(t_{j+1})$ a relation $r_j[t_j,t_{j+1}]$ □ $R[t_0,t_f]$ exists so that $\left(x_j^{t\,j+1}, x_{j+1}\left(t_{j+1}\right)\right)\in r_j^{t\,j+1}$, or functionally $r_j^{t\,j+1}\left(x_j^{t\,j+1}\right)=x_{j+1}\left(t_{j+1}\right)$, that is to say, the relationship r_j when acting on the behaviour x_j in $[t_j,t_{j+1}[$ produces the value of the behaviour x_{j+1} in t_{j+1}.

Said otherwise, two serial behaviours of the process are connected by means of the response-stimulus behavioural function because $b_j^{t\,j+1}\in g_D\left(x_{j+1}\left(t_{j+1}\right)\right)$. In an open system there is multitude of processes, even among the same behaviours, since it is possible that the connections are carried out by means of different relationships giving place therefore to different results.

We can identify the processes with the successions of real numbers, therefore, it makes sense to speak of *convergent and divergent* processes.

Definition 11: *A process* $P=\left\{r_i\left(x_i\left(t_i\right)\right)\right\}_{i=1}^{n}$ *is convergent if has one limit whose value will be the behaviour c in* t_f *such that* $\left(x_n^{t\,f}, c\left(t_f\right)\right)\in r_n^{t\,f}$ *functionally* $r_n^{t\,f}\left(x_n^{t\,f}\right)=c\left(t_f\right)$. *When the process is* $P=\left\{r_i\left(x_i\left(t_i\right)\right)\right\}_{i=1}^{n}$, *it will be convergent if has one limit, in which case it will coincide with the limit of the succession. In case that the process is not convergent we will say that it is divergent.*

Definition 12: *A process* $P=\left\{r_i\left(x_i\left(t_i\right)\right)\right\}_{i=1}^{n}$ *is a stimulus-response process iff:* $x_1(t_1)\in H_S[t_0,t_f]$, $x_2(t_2),...,x_n(t_n)\in H_{SR}[t_0,t_f]$ *and its limit* $c(t_f)\in H_R[t_0,t_f]$.

Definition 13: *A process* $P=\left\{r_i\left(x_i\left(t_i\right)\right)\right\}_{i=1}^{n}$ *is a transition process iff:* $x_1(t_1)\in H_S[t_0,t_f]$, $x_2(t_2),...,x_n(t_n)\in H_{SR}[t_0,t_f]$ *and its limit* $c(t_f)\in H_{SR}[t_0,t_f]$.

Definition 14: *A process* $P=\left\{r_i\left(x_i\left(t_i\right)\right)\right\}_{i=1}^{n}$ *is an internal transition process iff:* $x_1(t_1)$, $x_2(t_2),...,x_n(t_n)\in H_{SR}[t_0,t_f]$ *and its limit* $c(t_f)\in H_{SR}[t_0,t_f]$.

Definition 15: *A process* $P=\left\{r_i\left(x_i\left(t_i\right)\right)\right\}_{i=1}^{n}$ *is an internal response process iff:* $x_1(t_1)$, $x_2(t_2),...,x_n(t_n)\in H_{SR}[t_0,t_f]$ *and its limit* $c(t_f)\in H_R[t_0,t_f]$.

4. The Probabilistic Environmental Functions

Patten first proposition said (1978): “Every object H defines two environments: an input (stimulus) environment H’, and an output (response) environment H”.

The causal model of subject environment interaction leads not one, but two equally plausible and useful concepts of environment. The first is the stimulus environment H’ defined by H open system in the act of receiving energy matter or perceiving information. The second is the response environment of set of potential environments embodied in the states S of H.

In the original theory of the Environment, is not defined the deterministic or probabilistic character of the environments H' and H". However, certain characteristics of both environments make suppose that they can be considered as probabilistic spaces. In the enlarged revision that authors are making of the Patten’s theory, they suppose the existence of the spaces H', S and H" (stimulus, state and response respectively) as probabilistic spaces, that is to say, in them exist all the stimuli, states and possible responses, but not with same probability.

