

Neutrosophic and Modal Components of Relations in Complex Systems

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Abstract Semiotic components in the relations of complex systems depend on the Subject. There are two main semiotic components: Neutrosophic and Modal. Modal components are alethical and deontical. In this paper the authors applied the theory of Neutrosophy and Modal Logic to Deontical Impure Systems.

Keywords: *alethical component, alysidal set, deontical components, Deontical Impure System, modal logic, Neutrosophy*

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1. Introduction

In this work, we are going to deal with a very special type of complex systems: *Deontical Impure Systems* (Nescolarde-Selva et al, 2012^{a,b}; Nescolarde-Selva and Usó-Doménech, 2012; Nescolarde-Selva and Usó-Doménech, 2013^{a,b,c,d,e}; Usó-Doménech and Nescolarde-Selva, 2012; Usó-Doménech and Nescolarde-Selva, 2013). They are *Systems* because there are objects and relations among them. They are *Impure* because these objects are formed by material and/or energy beings. They are *Deontical* because between its relations it has, at least, one that fulfills at least one of the deontical modalities: *obligation, permission, prohibition, and faculty*. We are talking about the human societies. Not a particular society, but to any human society, at any time and place.

1. A **system** is an organization of the knowledge on the part of the subject S that fulfills the following conditions (Nescolarde-Selva and Usó-Doménech, 2013^b):

- The subjective condition.
- The condition of rationality.
- The external condition.
- S knows what there is a system.

The vision of a system interpreted by the set formed by different subjects within the system is determined by the belief system (Nescolarde-Selva and Usó-Doménech, 2013^{a,b,c,d,e}; Nescolarde-Selva, Usó-Doménech and Gash, 2014; Usó-Doménech and Nescolarde-Selva, 2012; Usó-Doménech and Nescolarde-Selva, 2013) that the subjects conceived as true, about themselves, the system and its environment.

2. An **impure set** is whose referential elements (absolute beings) are not counted as abstract objects and have the following conditions:

- They are real (material or energetic absolute beings).

- They exist independently of Subject.
- Subject develops perceptual significances¹.
- True things can be said about them.
- Subject can know these true things about them.
- They have properties that support a robust notion of mathematical truth.

3. An **impure system** is one whose set of elements is an impure set.

4. A **deontical system** is an organization of the knowledge on the part of the subject S that fulfills aforementioned conditions and the following others:

¹ In any process, we can distinguish that it has a signifier as an inherent property, and having significance when it is related to the rest of the processes of the perceived Reality that the Subject considers as a system. Significa esto que todas las estructuras tienen información? The existence of information is independent of the fact that there is a Subject able to decode the message, to which the Subject is attempting to communicate. A esta información objetiva la denominamos significante. This objective information is termed signifier. The information in a message acquires meaning if a Subject decodes the message. A esta información subjetiva la denominamos significancia. This subjective information is termed significance. Therefore, the signifier is an ontic property, considering that the significance will be a system of meaning. The signifier is absolute and infinite, the significance is relative and finite. The signifier comes from Absolute Being and significance generates the relative being. The signifier is interpreted as the material or physical form of the sign and is something that can be caught (perception) by some of the traditional senses of the human being. The significance, on the other hand is a mental construct. In our approach, the signifier has a truth value equal to 1, that is to say, $v(S) = 1$, whereas the

significance has as truth value a real positive number $v(s)$, between 0 and 1, with 0 corresponding to absolute ignorance of the signifier (therefore of the process) and 1 to absolute understanding, that is to say, $v(S) = v(s)$.

A-signifier (A- \square) or the first order signifier is the signifier that is inherent to beings, processes or phenomena of the referring context. B-signifier (B- \square), the second order signifier or connotation, is the signifier of significance s .

a. *Subjects* are the human beings. We can distinguish the subject as observer (subjectively outside the system) and, by definition, is the subject itself, or within the system. In this case, acquires category of object.

b. *Objects* (relative beings) are significances Nescolarde-Selva and Usó-Doménech, 2013; Nescolarde-Selva, Usó-Doménech am Gash, 2014), which are consequence of the perceptual beliefs on the part of the Subject of a material or energetic objects (absolute beings) with certain characteristics.

c. Some existing relations between elements have deontical modalities.

d. There is purpose (purposes).

5. **A Deontical Impure System (DIS)** is a system that meets the conditions of being both impure and deontical system.

The DIS (*Deontical Impure System*) approach is the following one:

1. Objects are *perceptual significances* (relative beings) of material or energetic objects (absolute beings).

2. The set of these objects will form an *impure set* of first order.

3. The existing relations between these relative objects will be of two classes: *transactions* of matter and/or energy and *inferential relations*.

4. Transactions have *alethic modality*: necessity, possibility, impossibility and contingency. They are ontic relations.

5. Ontic existence of possibility causes that inferential relations have *deontical modality*: obligation, permission, prohibition and faculty. They are human relations.

6. We distinguished between theorems (natural laws) and norms (ethical, legislative and customary rules of conduct).

7. Each relation has intensity and direction.

8. Between these relative objects it exists, not an only relation, but *sheaves* of relations and going in both directions, clockwise and no clockwise².

9. The sheaves also have intensity.

10. An inferential relation has *modal and neutrosophic components*.

11. In each sheaf there will be generating and generated relations.

12. Sheaves of both directions between two relative objects form a *freeway*³. Relative objects united by freeways form a chain (network).

13. This network is a chain of transmission of direct or indirect causality. Therefore in our approach network will be denominated chain⁴.

² A sheaf of relations and denoted as h_{ik} is the multiple relations existing between two variables x_i and x_k , $x_i, x_k \in M$. A sheaf is monorelational if there is a single relation between two variables. It is bi-relational, if there are two, and n-relational if there are n relations. The empty sheaf indicates the non-existence of relations between two variables.

