*American Journal of Numerical Analysis, 2014, Vol. 2, No. 3, 85-89*
Available online at http://pubs.sciepub.com/ajna/2/3/4
© Science and Education Publishing
DOI:10.12691/ajna-2-3-4

# One Class of Stable Difference Schemes for Hyperbolic System

**A.M. Blokhin[1], R.D. Alaev[2], M.U. Hudayberganov[2,*]**

[1]Differential Equations, Novosibirsk Institute of Mathematics, Novosibirsk, Russian
[2]Computations technologies and mathematical modeling, National University of Uzbekistan, Tashkent, Uzbekistan
*Corresponding author: mirzoali@mail.ru

*Received February 06, 2014; Revised March 03, 2014; Accepted April 02, 2014*

**Abstract** It is suggested one class of difference schemes for hyperbolic systems of equations, which have several forms of writing. Stability of suggested difference schemes is investigated with the help of technics of integrals energy. Numerical results of model problems are given.

***Keywords:*** *hyperbolic system, difference schemes, stability, energy norm*

**Cite This Article:** A.M. Blokhin, R.D. Alaev, and M.U. Hudayberganov, "One Class of Stable Difference Schemes for Hyperbolic System." *American Journal of Numerical Analysis*, vol. 2, no. 3 (2014): 85-89. doi: 10.12691/ajna-2-3-4.

## 1. Introduction

At present in descriptions of different physical phenomenons it is widely applied hydrodynamical models. For finding the approximate solutions of such models often used finite-difference methods. There are many different ways of formation of difference schemes for the hydrodynamical models (and, in particular, for equations of the gas dynamic). Some approaches to formation of difference scheme presented, for example, in [1]. In [1] is described one interesting approach, which based on possibility of writing of the system of gas dynamics equations in two variants. We shall explain meaning of this approach first for differential problem. As noted in [1] (see. also background work [4]), the system of equations of gas dynamics describing three dimensional motion of gas in assumption that gas is nonviscous not heat-conducting and in state of local thermodynamic balance (i.e. there exists equation of gas state), can be written in the form of symmetric $t$-hyperbolic system (by Friedrichs)

$$B^0\mathbf{Q}_t + \sum_{k=1}^{3} B^k \mathbf{Q}_{x_k} = 0 \qquad (1.1)$$

Here $B^0 = B^0(\mathbf{Q})$, $B^k = B^k(\mathbf{Q})$, $k = \overline{1,3}$ are symmetric matrices of order 5, and also $B^0 > 0$ (naturally, in physical justified requirements to smooth solution of the system (1.1)); $\mathbf{Q} = \begin{pmatrix} q_1 \\ \vdots \\ q_5 \end{pmatrix}$ is the vector of dependence variables.

Further, by special choosing of the vector of dependence variables $\mathbf{Q}$ (see. [1, 4]) on smooth solutions the system (1.1) can be rewritten in the following equivalent form:

$$\left(B^0\mathbf{Q}\right)_t + \sum_{k=1}^{3}\left(B^k\mathbf{Q}\right)_{x_k} = 0 \qquad (1.2)$$

These two equivalent (on smooth solutions) forms of writing of the initial system of equations of gas dynamics allow to obtain for it such called *local a priori estimation*. Let the system (1.1) has smooth solution $\mathbf{Q} = \mathbf{Q}(t,\mathbf{x})$, $(t,\mathbf{x}) \in \mathbb{R}^4_+ = \left\{(t,\mathbf{x}); t > 0, \mathbf{x} \in \mathbb{R}^3\right\}$, satisfying the condition:

$$(\mathbf{Q},\mathbf{Q}) \to 0 \text{ at } (\mathbf{x},\mathbf{x}) \to \infty,\ t \ge 0 \qquad (1.3)$$

and initial data for $t = 0$:

$$\mathbf{Q}(0,\mathbf{x}) = \mathbf{Q}_0(\mathbf{x}), \qquad (1.4)$$

here $\mathbf{x} = (x_1, x_2, x_3)$. Multiplying both sides of systems (1.1), (1.2) scalar to the vector $\mathbf{Q}$ and adding obtained expressions we have

$$\left(\mathbf{Q}, B^0\mathbf{Q}\right)_t + \sum_{k=1}^{3}\left(\mathbf{Q}, B^k\mathbf{Q}\right)_{x_k} = 0 \qquad (1.5)$$

