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Some Types of Integral Problems

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Abstract This paper uses the mathematical software Maple for the auxiliary tool to study four types of integrals. We can obtain the infinite series forms of these integrals by using binomial theorem and integration term by term theorem. In addition, we propose two examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple.

Keywords: *integrals, infinite series forms, binomial theorem, integration term by term theorem, Maple*

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1. Introduction

The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. As for the instructions and operations of Maple, we can refer to [1-7].

In calculus and engineering mathematics courses, we learnt many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, and so on. In this paper, we study the following four types of integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int \cosh[r\cos(\lambda x+\beta)]dx \tag{1}$$

$$\int \sinh[r\cos(\lambda x+\beta)]dx \tag{2}$$

$$\int \cosh[r\sin(\lambda x+\beta)]dx \tag{3}$$

$$\int \sinh[r\sin(\lambda x+\beta)]dx \tag{4}$$

where r,λ,β are real numbers, $r,\lambda\neq 0$. We can obtain the infinite series forms of these integrals by using binomial theorem and integration term by term theorem; these are the major results of this paper (i.e., Theorems 1,2). As for the study of related integral problems can refer to [8-24]. On the other hand, we provide some integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce some notations and some formulas used in this study.

2.1. Notations

2.1.1. Suppose m,n are positive integers, $m\leq n$.

Define $\binom{n}{m}=\dfrac{n(n-1)\cdots(n-m+1)}{m!}$ for all $m\leq n$, and $\binom{n}{0}=1$.

2.1.2. Assume r is a real number, the largest integer less than or equal to r is denoted by $[r]$.

2.2. Formulas

2.2.1. Euler's formula

$e^{iy} = \cos y + i\sin y$, where $i = \sqrt{-1}$, y is any real number.

2.2.2. Taylor series expression of the hyperbolic cosine function [25]

$$\cosh y = \sum_{n=0}^{\infty} \frac{1}{(2n)!} y^{2n}$$, where y is any real number.

2.2.3. Taylor series expression of the hyperbolic sine function [25]

$$\sinh y = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} y^{2n+1}$$, where y is any real number.

Next, we introduce two important theorems used in this paper.

2.3. Theorems

2.3.1. Binomial theorem

Suppose x,y are complex numbers, and n is any positive integer. Then $(x+y)^n = \sum_{m=0}^{n} \binom{n}{m} x^{n-m} y^m$.

2.3.2. Integration term by term theorem [26]

Suppose $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on an interval I. If $\sum_{n=0}^{\infty} \int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty} \int_I g_n$.

Before deriving the first major result of this study, we need a lemma.

2.4. Lemma 1

Suppose λ,β are real numbers, $\lambda \neq 0$, n is any positive integer, p is any non-negative integer, and C is a constant. Then the integrals

$$\int \cos^{2n}(\lambda x + \beta)dx$$
$$= \frac{1}{\lambda \cdot 2^{2n-1}} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \sin[(2n-2m)(\lambda x + \beta)] + \frac{\binom{2n}{n}}{2^{2n}} x + C \tag{5}$$

$$\int \cos^{2p+1}(\lambda x + \beta)dx$$
$$= \frac{1}{\lambda \cdot 2^{2p}} \cdot \sum_{m=0}^{p} \frac{\binom{2p+1}{m}}{2p-2m+1} \sin[(2p-2m+1)(\lambda x + \beta)] + C \tag{6}$$

Proof Because for any positive integer k,

$$\cos^k(\lambda x + \beta) = \left[\frac{1}{2}[e^{i(\lambda x+\beta)} + e^{-i(\lambda x+\beta)}]\right]^k$$

(By Euler's formula)

$$= \frac{1}{2^k} \cdot \sum_{m=0}^{k} \binom{k}{m} [e^{i(\lambda x+\beta)}]^{k-m} [e^{-i(\lambda x+\beta)}]^m$$

(By binomial theorem)

$$= \frac{1}{2^k} \cdot \sum_{m=0}^{k} \binom{k}{m} e^{i(k-2m)(\lambda x+\beta)}$$
$$= \frac{1}{2^k} \cdot \sum_{m=0}^{k} \binom{k}{m} \cdot \cos[(k-2m)(\lambda x+\beta)]$$
$$= \frac{1}{2^{k-1}} \cdot \sum_{m=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{m} \cdot \cos[(k-2m)(\lambda x+\beta)] + \frac{1+(-1)^k}{2} \cdot \frac{1}{2^k} \binom{k}{\lfloor k/2 \rfloor} \tag{7}$$