³ A freeway between two elements x_i and x_j denoted as Φ_{ij} is the set constituted by the sheaves $d - h_{ij}$, $l - h_{ij}$ and $r - h_{ij}$. We can represent it as $x_i \Leftrightarrow x_j$. Therefore, in a freeway Φ_{ij} there are sheaves of three directions: direct sheaf, reciprocal sheaf and inverse sheaf.

⁴ A chain \wp_i^k will be an abstract chain, the elements or variables of which are related by means of freeways, that is,

$$\wp_\omega^k = x_i \Leftrightarrow x_j \Leftrightarrow \dots \Leftrightarrow x_\omega$$

14. Being all the DIS' objects directly or an indirectly related to each other, it will be formed by a single chain with multiple ramifications.

15. An *Alysidal set* is one whose elements are chains⁵.

16. Coupling functions between Alysidal sets can be established.

17. Nodes are subject. These may be individuals or group of individuals (corporations, regions, states, etc.)

18. Special coupling function of recognition denominated *gnorpsic function* can be established.

19. Gnorpsic function allows operations of connection between systems. Gnorpsic functions involve knowledge and decision.

In a Deontical Impure System we distinguished two main semiotic components of relations: Neutrosophic and Modal components.

2. Neutrosophic components

Plato defines three abstract ideals that must guide the life of the men: kindness, beauty and truth. Both first they

We represent the chain like \wp_i^k , in where the subscript i represents the number of constituent variables (p-significances) of the chain, and the supraindex k an arbitrary number of identification.

In every chain \wp_i^k there will be a number of freeways equal to the number of variables which are components of the chain less one, that is, if the number of variables which are components of the chain is n, the number of freeways will be n - 1.

Each constituent variable will be a *node*.

Each freeway that leaves from a node will form a *branch*.

The initial node will be the *root node*.

The terminal will be an *apical node*. In a chain can have an single node root but several terminals.

The chain having more nodes will be denominated *trunk* and its terminal node will be *top apical node*.

Chains whose root node is connected by means of a freeway with the apical node are *cyclical chains*.

⁵ The alysidal set is the set whose elements are chains formed by relative beings united by freeways of inferential relations and/or transactions.

The alysidal sets has the following properties:

a. A relative object (p-significance) considers a monochain, that is to say, element of an alysidal set.

b. An alysidal set can be considered like a special class of system in where their elements (chains) are not interrelated.

c. Each alysidal element can be considered either as a system in itself or a subsystem.

d. There is emptiness alysidal set \emptyset .

e. For an alysidal set A_{al} , the difference $U - A_{al}$, where U is the universe of discourse, is called the complement of A and it is denoted by A_{al}^C .

Thus A_{al}^C is the set of everything that is not in A_{al} .

f. An ordered pair is a pair of alysidal elements with an order associated with them. If alysidal element are represented by \wp_i^k

and \wp_j^l , $i \begin{cases} = j \\ \neq j \end{cases}, k \neq l$, then we write the ordered pair

as (\wp_i^k, \wp_j^l) . Two ordered pairs (\wp_i^k, \wp_j^l) and (\wp_n^u, \wp_m^v) are equal if and only if $\wp_i^k = \wp_n^u$ and $\wp_j^l = \wp_m^v$.

g. Let A_{al} and B_{al} be two alysidal sets. The set of all ordered pairs (\wp_i^k, \wp_j^l) , where \wp_i^k is an element of A_{al} and \wp_j^l is an element of B_{al} , is called the Cartesian product of A_{al} and B_{al} and is denoted by $A_{al} \times B_{al}$.

are properties or qualities of the man and the things. Nevertheless, the truth is not a property. It is a characteristic or quality of the enunciations, judgments, propositions, theorems, laws, that are declarations as well. The truth is a semiotic property of the propositions. Propositions can be true (or false) of different ways or in different senses. It will depend on the type of established proposition. LeShan and Margeneau (1982) establish three types of propositions (and therefore of truths):

Empirical proposition: When the proposition and its associate truth are in agreement with the perception (*perceptual experience*). The empirical truth will depend on outer tests on the content of the propositions.

Analytical proposition: It is that fundamental consequence of certain axioms or assumptions. The veracity (truth values) is contained in the same proposition. The logical proposition belongs to this group, but also the theological ones. The axioms determine the veracity. Therefore, the truth is within the system of beliefs derived from that particular logic.

Scientific proposition: They are those that combine the analytical truth derived from reasonable axioms with the empirical truth. They derive from validated and accepted theories and that they are logical or mathematical constructions related, which have equipment connections with the perceptual experience through correspondence rules.

We are based on the denominated neutrosophic logic (Gershenson, 2001; Liu, 2001^{a,b}; Smarandache, 1999, 2003; Smarandache, Dezert, Buller, Khoshnevisan, Bhattacharya, Singh, Liu, Dinulescu-Campina, Lucas, Gershenson, 2001) whose characteristics are:

The Main Principle: *Between an idea <A> and its opposite <Anti-A>, there is a continuum-power spectrum of neutralities <Neut-A>.*

Definition 1 (Robinson, 1996): A number x is said to be infinitesimal iff for all positive integers n one has

$$|x| < \frac{1}{n}.$$

Let $\varepsilon > 0$ be such infinitesimal number.

Definition 2 (Robinson, 1996): The non-standard finite numbers $1^+ = 1 + \varepsilon$, a number where 1 is its standard part and ε its non-standard part.

The number 1^+ is infinitely small but greater than 1.

Definition 3 (Robinson, 1996): The non-standard finite numbers $\bar{0} = 0 - \varepsilon$ a number where 0 is its standard part and ε its non-standard part.