Integrating the identity (1.5) on domain $\mathbb{R}^3$, taking into account conditions (1.3) we get searching *a priori estimation*

$$I(t) = I(0),\ 0 < t < T < \infty, \qquad (1.6)$$

where

$$I(t) = \int_{\mathbb{R}^3}\left(\mathbf{Q}, B^0\mathbf{Q}\right)d\mathbf{x}.$$

In the process of obtaining of the estimation (1.6) we also supposed, that on given smooth solution $\mathbf{Q}(t,\mathbf{x})$, $(t,\mathbf{x}) \in \mathbb{R}^4_+$ the norms of the matrix $B^k(\mathbf{Q})$ are bounded.

Clearly, that the estimation (1.6) has local character, because the constant $T$ characterized time of existence of smooth solution (time up to occurrence of *gradient catastrophe*) of the problem (1.1), (1.4) and time of fulfillment of the inequality

$$B^0(\mathbf{Q}(t,\mathbf{x})) > 0, (t,\mathbf{x}) \in \mathbb{R}_+^4.$$

The fact, that the system of equations of gas dynamics can be written in the form (1.1) or (1.2), gives to the idea to use this by formation of finite-difference schemes for equations of the gas dynamics with the aim to get approximate solutions of *finite-difference analogue* of local a priori estimation (1.6). On terminology, accepted in [1], this means *adequacy* of the initial mathematical and computational models. In the next section we describe one class of such difference schemes, in formation which is used above mentioned circumstance (another examples given in [1], see also [3]).

**Remark 1.1.** *As noted in [1], the identity (1.5) is the another form of writing of additional law of the conservation to entropy (namely presence of such law and its nonuniqueness allows to present the system of the equations of gas dynamics in the form (1.1) or (1.2)).*

**Remark 1.2.** *We will digress from gas dynamical origin of the system (1.1), (1.2) and we assume, that initial symmetric $t$-hyperbolic system (1.1) can be rewritten in the form (1.2) on smooth solutions of the system (1.1)).*

The paper is organized as follows. In section 2 a class of difference schemes is constructed and the stability of difference schemes is proved. In section 3 it is given comparative analysis of numerical results. In conclusion it is briefly formulated obtained results.

## 2. One Class of "ctable" Difference Schemes for the System (1.1)

We formulate for the initial mathematical model (1.1), (1.4) the following *computational model*. In the domain $\mathbb{R}_+^4$ we construct grid with steps $\Delta = \Delta t$, $h_k = \Delta x_k$, $k = \overline{1,3}$ and introduce such designations (see [1, p.27]): $\mathbf{Q}_{\mathbf{a}}^n = \mathbf{Q}(n\Delta, \mathbf{ah}) = \mathbf{Q}_{\alpha_1\alpha_2\alpha_3}^n = \mathbf{Q}_{\mathbf{a}} = \mathbf{Q}_{\alpha_k} = \mathbf{Q}$ is the grid vector-function, $\mathbf{a} = (\alpha_1, \alpha_2, \alpha_3)$ is multi index, $\mathbf{h} = (h_1, h_2, h_3)$, $\mathbf{ah} = (\alpha_1 h_1, \alpha_2 h_2, \alpha_3 h_3)$, $n, |\alpha_k| = 0,1,\ldots$, $k = \overline{1,3}$; $\varphi$, $\psi_{\alpha_k}$, $\psi_{\alpha_k}^{-1}$, $k = \overline{1,3}$ is the shift operator: $\varphi\mathbf{Q} = \mathbf{Q}_{\mathbf{a}}^{n+1} = \widehat{\mathbf{Q}}$, $\psi_{\alpha_k}^{\pm 1}\mathbf{Q} = \mathbf{Q}_{\alpha_k \pm 1}^n$, $k = \overline{1,3}$, $\left(\psi_{\alpha_k}^{+1} = \psi_{\alpha_k}\right)$; $\tau$, $\xi_k$, $\overline{\xi_k}$, $k = \overline{1,3}$ are difference operators: $\tau = \varphi - 1$, $\xi_k = \psi_{\alpha_k} - 1$, $\overline{\xi_k} = 1 - \psi_{\alpha_k}^{-1}$.