Let H', S and H" be three probabilistic spaces referred to as the stimulus space, state space and response space, respectively and let $A_i^{'}, A_i^{s}, A_i{}''$ be three collections of sets belonging to H', S and H" respectively. Suppose x, s_t and y_t three behaviours that take their possible values from the H', S and H" respectively. Our hypothesis is a generalization of Dempster- Shafer (Dempster, 1967) theory based on the concept of a multivalued mapping that describes the compatibility relationship between two probability spaces.

4.1. The Creaon Probabilistic Function

The H open system (Holon) becomes orientated when its set of attributes is partitioned into stimulus and responses. The relation H on within system portion of H’ is thus explicit in the concept of stimulus environment. Behavioural attributes of the real world that not impact H as stimulus during its existence interval cannot influence the state of the object. They go unrecorded by H and consequently are not part of its environment. So basic is this environment defining function that this aspect of the open system is given (Patten, 1978) a special name, creaon.

A body of stimulus for the state space S is constituted by:

1) A set of processes that associate value of the two behaviours in the form *if* $x=h_i^{'}$ *then* s_t *is* A_i^{s}.

2) A probability distribution of the stimulus space H'.

A body of responses for the stimulus space H' is constituted by:

1) A set of processes that associate value of the two behaviours in the form *if* $s_t=h_i^{'}$ *then x is* $A_i^{'}$.

2) A probability distribution of the state space S.

A multivalued mapping from a probability space H' to a probability space S, which is referred to as the frame of behaviour, associates each element in H' with a set of elements in S, that is

$$c_D^{r}: H'\rightarrow 2^{S}$$

The image of an element h' in S under the mapping is denoted as the kernel of h', K(h').

Alternatively, the multivalued mapping can be viewed as a compatibility relationship between their elements

$$K(h') = \begin{Bmatrix} h \mid h \in H'', h' \text{ compatible with} \\ h / (h', h) \in r_t \end{Bmatrix}$$

Definition 16: *An element $h' \in H'$ is said to be compatible with an element $h \in S$, if it is possible that h' is a stimulus to H' and h is a response to S at the same interval of time* $\left[t_0, t_f\right]$.

Given a probability distribution of space H' and a comparability relation between H' and S, a basic probability assignment (bpa) of space S, denoted by *m:* $2^S \to [0,1]$, is induced

$$m\left(A^S\right) = \frac{\sum_{K\left(h_i'\right)=A^S} p\left(h_i'\right)}{1 - \sum_{K\left(h_i'\right)=\varnothing} p\left(h_i'\right)}$$

Subsets of H' and S with nonzero basic probabilities are called creaon focal elements (cfe).

Let be a subset *AS* of S and let be a subset A' of H'. The basic probability assignment m determines two functions, similar to belief and plausibility functions, that measure the minimal and maximal degree of fulfiment of a stimulus-state process are:

$$\min \Pr\left(A^S\right) = \sum_{A' \subseteq A^S} m(A')$$

$$\max \Pr\left(A^S\right) = \sum_{A' \cap A^S \neq \varnothing} m(A')$$

If m_1, m_2 are two basic probabiliy assignments induced by two independent sources of stimulus, they can be combined using Dempster theorem for the stimulus state process such as

$$m_1 \oplus m_2(C) = \frac{\sum_{A_i' \cap A_j' = C} m_1\left(A_i'\right) m_2\left(A_j'\right)}{1 - \sum_{A_i' \cap A_j' \neq \varnothing} m_1\left(A_i'\right) m_2\left(A_j'\right)}$$

4.2. The Genon Probabilistic Function

The states S are converted to responses through interaction of H with other objects. This is, to produce an actual response environment from potential environment implicit in the state structure S of H requires Holon production of potential attributes, then sequential creaon selections to achieve realization of these potentials. Output environment H'' is the resultant causality propagated from H as a network of direct and indirect causality. This environment generating property of H open systems (holons) is a probabilistic function that has name of genon.