The number $\bar{0}$ is infinitely small but less than 0.

Definition 4: The non-standard unit interval is the interval $]^{-}\bar{0}, 1^+ [$.

Numbers $\bar{0}$ and 1^+ belong to the non-standard unit interval.

The Fundamental Thesis of Neutrosophy: Any idea $\langle A \rangle$ is $T\%$ true, $I\%$ indeterminate, and $F\%$ false, where $T, I, F \subset]^{-}\bar{0}, 1^+ [$. and such as

- a) $T \subset]^{-}\bar{0}, 1^+ [$.
- b) $I \subset]^{-}\bar{0}, 1^+ [$
- c) $F \subset]^{-}\bar{0}, 1^+ [$

with

$$\begin{aligned} \sup T &= t_{\sup}, \inf T = t_{\inf} \\ \sup I &= i_{\sup}, \inf I = i_{\inf} \\ \sup F &= f_{\sup}, \inf F = f_{\inf} \\ n_{\sup} &= t_{\sup} + i_{\sup} + f_{\sup} \\ n_{\inf} &= t_{\inf} + i_{\inf} + f_{\inf} \end{aligned}$$

Although T, I, F can be intervals, any real sub-unitary subsets: discrete or continuous, single-element, finite or infinite, union or intersection of various subsets, etc, in the theory exposed here, we will consider them like intervals.

The Neutrosophic components T, I, F are at each instance dependant on many parameters, and therefore they can be considered set-valued vector functions or even operators. The parameters can be: time, space, etc. and of hidden or unknown variables, such as:

$$\begin{aligned} T(s, t, w_1, w_2, \dots, w_n), I(s, t, w_1, w_2, \dots, w_n), \\ F(s, t, w_1, w_2, \dots, w_n). \end{aligned}$$

T, I and F try to reflect the dynamics of ideas, significances and propositions.

T, I and F try to reflect the dynamics of ideas, significances and propositions.

Only in the third type of propositions one occurs: $(T_{sur}, I_{sur}, F_{sur})$, $\inf T_{sur} \geq 1$, $\sup F_{sur} \leq 0$, e.g., it

corresponds to Alethic modality of the necessity and to the surely probabilistic event. With respect to second classes, the analytical proposition, its truth will depend on its context, is to say of its logical system. In another logical system, it will lack true value. In the present state of our approach, we will not distinguish between the three truths and we will suppose each proposition (inferential relation) equipped with the three-neutrosophic components.

The Main Laws of Neutrosophy: Let $\langle a \rangle$ be an attribute, and $(T, I, F) \subset]^{-}\bar{0}, 1^+ [$. Then:

There is a proposition $\langle P \rangle$ and a referential system $\{R\}$, such that $\langle P \rangle$ is $T\%$ $\langle a \rangle$, $I\%$ indeterminate or $\langle Neut-a \rangle$, and $F\%$ $\langle Anti-a \rangle$.

For any proposition $\langle P \rangle$, there is a referential system $\{R\}$, such that $\langle P \rangle$ is $T\%$ $\langle a \rangle$, $I\%$ indeterminate or $\langle Neut-a \rangle$, and $F\%$ $\langle Anti-a \rangle$.

$\langle a \rangle$ is at some degree $\langle Anti-a \rangle$, while $\langle Anti-a \rangle$ is at some degree $\langle a \rangle$.

Let \aleph be the Reality, \beth being a part thereof, such that $\beth \subset \aleph$. Let S be a Subject, conceiving the Reality through his doxical filter, made up of the own beliefs system \mathbb{F} of his culture, and by a certain language L . Subject S is in a certain psychic state of organization of the Reality during a determined objective temporary interval $[t_0, t_n]$. In our approach:

All inferential relation in a referential system (DIS) Σ is a proposition $\langle P \rangle$.

The proposition $\langle P \rangle$ is $T\%$, $I\%$ indeterminate, and $F\%$.

This representation characterizes the imprecision of knowledge or linguistic inexactitude, due to the *Principle of Semiotic Incompleteness*⁶, received by one or various Subjects. The sources of uncertainty can be:

⁶ Semantic Incompleteness Principle (Nescolarde-Selva and Usó-Domènech. 2013^{a,b}; Nescolarde-Selva, Usó-Domènech and Gash, 2014;

Stochasticity: the case of intrinsic imperfection where a typical and single value does not exist.

Incomplete knowledge: ignorance of the totality, linguistic inexactitude, limited view on a system because of its complexity.

Acquisitions errors: intrinsically imperfect observations, the quantitative errors in measures.

In addition, it leads us to the own probability:

The *objective probability* process uncertainty of random type (stochastic) introduced by the chance.

We will interpret, of intuitive way, the *subjective probability* of an event like the belief degree in that this one happens when the random experiment is made. Nevertheless, it has been considered often that the probability is simply the belief degree that is due to assign to a proposition. The probability of occurrence of an event is the degree of belief on the part of an individual that an event happens, based on all the evidence to its disposition. Under this premise it is possible to be said that this approach is adapted when single is an opportunity of occurrence of the event. E.g., that the event will happen or it will not happen that single time. The value of probability under this approach is a personal judgment.

Vagueness is another form of uncertainty is the character of those which contours or limits lacking precision, clearness, etc.

Definition 5: The indeterminacy I is the degree of uncertainty, vagueness, imprecision, undefined, unknown, inconsistency and redundancy.

Consequence 1: The subjective probability will measure indeterminacy.