As known, for symmetric matrices $B^k$, $k = \overline{1,3}$ it is possible to realize the following representation:

$$B^k = B_+^k - B_-^k, k = \overline{1,3}, \tag{2.1}$$

where $B_+^k, B_-^k \geq 0$ are symmetric matrices.

It is easy to be sure in correctness of (2.1), if as $B_+^k, B_-^k$ take the following matrices:

$$B_+^k = \frac{1}{2} R_k^* \left(\Lambda_k + |\Lambda_k|\right) R_k,$$

$$B_-^k = \frac{1}{2} R_k^* \left(-\Lambda_k + |\Lambda_k|\right) R_k.$$

Here $R_k$ is the orthogonal matrix, that reduce the matrix $B^k$ to diagonal form: $B^k = R_k^* \Lambda_k R_k$, where $R_k^*$ is conjugate matrix to $R_k$, $\Lambda_k = diag(\lambda_1^{(k)}, \ldots, \lambda_5^{(k)})$, $\lambda_i^{(k)}$, $i = \overline{1,5}$ are eigenvalues of the matrix $B^k$, $|\Lambda_k| = diag(|\lambda_1^{(k)}|, \ldots, |\lambda_5^{(k)}|)$, $k = \overline{1,3}$.

For finding of numerical solution of the initial mathematical model (1.1), (1.4) it is suggested the following finite-difference scheme:

$$\begin{aligned} &\widehat{B}^0 \tau\mathbf{Q} + \widehat{D}^{-1} D\tau(B^0\mathbf{Q}) + \\ &+ \sum_{k=1}^{3} r_k \left\{ \begin{array}{l} B_+^k(\mathbf{U}^{(k)})\overline{\xi_k}\widehat{\mathbf{Q}} + \widehat{D}^{-1}\widehat{D}_{\alpha_k - 1}\overline{\xi_k}(B_+^k(\mathbf{U}^{(k)})\widehat{\mathbf{Q}}) \\ -B_-^k(\mathbf{V}^{(k)})\xi_k\widehat{\mathbf{Q}} - \widehat{D}^{-1}\widehat{D}_{\alpha_k + 1}\xi_k(B_-^k(\mathbf{V}^{(k)})\widehat{\mathbf{Q}}) \end{array} \right\} = 0, \end{aligned} \tag{2.2}$$

$n, |\alpha_k| = 0,1,\ldots$, $k = \overline{1,3}$, with initial data

$$\mathbf{Q}_{\mathbf{a}}^0 = \mathbf{Q}_0(\mathbf{ah}), \quad |\alpha_k| = 0,1,\ldots, k = \overline{1,3}. \tag{2.3}$$

Here $B^0 = B^0(\mathbf{Q}_{\mathbf{a}}^n)$, $\widehat{B}^0 = B^0(\widehat{\mathbf{Q}})$, $D = diag(q_1, \ldots, q_5)$, $D^{-1} = diag(\frac{1}{q_1}, \ldots, \frac{1}{q_5})$, $r_k = \frac{\Delta}{h_k}$, $k = \overline{1,3}$, $\mathbf{U}^{(k)}$, $\mathbf{V}^{(k)}$ are "intermediate" values of the vector $\mathbf{Q}$, $k = \overline{1,3}$.