A body of stimulus for the response space H'' is constituted by:

1) A set of processes that associate value of the two behaviours in the form $if\ s_t = h_i\ then\ y_t\ is\ A_i''$.

2) A probability distribution of the state space S.

A body of responses for the state space S is constituted by:

1) A set of processes that associate value of the two behaviours in the form $if\ y_t = h_i''\ then\ s_t\ is\ A_i^s$.

2) A probability distribution of the response space H''.

A multivalued mapping from a probability space S to a probability space H'', which is referred to as the frame of behaviour, associates each element in S with a set of elements in H'', that is

$$g_D^r : S \to 2^{H''}$$

The image of an element h in H'' under the mapping is denoted as the kernel of h, K(h).

Definition 17: *An element $h \in S$ is said to be compatible with an element $h'' \in H''$, if it is possible that h' is a stimulus to S and h is a response to H'' at the same interval of time* $\left[t_0, t_f\right]$.

Alternatively, the multivalued mapping can be viewed as a compatibility relationship between their elements

$$K(h) = \begin{Bmatrix} h'' \mid h'' \in H'', h' \text{ compatible with} \\ h'' / (h, h'') \in r_t \end{Bmatrix}$$

Given a probability distribution of space S and a comparability relation between S and H'', a basic probability assignment (bpa) of space H'', denoted by *m*': $2^{H''} \to [0,1]$ is induced

$$m'(A'') = \frac{\sum_{K\left(h_i\right)=A''} p\left(h_i\right)}{1 - \sum_{K\left(h_i\right)=\varnothing} p\left(h_i\right)}$$

Subsets of S and H'' with nonzero basic probabilities are called genon focal elements (gfe).

Let be a subset A'' of H'' and let be a subset *AS* of S. The basic probability assignment m determines two functions, that measure the minimal and maximal degree of fulfiment of a state-response process are:

$$\min \Pr(A'') = \sum_{A^S \subseteq A''} m\left(A^s\right)$$

$$\max \Pr(A'') = \sum_{A^S \cap A'' \neq \varnothing} m\left(A^s\right)$$

If m_3, m_4 are two basic probabiliy assignments induced by two independent sources of stimulus, they can be combined using Dempster theorem for the state response process such as

$$m_3 \oplus m_4(C) = \frac{\sum_{A_i^s \cap A_j^s = C} m_3\left(A_i^s\right) m_4\left(A_j^s\right)}{1 - \sum_{A_s^s \cap A_j^s \neq \varnothing} m_3\left(A_i^s\right) m_4\left(A_j^s\right)}$$

5. Further Remarks

The theory of the Environment, supposes a systemic conception of the reality on the part of Observer. The object receives and believes two ambient means: the environment stimulus and response, respectively. Stimuli, transitions and responses are physical processes reflected in the Observer's mind, but there is not absolute certainty that this happens this way. Probabilities exist in all the processes. At the same time all the variables that are managed are variable linguistic that can be defined as variables whose you value it plows words or sentences in natural or artificial languages (Grzymala-Busse, 1991; Sastre- Vazquez, 1999; Uso-Domenech, 2000a, b; Villacampa & Uso-Domenech, 1999; Villacampa et al., 1999 a, b). The fulfiment functions stimulus-state and state-response process plows belief functions. The interpretation of a theory of Environment from this point of view is exciting and opens methodological epistemological perspectives. The authors have undertaken this road as part of their mathematical investigation.

Acknowledgments

The authors thank Professor B.C. Patten (Institute of Ecology, University of Georgia, Athens, GA) by the suggestions and interest demonstrated in the applied mathematical development of his own theory of Environment and Professor Daniel Berend (Department of Mathematics and Computer Sciences, University Ben Gurion of the Negev, Israel), for the facilities offered in the theory of belief functions.

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