Let R be generated relation and r_i the n generating relations. T, I and F they are respectively the probabilities really, indetermination and falsification of one relation. Applying the theorem of Bayes, we will be able to obtain the respective probabilities of the generated relation that is conditioned by the generating relations, independent between it.

$$T(R) = \sum_{i=1}^n T(R|r_i)I(r_i),$$

$$I(R) = \sum_{i=1}^n I(R|r_i)I(r_i),$$

$$F(R) = \sum_{i=1}^n F(R|r_i)I(r_i).$$

Therefore we will have each generated relation will have the three neutrosophic components

$$\begin{aligned} & (T(R), I(R), F(R)) \\ &= \left(\begin{array}{l} \sum_{i=1}^n T(R|r_i)T(r_i), \sum_{i=1}^n I(R|r_i)I(r_i), \\ \sum_{i=1}^n F(R|r_i)F(r_i) \end{array} \right) \end{aligned}$$

and so that.

$$(\inf T(R) + \inf I(R) + \inf F(R)) \geq -0$$

$$(\sup T(R) + \sup I(R) + \sup F(R)) \leq 3^+$$

Let R_1, R_2 be two independent relations of same sheaf h, so that their neutrosophic probability is

$$P(R_1) = (T(R_1), I(R_1), F(R_1));$$

$$P(R_2) = (T(R_2), I(R_2), F(R_2))$$

Then:

$$\begin{aligned} & (T(R_1), I(R_1), F(R_1)) \oplus (T(R_2), I(R_2), F(R_2)) \\ &= (T(R_1) \oplus T(R_2), I(R_1) \oplus I(R_2), F(R_1) \oplus F(R_2)) \\ & (T(R_1), I(R_1), F(R_1)) \langle - \rangle (T(R_2), I(R_2), F(R_2)) \\ &= (T(R_1) \langle - \rangle T(R_2), I(R_1) \langle - \rangle I(R_2), F(R_1) \langle - \rangle F(R_2)) \\ & (T(R_1), I(R_1), F(R_1)) \otimes (T(R_2), I(R_2), F(R_2)) \\ &= (T(R_1) \otimes T(R_2), I(R_1) \otimes I(R_2), F(R_1) \otimes F(R_2)) \end{aligned}$$

and

$$P(R_1 \cap R_2) = P(R_1) \otimes P(R_2)$$

$$= \left(\begin{array}{l} T(R_1) \otimes T(R_2), \\ I(R_1) \otimes I(R_2), \\ F(R_1) \otimes F(R_2) \end{array} \right)$$

$$P(R_1 \cup R_2) = P(R_1) \oplus P(R_2)$$

$$\langle - \rangle P(R_1) \otimes P(R_2)$$

$$= \left[\begin{array}{l} T(R_1) \oplus T(R_2) \langle - \rangle T(R_1) \otimes T(R_2), \\ I(R_1) \oplus I(R_2) \langle - \rangle I(R_1) \otimes I(R_2), \\ F(R_1) \oplus F(R_2) \langle - \rangle F(R_1) \otimes F(R_2) \end{array} \right]$$

Let us suppose the case of sheaf h formed by three independent relations R_1, R_2, R_3 . Then:

$$P(h) = (T(h), I(h), F(h)) = P(R_1 \cup R_2 \cup R_3)$$

Then

$$T(h) = T(R_1) \oplus T(R_2) \oplus T(R_3) \langle - \rangle T(R_1)$$

$$\otimes T(R_2) \langle - \rangle T(R_1) \otimes T(R_3) \langle - \rangle T(R_2)$$

$$\otimes T(R_2) \oplus T(R_1) \otimes T(R_2) \otimes T(R_3)$$

$$I(h) = I(R_1) \oplus I(R_2) \oplus I(R_3) \langle - \rangle I(R_1)$$

$$\otimes I(R_2) \langle - \rangle I(R_1) \otimes I(R_3) \langle - \rangle I(R_2)$$

$$\otimes I(R_2) \oplus I(R_1) \otimes I(R_2) \otimes I(R_3)$$

$$F(h) = F(R_1) \oplus F(R_2) \oplus F(R_3) \langle - \rangle F(R_1)$$

$$\otimes F(R_2) \langle - \rangle F(R_1) \otimes F(R_3) \langle - \rangle F(R_2)$$

$$\otimes F(R_2) \oplus F(R_1) \otimes F(R_2) \otimes F(R_3)$$

Generalizing for sheaf h constituted by n independent relations:

$$h = \bigcup_{k=1}^n T(R_k),$$

then

Usó-Domènech and Nescolarde-Selva. 2012;): It is not possible to totally characterize a structure of objects or processes with a language (formal or not), or to completely present a portion of "truth" that this language can express about these objects or processes through deductive operation.

$$\begin{aligned}
T(h) &= \sum_{k=1}^n T(R_k) \langle - \rangle \sum_{i<j}^n (T(R_i) \otimes T(R_j)) \\
&\oplus \sum_{i<j<k}^n (T(R_i) \otimes T(R_j) \otimes T(R_k)) \\
&\langle - \rangle \dots \oplus (-1)^{n+1} \prod_{k=1}^n T(R_k) \\
I(h) &= \sum_{k=1}^n I(R_k) \langle - \rangle \sum_{i<j}^n (I(R_i) \otimes I(R_j)) \\
&\oplus \sum_{i<j<k}^n (I(R_i) \otimes I(R_j) \otimes I(R_k)) \\
&\langle - \rangle \dots \oplus (-1)^{n+1} \prod_{k=1}^n I(R_k) \\
F(h) &= \sum_{k=1}^n F(R_k) \langle - \rangle \sum_{i<j}^n (F(R_i) \otimes F(R_j)) \\
&\oplus \sum_{i<j<k}^n (F(R_i) \otimes F(R_j) \otimes F(R_k)) \\
&\langle - \rangle \dots \oplus (-1)^{n+1} \prod_{k=1}^n F(R_k)
\end{aligned}$$

This probability of relations and sheaves uses a subset-approximation for the truth-value like imprecise probability, but also subset-approximation for indeterminacy and falsity values. Also, it makes a distinction between *relative sure relation*, relation which is sure only in some particular world (s): $P(\text{rsr}) = 1$, and *absolute sure relation*, relation which is sure in all possible worlds: $P(\text{asr}) = 1^+$; similarly for *relative impossible relation* and *absolute impossible relation* and for *relative indeterminate relation* and *absolute indeterminate relation*.