**Remark 2.1.** *The difference scheme (2.2) do not approximate initial system (1.1), it approximate corollary of (1.1) (which obtained from (1.1), (1.2)):*

$$B^0\mathbf{Q}_t + \left(B^0\mathbf{Q}\right)_t + \sum_{k=1}^{3}\left\{B^k\mathbf{Q}_{x_k} + \left(B^k\mathbf{Q}\right)_{x_k}\right\} = 0. \tag{2.4}$$

**Remark 2.2.** *We assume that the solution of the finite-difference model (2.2), (2.3) satisfy the condition*

$$\left(\mathbf{Q}_{\mathbf{a}}^n, \mathbf{Q}_{\mathbf{a}}^n\right) \to 0 \quad by \left(\sum_{k=1}^{3}\alpha_k^2\right) \to \infty, n = 0,1,\ldots \tag{2.5}$$

We also suppose, that norms of matrices $B_+^k(\mathbf{U}^{(k)})$, $B_-^k(\mathbf{V}^{(k)})$, $k = \overline{1,3}$ are bounded.

**Remark 2.3.** *Suggested difference scheme (2.2) is implicit and has to be realized by iteration of nonlinearity.*

**Remark 2.4** *We will not concretize choice of vectors $\mathbf{U}^{(k)}$, $\mathbf{V}^{(k)}$, $k = \overline{1,3}$, because it is possible different variants of such choice ("intermediate" means averaged value of the vector Q in the difference cell, such many variance may be useful for numerical calculations of applied problems).*

The following theorem is true

**Theorem.** *Let on solutions of the finite-difference model (2.2), (2.3) the condition (2.5) and the following inequality are fulfilled:*

$$B^0(\mathbf{Q}_{\mathbf{a}}^n) > 0, \quad n, |\alpha_k| = 0,1,\ldots, k = \overline{1,3}.$$

*Then the difference model (2.2), (2.3) is "stable" in the energy norm $\sqrt{J_n}$, where*

$$J_n = \bar{h}\sum_{k=1}^{3}\sum_{\alpha_k=-\infty}^{\infty}\left(\mathbf{Q}^n_{\alpha_k}, B^0(\mathbf{Q}^n_{\mathbf{a}})\mathbf{Q}^n_{\alpha_k}\right), \qquad (2.6)$$

$\bar{h} = h_1 h_2 h_3$.

**Proof.** We multiply both sides of the system (2.2) scalar to the vector $\hat{\mathbf{Q}}$

$$(\hat{\mathbf{Q}}, \hat{B}^0\tau\mathbf{Q}) + (\hat{\mathbf{Q}}, \hat{D}^{-1}D\tau(B^0\mathbf{Q})) + \\ +\sum_{k=1}^{3} r_k \begin{Bmatrix} (\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\overline{\xi_k}\hat{\mathbf{Q}}) \\ +(\hat{\mathbf{Q}}, \hat{D}^{-1}\hat{D}_{\alpha_k-1}\overline{\xi_k}(B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}})) \\ -(\hat{\mathbf{Q}}, B^k_-(\mathbf{V}^{(k)})\xi_k\hat{\mathbf{Q}}) \\ -(\hat{\mathbf{Q}}, \hat{D}^{-1}\hat{D}_{\alpha_k+1}\xi_k(B^k_-(\mathbf{V}^{(k)})\hat{\mathbf{Q}})) \end{Bmatrix} = 0, \qquad (2.7)$$

The following relations are true:

1) $(\hat{\mathbf{Q}}, \hat{B}^0\tau\mathbf{Q}) + (\hat{\mathbf{Q}}, \hat{D}^{-1}D\tau(B^0\mathbf{Q})) = \tau(\mathbf{Q}, B^0\mathbf{Q})$,

2) $(\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\overline{\xi_k}\hat{\mathbf{Q}}) + (\hat{\mathbf{Q}}, \hat{D}^{-1}\hat{D}_{\alpha_k-1}\overline{\xi_k}(B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}}))$
$= \overline{\xi_k}(\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}})$, $k = \overline{1,3}$,

3) $(\hat{\mathbf{Q}}, B^k_-(\mathbf{V}^{(k)})\xi_k\hat{\mathbf{Q}}) + (\hat{\mathbf{Q}}, \hat{D}^{-1}\hat{D}_{\alpha_k+1}\xi_k(B^k_-(\mathbf{V}^{(k)})\hat{\mathbf{Q}}))$
$= \xi_k(\hat{\mathbf{Q}}, B^k_-(\mathbf{V}^{(k)})\xi_k\hat{\mathbf{Q}})$, $k = \overline{1,3}$.