It follows that the integral

$$\int \cos^{2n}(\lambda x + \beta)dx$$
$$= \frac{1}{2^{2n-1}} \cdot \sum_{m=0}^{n-1} \binom{2n}{m} \int \cos[(2n-2m)(\lambda x + \beta)]dx + \frac{1}{2^{2n}} \binom{2n}{n} x$$
$$= \frac{1}{\lambda \cdot 2^{2n-1}} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \sin[(2n-2m)(\lambda x + \beta)] + \frac{\binom{2n}{n}}{2^{2n}} x + C$$

And

$$\int \cos^{2p+1}(\lambda x + \beta)dx$$
$$= \frac{1}{2^{2p}} \cdot \sum_{m=0}^{p} \binom{2p+1}{m} \int \cos[(2p-2m+1)(\lambda x + \beta)]dx$$
$$= \frac{1}{\lambda \cdot 2^{2p}} \cdot \sum_{m=0}^{p} \frac{\binom{2p+1}{m}}{2p-2m+1} \sin[(2p-2m+1)(\lambda x + \beta)] + C$$

In the following, we obtain the infinite series forms of the integrals (1) and (2).

2.5. Theorem 1

Let r,λ,β be real numbers, $r,\lambda \neq 0$.and C is a constant.

(1) The integral

$$\int \cosh[r\cos(\lambda x + \beta)]dx$$
$$= \frac{1}{\lambda} \cdot \sum_{n=1}^{\infty} \frac{r^{2n}}{2^{2n-1}(2n)!} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \sin[(2n-2m)(\lambda x + \beta)] + \sum_{n=0}^{\infty} \frac{r^{2n}}{2^{2n}(n!)^2} \cdot x + C \tag{8}$$

(2) The integral

$$\int \sinh[r\cos(\lambda x + \beta)]dx$$
$$= \frac{1}{\lambda} \cdot \sum_{n=0}^{\infty} \frac{r^{2n+1}}{2^{2n}(2n+1)!} \cdot \sum_{m=0}^{n} \frac{\binom{2n+1}{m}}{2n-2m+1} \sin[(2n-2m+1)(\lambda x + \beta)] + C \tag{9}$$

Proof (1) The integral

$$\int \cosh[r\cos(\lambda x+\beta)]dx$$

$$=\int \sum_{n=0}^{\infty}\frac{1}{(2n)!}[r\cos(\lambda x+\beta)]^{2n}dx$$

(By Formula 2.2.2.)

$$=\sum_{n=0}^{\infty}\frac{r^{2n}}{(2n)!}\int \cos^{2n}(\lambda x+\beta)dx$$

(By integration term by term theorem)

$$=\sum_{n=1}^{\infty}\frac{r^{2n}}{(2n)!}\int \cos^{2n}(\lambda x+\beta)dx+x+C$$

$$=\frac{1}{\lambda}\cdot\sum_{n=1}^{\infty}\frac{r^{2n}}{2^{2n-1}(2n)!}\cdot\sum_{m=0}^{n-1}\frac{\binom{2n}{m}}{2n-2m}\sin[(2n-2m)(\lambda x+\beta)]+\sum_{n=0}^{\infty}\frac{r^{2n}}{2^{2n}(n!)^2}\cdot x+C \quad \text{(By (5))}$$

(2) The integral

$$\int \sinh[r\cos(\lambda x+\beta)]dx$$

$$=\int \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}[r\cos(\lambda x+\beta)]^{2n+1}dx$$

(By Formula 2.2.3.)

$$=\sum_{n=0}^{\infty}\frac{r^{2n+1}}{(2n+1)!}\int \cos^{2n+1}(\lambda x+\beta)dx$$

(By integration term by term theorem)

$$=\frac{1}{\lambda}\cdot\sum_{n=0}^{\infty}\frac{r^{2n+1}}{2^{2n}(2n+1)!}\cdot\sum_{m=0}^{n}\frac{\binom{2n+1}{m}}{2n-2m+1}\sin[(2n-2m+1)(\lambda x+\beta)]+C \quad \text{(By (6))}$$

To prove the second major result of this study, we also need a lemma.