3. Modal Components

The inferential relations express the logical relation denominated inference, e.g., they indicate that the sequence in which it is integrated, will really have a value as long as the expressed thing in the previous sequence is fulfilled. Halliday and Hassan (1976) formulate it of the following way: *'possibly a if it is thus, then b'*. The hypothetical inference has not necessary but merely probable character, and is also a type of synthetic or enlarging reasoning. Hypotheses can very be varied, but they have in common the one that are formulated *to explain* an observed phenomenon. Peirce (Haack, 1993; Murphey, M.G. 1993; Peirce, C.S., 1870) establishes at least three types:

About organizations or facts no observed at the moment for formulating the hypothesis, but observable in the future verifying it.

About organizations or facts that somebody could observe, although at the moment it is impossible to repeat the observation, since they are done of the past. They are observable in principle, but inobservables organizations or facts actually to belong to the past. It is a frequent case in sciences of the nature. But the hypothesis is not a type of

exclusive reasoning of natural sciences. In human sciences also hypotheses on the past explaining what are formulated we know of the present.

About organizations or facts that are inobservables actually and also in principle, because they are beyond the perceivable thing directly by the senses. In agreement with Peirce, therefore, the scientific activity does not respond to an exclusively positivist model that it only admits like organizations or real facts those that are directly observable. The scientist resorts constantly to hypothesis about inobservables realities to explain the observed realities, so that, without losing the connection with the sensible experience, he extends looking for it his rationality.

Induction and hypothesis look like in their enlarging character, as soon as that both extend the knowledge beyond merely observed: individuals or characters (induction and hypothesis respectively). In that they are distinguished of the deduction that has explanatory character merely. However, induction and hypotheses are two different ways of enlarging reasoning. By means of the induction, we concluded that made similar to the observed facts they are true in no examined cases. By means of the hypothesis, we concluded the existence of a fact very different from the entire observed one, from which, according to the known laws, would be necessarily something observed. The first one is a reasoning of the individuals to the general law; the second, of the effect to the cause. The first one classifies, the second explains. Induction and hypothesis are separated forms of inference: it is impossible to infer hypothetical conclusions inductively.

Inferential relations imply ontic signs and flows of signals which take semantic meaning within the established habitual epistemic forms between interactive pairs from s-impure object set. Se entiende por categorías los *géneros supremos o más universales de los entes que se pueden predicar de algún sujeto*. Categories are understood to be the supreme or universal genres of the entities, which may be predicated from any subject. De manera que cada categoría viene a ser una idea universal debajo de la cual se contienen varias ideas relacionadas contenidas bajo la primera. So that each category is a universal idea beneath which various related ideas are contained under the first. De aquí se infiere que la categoría puede tomarse, o bien por el género supremo de una clase determinada de seres, o bien por la serie o colección de géneros y especies que se contienen y colocan bajo un género supremo. From this it may be inferred that the category may be taken, either by the supreme genre of a specific class of beings or either by the series or collection of genres and species, which are contained and placed under a supreme genre. Toda vez que las categorías no son otra cosa en el fondo sino las varias clases de seres o realidades que pueblan y constituyen la Realidad, se sigue de aquí: As the categories are simply nothing more in fact than various classes of beings or realities which people and constitute Reality, it follows from here that

Las categorías son divisiones del ente actual creado. Categories are divisions of the present entity created. En todas las categorías hay algo en que convienen, y algo en que se diferencian. In all categories there is something on which they agree and something on which

they differ: convienen entre sí en cuanto que toda categoría significa una *realidad objetiva, una cosa* con ella, pero menos universales, formando una serie o colección ordenada de (*res*) una *esencia real*: They agree in that every category means an *objective reality, a thing* with it, yet less universal, forming an ordered collection or series of. a *real essence (res)*: se diferencian entre sí, en cuanto que cada esencia categórica tiene un *modo de ser especial*. They differ from each other in that each categorical essence has a *means of being special*.

El estudio de la lógica modal ha tenido un enorme desarrollo y se ha ensanchado el campo de lo que haya de interpretarse como su tema propio. G.H. von Wright (1971) distingue varias "*familias*" de conceptos modales, sugiriendo que el campo de la comprensión de la modalidad está en crecimiento. The study of modal logic has developed enormously and has broadened the field of what should be interpreted as its own subject. Von Wright (1971) distinguishes various "*families*" of modal concepts, suggesting that the field of comprehension of modality is growing. Distinguiremos: We shall distinguish:

Los modos *aléticos* ("*posible-necesario*"). *Alethical modes (possible-necessary-impossible-contingent)*.

Los modos *deónticos* (*obligación-permisión-prohibición*). *Deontical modes (obligation-permission-prohibition-faculty)*.

Las ideas *dóxicas* (*conocimiento-duda-creencia-incertidumbre*). *Doxical modes (knowledge-doubt-belief-uncertainty)*.

Los modos *epistémicos* (*verificado, no decidido, falsificado*). *Epistematical modes (verified- undecided-falsified)*.

All the families having these structural affinities could be termed modal concepts and it is possible to speak of their formal study as generalised modal logic. In the same way we could speak of modal systems being those which in any of their relations have at least one of those categories or that the Subject conceiving it should use modal concepts. In our approach we will distinguish two main classes of modality: alethical (ontic) and deontical (semiotic).