Taking into account 1)-3) from the equality (2.7) we get

$$\tau(\mathbf{Q}, B^0\mathbf{Q}) + \sum_{k=1}^{3} r_k\left\{\overline{\xi_k}(\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}}) - \right. \\ \left. -\xi_k(\hat{\mathbf{Q}}, B^k_-(\mathbf{V}^{(k)})\hat{\mathbf{Q}})\right\} = 0. \qquad (2.7')$$

Multiplying (2.7') by $\bar{h} = h_1 h_2 h_3$ and adding obtained results by $\alpha_k$ from $-\infty$ up to $\infty$, taking into account (2.5) finally we get searching estimation

$$J_{n+1} = J_n,\ n = 0,1,\ldots,$$

i.e.

$$J_n = J_0,\ n = 1,2,\ldots, \qquad (2.8)$$

where $J_n$ is defined by formula (2.6).

The equality (2.8) is finite-difference analogue of a priori estimation (1.6). From (2.8) also follows "stability" of the difference model (2.2), (2.3) in energy norm $\sqrt{J_n}$.

**Remark 2.5.** *We have taken the word stability in quotation, because strictly saying, it is required to prove, that the inequality*

$$B^0(\mathbf{Q}^n_{\mathbf{a}}) > 0, \quad n, |\alpha_k| = 0,1,\ldots,\ k = \overline{1,3}.$$

*and the condition (2.5) are fulfilled on solutions of finite-difference computational model (2.2), (2.3).*

**Remark 2.6.** *In the process of obtaining of (2.8) we used the following calculations:*

$$\sum_{\alpha_k=-\infty}^{\infty}\overline{\xi_k}(\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}})$$

$$= \lim_{M_k\to\infty}\sum_{\alpha_k=-M_k+1}^{M_k}\overline{\xi_k}(\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}})$$

$$= \lim_{M_k\to\infty}\left\{(\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}})|_{\alpha_k=M_k} \right. \\ \left. -(\hat{\mathbf{Q}}, B^k_+(\mathbf{U}^{(k)})\hat{\mathbf{Q}})|_{\alpha_k=-M_k}\right\} = 0,\ k = \overline{1,3},$$

*which is obtained by virtue of (2.5).*

## 3.Some Simple Examples

As simple and also interesting examples we consider simple hyperbolic equations

$$q_t + bq_x = 0,\ b = const > 0, \qquad (3.1)$$

$$q_t + qq_x = 0. \qquad (3.2)$$

For the equation (3.1) we consider the following problem:

$$q_t + bq_x = 0,\ 0 < t < t_*,\ |x| < 1, \\ q(0,x) = q_0(x) = \begin{cases} \frac{3}{2}, -1 < x < 0 \\ \frac{1}{2}, 0 < x < 1, \end{cases} \qquad (3.3) \\ q(t,-1) = \frac{3}{2},\ 0 < t < t_*,$$

where $t_* = \frac{1}{b}$.

The problem (3.3) illustrates gas-dynamical situation with the generated gasdynamic shock transition.

The difference scheme (2.2) for the equation (3.1) has following form:

$$\hat{q} = q + \frac{br}{\hat{q}+q}\left(\hat{q}_{\alpha-1} - \hat{q}\right)\left(\hat{q} + \hat{q}_{\alpha-1}\right). \qquad (3.4)$$

Here $q = q(\Delta n, \alpha h) = q^n_\alpha = q_\alpha$, $\hat{q} = q^{n+1}_\alpha$,

$$n = 0,\ldots,\frac{t_* - \Delta l}{\Delta}, |\alpha| = 0,\ldots,\frac{1}{h},$$

$r = \frac{\Delta}{h}$, $l > 1$ is some integer number.