2.6. Lemma 2

If the assumptions are the same as Lemma 1, then the integrals

$$\int \sin^{2n}(\lambda x+\beta)dx=\frac{1}{\lambda\cdot 2^{2n-1}}\cdot\sum_{m=0}^{n-1}\frac{\binom{2n}{m}}{2n-2m}\sin\left[(2n-2m)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]+\frac{\binom{2n}{n}}{2^{2n}}x+C \quad (10)$$

$$\int \sin^{2p+1}(\lambda x+\beta)dx=\frac{1}{\lambda\cdot 2^{2p}}\cdot\sum_{m=0}^{p}\frac{\binom{2p+1}{m}}{2p-2m+1}\sin\left[(2p-2m+1)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]+C \quad (11)$$

Proof Because for any positive integer k,

$$\sin^k(\lambda x+\beta)=\cos^k\left(\lambda x+\beta-\frac{\pi}{2}\right)$$

$$=\frac{1}{2^{k-1}}\cdot\sum_{m=0}^{\left\lfloor\frac{k-1}{2}\right\rfloor}\binom{k}{m}\cdot\cos\left[(k-2m)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]+\frac{1+(-1)^k}{2}\cdot\frac{1}{2^k}\binom{k}{\lfloor k/2\rfloor} \quad \text{(By (7))}$$

It follows that the integral

$$\int \sin^{2n}(\lambda x+\beta)dx$$

$$=\frac{1}{2^{2n-1}}\cdot\sum_{m=0}^{n-1}\binom{2n}{m}\int \cos\left[(2n-2m)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]dx+\frac{1}{2^{2n}}\binom{2n}{n}x$$

$$=\frac{1}{\lambda\cdot 2^{2n-1}}\cdot\sum_{m=0}^{n-1}\frac{\binom{2n}{m}}{2n-2m}\sin\left[(2n-2m)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]+\frac{\binom{2n}{n}}{2^{2n}}x+C$$

And

$$\int \sin^{2p+1}(\lambda x+\beta)dx$$

$$=\frac{1}{2^{2p}}\cdot\sum_{m=0}^{p}\binom{2p+1}{m}\int \cos\left[(2p-2m+1)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]dx$$

$$=\frac{1}{\lambda\cdot 2^{2p}}\cdot\sum_{m=0}^{p}\frac{\binom{2p+1}{m}}{2p-2m+1}\sin\left[(2p-2m+1)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]+C$$

Finally, we determine the infinite series forms of the integrals (3) and (4).

2.7. Theorem 2

If the assumptions are the same as Theorem 1.

(1) The integral

$$\int \cosh[r\sin(\lambda x+\beta)]dx$$

$$=\frac{1}{\lambda}\cdot\sum_{n=1}^{\infty}\frac{r^{2n}}{2^{2n-1}(2n)!}\cdot\sum_{m=0}^{n-1}\frac{\binom{2n}{m}}{2n-2m}\sin\left[(2n-2m)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]+\sum_{n=1}^{\infty}\frac{r^{2n}}{2^{2n}(n!)^2}\cdot x+C \quad (12)$$

(2) The integral

$$\int \sinh[r\sin(\lambda x+\beta)]dx$$

$$=\frac{1}{\lambda}\cdot\sum_{n=1}^{\infty}\frac{r^{2n+1}}{2^{2n}(2n+1)!}\cdot\sum_{m=0}^{n}\frac{\binom{2n+1}{m}}{2n-2m+1}\sin\left[(2n-2m+1)\left(\lambda x+\beta-\frac{\pi}{2}\right)\right]+C \quad (13)$$

Proof (1) The integral

$$\int \cosh[r\sin(\lambda x+\beta)]dx$$

$$=\int \sum_{n=0}^{\infty}\frac{1}{(2n)!}[r\sin(\lambda x+\beta)]^{2n}dx$$

$$=\sum_{n=1}^{\infty}\frac{r^{2n}}{(2n)!}\int \sin^{2n}(\lambda x+\beta)dx$$

$$= \frac{1}{\lambda} \cdot \sum_{n=1}^{\infty} \frac{r^{2n}}{2^{2n-1}(2n)!} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \sin\left[(2n-2m)\left(\lambda x + \beta - \frac{\pi}{2}\right)\right] + \sum_{n=1}^{\infty} \frac{r^{2n}}{2^{2n}(n!)^2} \cdot x + C \quad \text{(By (10))}$$