3.1. Alethical components

Alethical modal components constitute the bottom drop curtain or substratum of the DIS. They are "natural" modalities, in the form sense they leave from the theorems or natural laws. Alethical modality constitutes an only concept, that it is possible to be outlined of the following way:

Necessity (n) \square Impossibility (i) $\neg\Diamond$

Contingency (c) $\neg\square$ Possibility (p) \Diamond

The two modalities of each column (n and c, i and p) form a *modal alethical opposition*, e.g., they are excluded in extension and they are implied in comprehension.

Let \Updownarrow be the Mutual true exclusion and \Leftrightarrow be the Reciprocal implication. Then:

$$\begin{aligned} \square r \Updownarrow \neg\square r, \square r \Leftrightarrow \neg\square r \text{ sphere of the necessity} \\ \neg\Diamond r \Updownarrow \Diamond r, \neg\Diamond r \Leftrightarrow \Diamond r \text{ sphere of the possibility} \end{aligned}$$

Both component of a line (n and i, c and p) they do not constitute an opposition. Forward edge (n and i) belongs to *the sphere of the necessity*. If r constitutes an event, a fact of the phenomenon, a property of the object or an inferential relation in our theory, we have in classic logic: $r_{\neg\Diamond} \Leftrightarrow \neg r_{\square}, r_{\square} \Leftrightarrow \neg r_{\neg\Diamond}$. That is, the impossibility of r is equivalent to the necessity of no-r. With respect to the second line (c and p), it belongs to *the sphere of the possibility* sight that *the contingency implies the pluripossibility*. And, therefore, the compossibility of r and no-r: $r_{\neg\square} \Leftrightarrow (r_{\Diamond} \wedge \neg r_{\Diamond})$

Inversely, the possibility of r or goes jointly with the one of no-r, and r is contingent, or no, to knowing no-r is impossible, and then r is necessary by virtue of $\neg r \Leftrightarrow r_{\square}$.

$$\Diamond r \Leftrightarrow (\square r \Updownarrow \neg\square r)$$

Let us see the first diagonal (n and p). The necessity of r (excluding the one from no-r), is equivalent to *the unipossibility* of r, therefore *necessity implies classically possibility*: $\square r \Rightarrow \Diamond r$.

It is a univalent possibility, against the pluripossibility of the contingency. Inversely, the possibility is against weakly to the necessity due to partial consubstantiality with the contingency. On the other hand, the possibility is also against weakly to the contingency by its partial identity with the necessity..

With respect to the second diagonal (i and c), it contains a strong modal opposition: the impossibility, whereas negative necessity is totally opposite to the contingency. In short, the impossibility strongly is against the other three poles of the concept: *an impossible thing is expelled from the Reality* whereas the other three poles stay within the Reality. In addition, this last ontic opposition, is not own of the classic logic.

Whereas category, and in agreement with Hegel, the necessity implies the contingency already because it forms a bipole, because the synonymous of the necessity is not-contingent and reciprocally.

We will notice that *the dominion of compossibility and its paper of contingency, determined accurately the limited and determined necessity, when drawing up the border that separates it of the impossibility*. Reciprocally, *all concrete contingencies imply necessities that determine their field of compossibility rigorously*. Possibility is *compossibility, e.g., compatibility of A with other terms or connections of terms taken like reference..*

The same negative definition of the possibility idea as "*absence of contradiction*" only in this context reaches some sense, because a "*absence of contradiction*", thought absolutely, does not mean anything; nor, therefore, the call means nothing "*logical possibility*" that many define indeed by the "*absence of contradiction*". It has to sobrentender itself like "*absence of contradiction of something*" (of A); but this something must be given like complex. Otherwise: absence of contradiction, since everything what can be thought is complex, stops being a negative-absolute concept and it are pronounced like contextual.

The "*absolute possibility*" is therefore a development limit of the idea of compossibility (*compossibility of A with same itself*) that will only have a differential meaning if it assumes that Á is simple and therefore, unthinkable; then

if A is complex, when "relating it to same itself" we are unavoidably inserting it in outer contexts, through multiple components. The idea of possibility is, therefore, based on the operations by which we constructed the concept of A; but this is not applied to the operations, but to the constructed objects and in relation to other objects, as *system*. E.g., the possibility is *objective*. The formal-modal logical concept of possibility is obtained applying this same idea of compossibility, and with no need to appeal (at the moment, and at least) to possible worlds.

Let r be an inferential relation. We will define the following axioms:

Axiom 1: The possibility of an inferential relation implies its existence: $\diamond r \rightarrow r$.

Axiom 2: The necessity of an inferential relation implies its existence: $\Box r \rightarrow r$.

Axiom 3: The possibility of an inferential relation implies the necessity of its possibility: $\diamond r \rightarrow \Box \diamond r$.

Axiom 4: The necessity of an inferential relation implies its possibility. $\Box r \rightarrow \diamond r$.

Axiom 5: The not-possibility of an inferential relation implies the necessity of its not-existence: $\neg \diamond r \rightarrow \Box \neg r$.

Axiom 6: The not-possibility of the not-existence an inferential relation implies the necessity of its existence: $\neg \diamond \neg r \rightarrow \Box r$.

Axiom 7: The not-necessity of the existence an inferential relation implies the possibility of its not-existence: $\neg \Box r \rightarrow \diamond \neg r$.

Axiom 8: The not-necessity of the not-existence of an inferential relation implies the possibility of its existence: $\neg \Box \neg r \rightarrow \diamond r$.