Lax-Friedrichs difference scheme for the equation (3.1) according to [2] has the following form:

$$\hat{q} = \frac{q_{\alpha+1} + q_{\alpha-1}}{2} - \frac{br}{2}\left(q_{\alpha+1} - q_{\alpha-1}\right). \qquad (3.5)$$

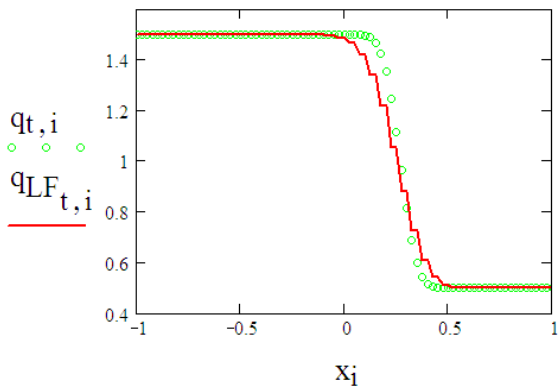


**Figure 1.** $t = 0.21$, ○○○ -numerical solution by scheme (3.4); −− -numerical solution by Lax-Friedrichs

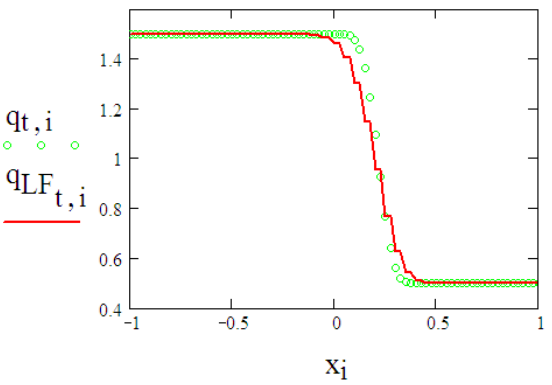


**Figure 2.** $t = 0.42$, ∘∘∘ -numerical solution by scheme (3.4); −− - numerical solution by Lax-Friedrichs scheme.

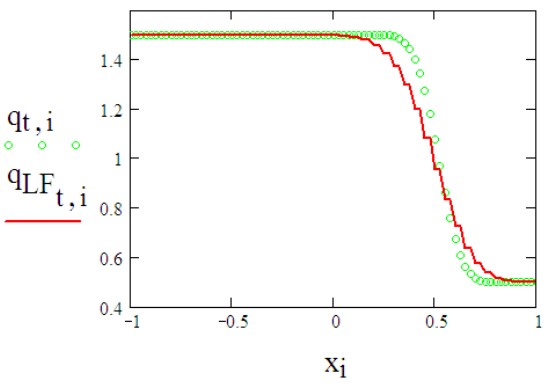


**Figure 3.** $t = 0.50$, ∘∘∘ - numerical solution by scheme (3.4); −− - numerical solution by Lax-Friedrichs scheme

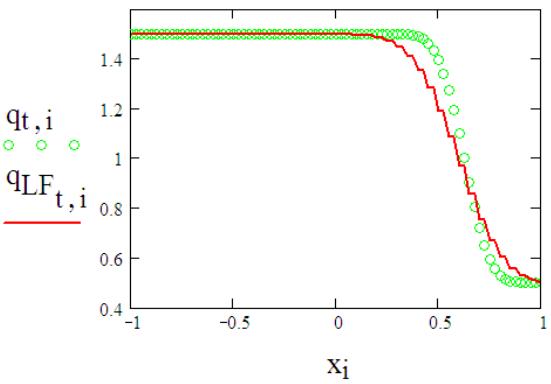


**Figure 4.** $t = 0.61$, ∘∘∘ -numerical solution by scheme (3.4); −− - numerical solution by Lax-Friedrichs scheme

In Figure 1- Figure 4 are given comparisons of the calculation results by scheme (3.4) with numerical calculation by Lax-Friedrichs scheme for $b = 1$, $\Delta = 0.01$, $h = 0.025$ and for different values of time $t$.