(2) The integral

$$\int \sinh[\, r \sin(\lambda x + \beta)] dx$$

$$= \int \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} [r \sin(\lambda x + \beta)]^{2n+1} dx$$

$$= \sum_{n=0}^{\infty} \frac{r^{2n+1}}{(2n+1)!} \int \sin^{2n+1}(\lambda x + \beta) dx$$

$$= \frac{1}{\lambda} \cdot \sum_{n=1}^{\infty} \frac{r^{2n+1}}{2^{2n}(2n+1)!} \cdot \sum_{m=0}^{n} \frac{\binom{2n+1}{m}}{2n-2m+1} \sin\left[(2n-2m+1)\left(\lambda x + \beta - \frac{\pi}{2}\right)\right] + C \quad \text{(By (11))}$$

3. Examples

In the following, for the four types of integral problems in this study, we provide two examples and use Theorems 1,2 to determine the infinite series forms of these integrals. On the other hand, we evaluate some related definite integrals and employ Maple to calculate the approximations of these definite integrals and their solutions for verifying our answers.

3.1. Example 1

3.1.1. Using (8) of Theorem 1, we obtain

$$\int \cosh\left[5\cos\left(3x + \frac{\pi}{4}\right)\right] dx$$

$$= \frac{1}{3} \cdot \sum_{n=1}^{\infty} \frac{5^{2n}}{2^{2n-1}(2n)!} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \sin\left[(2n-2m)\left(3x + \frac{\pi}{4}\right)\right] + \sum_{n=0}^{\infty} \frac{5^{2n}}{2^{2n}(n!)^2} \cdot x + C \tag{14}$$

Therefore, we can determine the definite integral

$$\int_{\pi/2}^{\pi} \cosh\left[5\cos\left(3x + \frac{\pi}{4}\right)\right] dx$$

$$= \frac{2}{3} \cdot \sum_{n=1}^{\infty} \frac{5^{2n}}{2^{2n-1}(2n)!} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \cdot \sin\frac{(n-m)\pi}{2} + \frac{\pi}{2} \cdot \sum_{n=0}^{\infty} \frac{5^{2n}}{2^{2n}(n!)^2} \tag{15}$$

Next, we use Maple to verify the correctness of (15).

>evalf(int(cosh(5*cos(3*x+Pi/4)),x=Pi/2.Pi),18);

54.2832471573584164

>evalf(2/3*sum(5^(2*n)/(2^(2*n-1)*(2*n)!)*sum((2*n)!/(m!*(2*n-m)!*(2*n-2*m))*sin((n-m)*Pi/2),m=0.(n-1)),n=1.infinity)+Pi/2*sum(5^(2*n)/(2^(2*n)*(n!)^2),n=0.infinity),18);

54.2832471573584164

3.1.2. By (9) of Theorem 1, we obtain

$$\int \sinh\left[7\cos\left(6x - \frac{2\pi}{3}\right)\right] dx$$

$$= \frac{1}{6} \cdot \sum_{n=1}^{\infty} \frac{7^{2n+1}}{2^{2n}(2n+1)!} \cdot \sum_{m=0}^{n} \frac{\binom{2n+1}{m}}{2n-2m+1} \sin\left[(2n-2m+1)\left(6x - \frac{2\pi}{3}\right)\right] + C \tag{16}$$

Thus, the definite integral

$$\int_{0}^{\pi/6} \sinh\left[7\cos\left(6x - \frac{2\pi}{3}\right)\right] dx$$

$$= \frac{1}{6} \cdot \sum_{n=0}^{\infty} \frac{7^{2n+1}}{2^{2n}(2n+1)!} \cdot \sum_{m=0}^{n} \frac{\binom{2n+1}{m}}{2n-2m+1} \left[\sin\frac{(2n-2m+1)\pi}{3} + \sin\frac{(4n-4m+2)\pi}{3}\right] \tag{17}$$

Using Maple to verify the correctness of (17) as follows:

>evalf(int(sinh(7*cos(6*x-2*Pi/3)),x=0.Pi/6),18);

87.4180616135055458

>evalf(1/6*sum(7^(2*n+1)/(2^(2*n)*(2*n+1)!)*sum((2*n+1)!/(m!*(2*n-m+1)!*(2*n-2*m+1))*(sin((2*n-2*m+1)*Pi/3)+sin((4*n-4*m+2)*Pi/3)),m=0.n),n=0.infinity),18);

87.418061635055493 – 5.99593039950345028·10^{-17}I

The above answer obtained by Maple appears I ($=\sqrt{-1}$), it is because Maple calculates by using special functions built in. The imaginary part is very small, so can be ignored.