3.2. Deontical components

Deontical modal components are own, in first instance, of the existence of the life, at least of organized life and developed to the end, of the existence of the human being. Let r be an inferential relation. The operator O who means "obligatory" is that it does possible to describe acts or propositions like obligatory. From the operator of obligation and the logical negation it is possible to define the operators of prohibition (Ph) and permission (P):

$$Or \equiv Ph\neg r \equiv \neg P\neg r$$

Whose reading is: "(Obligatory r) iff (prohibited non- r) iff (not allowed non- r)".

We may represent this last phrase of the following way (where G is a constant that means, "influences", it is an individual of which the previous thing is preached and \rightarrow it is the conditional material) $Or \rightarrow \Box(Ga \rightarrow r)$.

If S means the fact that the norm determined in the inferential relation has been violated, then: $Op \rightarrow \Box(\neg p \rightarrow S)$.

The rule of not monotony is the coherence exigency according to which a valid inference is not less valid by the addition of new premises:

$$\frac{Or_1 \rightarrow Or_2 \quad Or_1 \wedge P_{r3}}{Or_3}$$

The operator of faculty $Fr \equiv Pr \wedge P\neg$ is interpreted like "(Facultative r) iff (Allowed r and allowed not r)".

The operator of faculty seems more suitable to express the following consideration: "Subject S is free to consider

the inferential relation r ". It would be: "the conduct to consider the inferential relation r is facultative" or "It is facultative that is expressed the inferential relation r " or, which is the same, "they are allowed both conducts: considering and not considering the inferential relation r ".

We will establish the following table of equivalences:

Table 1.

$Or \equiv Ph\neg r \equiv \neg P\neg r$
$O\neg r \equiv Phr \equiv \neg Pr$
$\neg O\neg r \equiv \neg Phr \equiv Pr$
$\neg Or \equiv \neg Ph\neg r \equiv P\neg r$

Principle of permission: $Pr \vee P\neg r$ and it is interpreted as about an act, on the part of the Subject, to infer a relation (or a proposition concerning an inferential act), either this one is allowed or allowed its negation.

Principle of deontical distribution: $P(r_1 \vee r_2) \equiv Pr_1 \vee Pr_2$ and it is interpreted as the statement according to which the disjunction of two acts to infer a relation on the part of the Subject is allowed is equivalent, as well, to the disjunction of two statements: the one that affirms that the first act is allowed and the one that affirms that the second act is allowed.

This last principle is written sometimes: $O(r_1 \wedge r_2) \equiv Or_1 \wedge Or_2$.

Table 2.

$\neg r_1 \rightarrow (r_1 \rightarrow Or_2)$
$Or_1 \rightarrow (r_2 \rightarrow Or_1)$
$O\neg r_1 \rightarrow O(r_1 \rightarrow r_2)$
$Or_1 \rightarrow O(r_2 \rightarrow r_1)$

Table 3.

$\neg r_1 \rightarrow \neg r_1 \vee Or_2$
$Or_1 \rightarrow \neg r_2 \vee Or_1$
$O\neg r_1 \rightarrow O(\neg r_1 \vee r_2)$
$Or_1 \rightarrow O(\neg r_2 \vee r_1)$

3.3. Relation between Alethic and deontical components

Strictly speaking, *the obligatory thing cannot be necessary* according to the sense of the necessary thing previously expressed. The obligatory concept belongs to the semantic constellation of the ethics, moral, etc, e.g., of the ideological belief systems to which the Subject belongs, and that nothing has to do with the expressed synthetic identities in a theorem. The dichotomy between the semantic and ontic plane must be dissolved, because all semantic is ontic since the words (or the signs) also are made physical, although "artificia", worked and selected by the human species. It makes no sense to force planets to draw ellipses around the sun. It does not have sense either to say that the planets describe those orbits forced by the law of the gravitation, but that the law of the gravitation, in any case, explains, *propter quid*, a phenomenon that already was well-known previously (Kepler) to the formulation of this law. Possible solution to this type of arguments happens to establish a mixed, Alethic-deontical logic in where some - all alethical axioms have not deontical costories that can also continue staying like principles in the deontical context. The reason, *ad hoc* elaborated is that, in deontical logic is not necessary to admit like axiom that *the obligation must be allowed*, which, in alethical terms, is absurd: *The necessity*

implies the possibility. This incongruence has not to be understood like paradox, or like mere gratuitous reconstruction and *ad hoc*. The true reason sublies in the necessity to save the phenomena, in this case: the analogy of which part between Alethic and deontical terms. But the possibility of denying the same analogy is not cancelled this way. The correspondence (of Alethic and deontical terms) between the worldly uses of these two classes of concepts (*ontic and semiotics*) does not constitute, seems to us either, a reason sufficient to maintain the analogies at all costs to begin with. *Necessity* no longer talks about the property of the parts of a discourse, but to the property added to the real existence of a cognoscible being if we come *regressively* from finite and contingent beings.

On the other hand, if this analogy between necessity and obligation is subadded in an inequality analogy: the one that it mediates between natural (ontic) laws and

normative rules (*univocal from the perspective logical, ambiguous from the philosophical one*). Then, not even it is such analogy: Cannot be disobeyed ontic laws (theorems) but, in any case, be controlled by means of other laws, also ontic. Normative rules (norms, no theorems) estimate, of necessary way, the possibility of failing to fulfill them. *Normative Law* (rules) and *Natural Law* (theorems) is not analogous, but sintagmas including an ambiguous concept, no analogous to that, granting much, we can metaphorically interpret.