For the equation (3.2) consider such problem:

$$q_t + qq_x = 0,\ 0 < t < t_*,\ -1 < x < a,$$
$$q(0,x) = q_0(x) = \begin{cases} 1, -1 < x < -\varepsilon \\ \rho(x), -\varepsilon < x < \varepsilon, \\ \delta, \varepsilon < x < a, \end{cases} \qquad (3.6)$$
$$q(t,-1) = 1,\ 0 < t < t_*,$$

where $\rho(x) = \dfrac{1-\delta}{4\varepsilon^3}x^3 - \dfrac{3}{4}\dfrac{1-\delta}{\varepsilon}x + \dfrac{1+\delta}{2}$, $\varepsilon > 0$, $\delta(0 < \delta < 1)$, $a > 0$, $t_*(t_* > t_c > 0)$ are some parameters, $t_c = \dfrac{4\varepsilon}{3(1-\delta)}$ is the time of coming of gradient catastrophe.

The problem (3.5) well illustrates gas-dynamical situation of beginning and evolutions of the gasdynamic shock transition.

Though the equation (3.2) is not written in the form (1.2), but it can be represented such (see. formula (2.4)):

$$\frac{1}{2}q_t + \left(\frac{1}{2}q\right)_t + \frac{1}{3}qq_x + \left(\frac{1}{3}q^2\right)_x = 0$$

and consequently the difference scheme (2.2) for the equation (3.2) has the following form:

$$\hat{q} = q + \frac{2r}{3(\hat{q}+q)}\left\{\hat{q}(\hat{q}_{\alpha-1} - \hat{q})U + \right.$$
$$\left. + \hat{q}_{\alpha-1}(U_{\alpha-1}\hat{q}_{\alpha-1} - U\hat{q}\right\}. \qquad (3.7)$$

Here $U$ is "intermediate" value $q$ (in this example as "intermediate" vale is taken $q$), $n = 0,\ldots,\dfrac{t_*}{\Delta}$, $\alpha = -\dfrac{1}{h},\ldots,0,1,\ldots,\dfrac{a}{h}$.

The Lax-Friedrichs difference scheme for the equation (3.2) according to [2] has the following form:

$$\hat{q} = \frac{q_{\alpha+1} + q_{\alpha-1}}{2} - \frac{r}{4}\left(q_{\alpha+1}^2 - q_{\alpha-1}^2\right). \qquad (3.8)$$

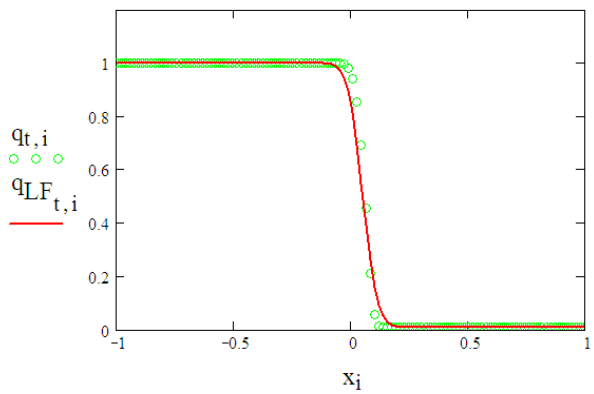


**Figure 5.** $t = 0.135$ (time of beginning of gradient catastrophe), ∘∘∘ -numerical solution by scheme (3.7); −− - numerical solution by Lax-Friedrichs scheme