3.2. Example 2

3.2.1. By (12) of Theorem 2, we have

$$\int \cosh\left[9\sin\left(2x + \frac{5\pi}{8}\right)\right] dx$$

$$= \frac{1}{2} \cdot \sum_{n=1}^{\infty} \frac{9^{2n}}{2^{2n-1}(2n)!} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \sin\left[(2n-2m)\left(2x + \frac{\pi}{8}\right)\right] + \sum_{n=1}^{\infty} \frac{9^{2n}}{2^{2n}(n!)^2} \cdot x + C \tag{18}$$

Hence, the definite integral

$$\int_{\pi/4}^{\pi/2} \cosh\left[9\sin\left(2x + \frac{5\pi}{8}\right)\right] dx$$

$$= \frac{1}{2} \cdot \sum_{n=1}^{\infty} \frac{9^{2n}}{2^{2n-1}(2n)!} \cdot \sum_{m=0}^{n-1} \frac{\binom{2n}{m}}{2n-2m} \cdot \left[\sin\frac{(n-m)\pi}{4} - \sin\frac{(5n-5m)\pi}{4}\right] + \frac{\pi}{4} \cdot \sum_{n=0}^{\infty} \frac{9^{2n}}{2^{2n}(n!)^2} \tag{19}$$

>evalf(int(cosh(9*sin(2*x+5*Pi/8)),x=Pi/4.Pi/2),18);

1503.11190390538857

>evalf(1/2*sum(9^(2*n)/(2^(2*n-1)*(2*n)!)*sum((2*n)!/(m!*(2*n-m)!*(2*n-2*m))*(sin((n-m)*Pi/4)-sin((5*n-5*m)*Pi/4)),m=0.(n-1)),n=1.infinity)+Pi/4*sum(9^(2*n)/(2^(2*n)*(n!)^2),n=0.infinity),18);

1503.11190390538857

3.2.2. Using (13) of Theorem 2, we can determine

$$\int \sinh\left[8\sin\left(10x-\frac{7\pi}{6}\right)\right]dx$$
$$=\frac{1}{10}\cdot\sum_{n=1}^{\infty}\frac{8^{2n+1}}{2^{2n}(2n+1)!}\cdot\sum_{m=0}^{n}\frac{\binom{2n+1}{m}}{2n-2m+1}\sin\left[(2n-2m+1)\left(10x-\frac{5\pi}{3}\right)\right]$$
$$+C \qquad (20)$$

Therefore, the definite integral

$$\int_{\pi/3}^{2\pi/3}\sinh\left[8\sin\left(10x-\frac{7\pi}{6}\right)\right]dx$$
$$=-\frac{1}{10}\cdot\sum_{n=0}^{\infty}\frac{8^{2n+1}}{2^{2n}(2n+1)!}\cdot\sum_{m=0}^{n}\frac{\binom{2n+1}{m}}{2n-2m+1}$$
$$\cdot\sin\frac{(10n-10m+5)\pi}{3} \qquad (21)$$

>evalf(int(sinh(8*sin(10*x-7*Pi/6)),x=Pi/3.2*Pi/3),18);

66.7890398786811806

>evalf(-1/10*sum(8^(2*n+1)/(2^(2*n)*(2*n+1)!)*sum((2*n+1)!/(m!*(2*n-m+1)!*(2*n-2*m+1))*sin((10*n-10*m+5)*Pi/3),m=0.n),n=0.infinity),18);

66.789039876811832-6.84850388804298574·10^-17 I

Also, the imaginary part of the above answer obtained by Maple is very small, so can be ignored.

4. Conclusion

In this paper, we provide a new technique to solve some integrals. We hope this technique can be applied to evaluate another integral problems. On the other hand, the binomial theorem and the integration term by term theorem play significant roles in the theoretical inferences of this study. In fact, the applications of these two theorems are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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