Ontic possibility (Alethic modality) creates deontic modalities. In the human individual, the free will needs two components: possibility and decision (faculty). Human colectivity is the interaction between multiple individualities, and in there decisions these two modalities sublies. We are going to summarize this fact in the following figure (figure 1):

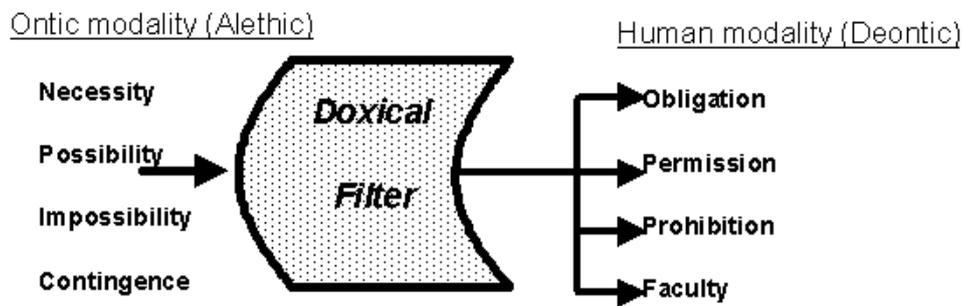


Figure 1. Alethical and deontical components

It is the field of the possibility, where the Subject S conceives the deontical components, and where it infers the relations that characterize their peculiar vision of the Reality conceived like system.

In a freeway, we will find transactions and inferential relations. We will have to distinguish between two classes of transactions: necessary transactions and allowed transactions.

The first one is not influenced by human decision: we cannot prevent that the Sun illuminates the Earth or the continue bombing of cosmic rays. Theorems (natural laws) are strictly necessary. We can break neither the law of gravity nor the second principle of thermodynamics.

In the second one (*allowed transactions*), its necessity is in conditional favour of deontic modality. For example, processes that are made within an atomic reactor in a nuclear power station are natural laws (theorems). Nevertheless, so that it happens will depend that a government forces the construction of the power station, or allows or prohibits it. Or of the facultative decision to ignite or not the reactor. And thus many examples.

3.4. Relation between Semiotic Components

It is possible to establish a relation between Alethic, deontical and neutrosophic components (Table 4):

Table 4.

Alethic components	Deontical components	Probability theory
Necessity	Obligation	Sure event
<i>Impossibility</i>	<i>Prohibition</i>	<i>Impossible event</i>
<i>Possibility</i>	<i>Permission</i>	<i>Totally indeterminate event</i>
<i>Contingence</i>	<i>Faculty</i>	<i>Chance</i>

4. Conclusions

A Deontical Impure System will have the following properties:

They are objectively diachronic, that is to say, they are born, evolve and die in a Newtonian period $[t_0, t]$.

They are subjectively diachronic, that is to say, it exists an own subjective time of the system $[t_0, t]_S$, and so that $[t_0, t] \subset [t_0, t]_S$.

DIS are open systems, that is to say, two exist environments, stimulus environment H' and response environment H'.

Both environments are systems also, but for subjects pertaining to DIS avoid this structure, for that reason they are possible to be considered like Alysidal sets.

Interactions between the system and its environments exist. These interactions will be transactions and inferential relations. The transactions will be necessary, distinguishing between the ontically necessary transactions and deontically necessary transactions.

Some of these transactions are contingent. Then, phenomena of fortuitous interaction of unforeseeable consequences for the DIS. take place.

We have used, among other tools, the modal logics (*alethical and deontic*), Neutrosophic and epistemic logic (*beliefs*). However, the subject is very far from being

closed. In addition, we expose the following considerations:

The permission (or the obligation) of a response depends of the relationships among the objects, the state and the knowledge about this state.

The value of a certain response, not only depends on the denoted response and the meaning of allowed, but also of the moment when this response is expressed. The response will be probably allowed today, but that do not mean to be always accepted⁷. We should still guarantee the complete formalization of this interpretation with a formal semantics such as 'possible worlds'.

It is not always necessary that a Subject S be able to say if a response is allowed or forbidden with regard to certain state of the system. A language should not be reduced to a single function of referring with regard to a factual or counterfactual world. Formalization of DIS by means of logical language demands that this last one be sufficiently expressive to reflect all the subtleties of the reality. In other words, these logical languages should be able of reflecting all the extra-referential functions of the system.

A semiotic theory of systems derived from the language (DIS belongs to this class) would have therefore the purpose of classifying all the systems of linguistic expression: philosophy, ideology, myth, poetry, art, as much as the dream, lapsus, the free association in a pluridimensional matrix where will be interfered much diversified fields. In each one of these discourses is necessary, in effect, to consider a plurality of questions, the essence of which will only be comprehensible by the sum of all; it will be necessary to ask, in the first place, as it will be the purpose of this language, the function that fulfills, the reason by which has been constructed.

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⁷ Theorem of Non Wished Effects (NWET) (Usó-Doménech and Nescolarde-Selva, 2012, 2013) derived of Gödel theorem had demostred that the goal of reducing Reality to systemic conception (models) cannot be totally reached. For each constructed systemic conception, can happen to it one of the two following things:

Either some allowed responses are not produced or
Else some forbidden responses are produced.

What would it mean to say that Reality is reduced to a given systemic conception? It would mean that system produces as response each allowed response of the Reality, but also forbidden responses for the system. That is to say: any allowed response is produced from the system but that forbidden response is so produced. To the forbidden responses produced by the system we will denominate nonwished effects.

In economics are often called "perverse effects". In the social sciences are *unintended consequences* (sometimes unanticipated consequences or unforeseen consequences) are outcomes that are not the ones intended by a purposeful action. The concept has long existed but was named and popularized in the twentieth century by American sociologist Robert K. Merton. As example prohibition in the 1920s the USA, originally enacted to suppress the alcohol trade, drove many small-time alcohol suppliers out of business and consolidated the hold of large-scale organized crime over the illegal alcohol industry. Since alcohol was still popular, criminal organizations producing alcohol were well funded and hence also increased their other activities. Similarly, the war on drugs, intended to suppress the illegal drug trade, instead consolidates the profitability of drug cartels

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