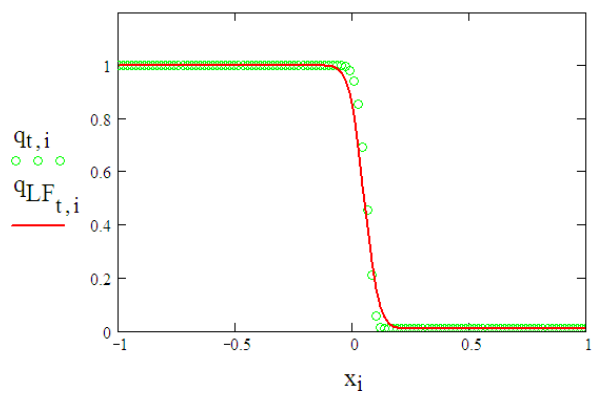


**Figure 6.** $t = 0.56$, ∘∘∘ -numerical solution by scheme (3.7); −− - numerical solution by Lax-Friedrichs scheme

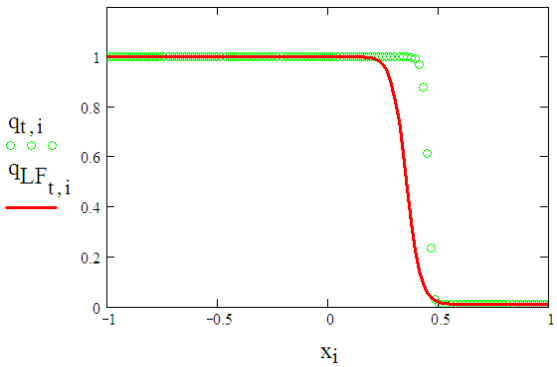


**Figure 7.** $t = 0.7$, ∘∘∘ -numerical solution by scheme (3.7); −− - numerical solution by Lax-Friedrichs scheme.

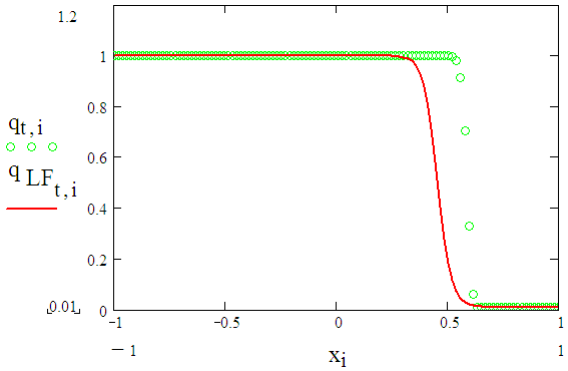


**Figure 8.** $t = 0.9$, ∘∘∘ -numerical solution by scheme (3.7); −− - numerical solution by Lax- Friedrichs scheme

The comparisons numerical calculations by scheme (3.7) with numerical calculations by Lax-Friedrichs difference scheme given in Figure 5 - Figure 8 for different values of time $t$.

The general conclusion of numerical experiments is that the suggested scheme gives better both profile and origin of the shock wave.

## 4. Conclusion

So in the present paper it is constructed a class of difference schemes for systems of hyperbolic system which have different forms of writing. It is proved stability of such schemes in the energy norm $\sqrt{J_n}$ (see.(2.6)). Numerical experiments are done on model examples. Results of numerical calculations showed capacity for work, and also competitive ability of suggested difference schemes comparatively with known schemes.

## References

[1] Blokhin A.M., Aloev R.D. Energy integrals and thier applications to investigation of stability of difference schemes. Novosibirsk, 1993. 224 p.

[2] Kulikovskii A.G., Pogorelov N.V., Semenov A.Yu. Mathematical problems of numerical solution of hyperbolic systems. M.:Physics and mathematics publishers, 2001, 608 p.

[3] Blokhin A.M., Sokovikov I.G. About one approach to formulation of difference schemes for quasi-linear equations of gas dynamics. Siberian Mathematical Journal, 1999, V.40, № 6, Pp.1236-1243.

[4] Harten A. On the symmetric form of systems of conservation laws with enthropy. J. Comput. Phys. 1983. V. 49, № 1, p.151-164.