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Some Fixed Point Theorems and Cyclic Contractions in Dislocated and Dislocated Quasi-Metric Spaces

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Abstract In this paper, we established some common fixed point theorems for types of cyclic contractions in the setting of dislocated metric spaces. Using type of contraction introduced by Geraghty [19] and a class of continuous functions G_3 in [10] we extend, generalize and unify some results in the existing literature.

Keywords: *cyclic map, cyclical contraction, dislocated quasi-metric, common fixed point*

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1. Introduction

Notion of dislocated metric spaces was introduced by Hitzler and Seda in 2000 as a generalization of metric space. They generalized the Banach Contraction Principle in such spaces. These metrics play a very important role not only in topology but also in other branches of science involving mathematics especially in logic programming and electronic engineering. Fixed point theory has been a subject of growing interest of many researchers for various types of well known contractions in these spaces.In 2003 Kirk et al [18] introduced cyclic contractions in metric spaces and investigated the existence of proximity points and fixed points for cyclic contraction mappings, Since then many authors has given results in this field.

In this paper we introduced the notion of Geraghty type dq - cyclic contraction and derive the existence of common fixed point theorems in the framework of dislocated metric spaces. Our main theorem extends and unifies existing results in the recent literature.

Definition 1.1 [12,20] Let X be a non-empty and let $d : X \times X \to \mathbb{R}^+$ be a function, called a distance function if for all $x, y, z \in X$, satisfies:

$$d_1 : d(x,x) = 0$$
$$d_2 : d(x,y) = d(y,x) = 0 \Rightarrow x = y$$
$$d_3 : d(x,y) = d(y,x)$$
$$d_4 : d(x,y) \le d(x,z) + d(z,y).$$

If d satisfies the condition $d_1 - d_4$, then d is called a metric on X. If it satisfies the conditions d_1, d_2 and d_4 it is called a quasi-metric space. If d satisfies conditions d_2, d_3 and d_4 it is called a dislocated metric (or simply d-metric). If d satisfies only d_2 and d_4 then d is called a dislocated quasi-metric (or simply dq-metric) on X.

Definition 1.2 [20] A sequence $(x_n)_{n\in\mathbb{N}}$ in a dq-metric space (X,d) dislocated quasi-converges (for short, dq-converges) to $x \in X$ if $\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$. In this case x is called a dq-limit of $(x_n)_{n\in\mathbb{N}}$ and we write $x_n \to x$.

Definition 1.3 [20] A sequence $(x_n)_{n\in\mathbb{N}}$ in a dq-metric space (X,d) is said to be Cauchy if for every $\varepsilon > 0$, $\exists n_0 \in N$ such that $\forall m,n \ge n_0$, $d(x_m, x_n) < \varepsilon$ and $d(x_n, x_m) < \varepsilon$.

Definition 1.4 [20] A dq-metric space (X,d) is complete if every Cauchy sequence in it is dq-convergent in X.

Example 1.5 Let $X = [0,1]$ and $d(x,y) = \max\{x,y\}$. Then the pair (X,d) is a dislocated metric space, but it is not a metric space.

Lemma 1.6 [20] Every subsequence of dq-convergent sequence to a point x_0 is dq-convergent to x_0.

Definition 1.7 [20] Let (X,d) be a dq-metric space. A mapping $T : X \to X$ is called contraction if there exists $0 \le \lambda < 1$ such that: $d(Tx,Ty) \le \lambda d(x,y)$ for all $x, y \in X$.

Lemma 1.8 [20] dq-limit in a dq-metric space is unique.

Definition 1.9 [7] Let A and B be nonempty subsets of a metric space (X,d) and $T: A\cup B \to A\cup B$. T is called a cyclic map iff $T(A)\subseteq B$ and $T(B)\subseteq A$.

Definition 1.10 [18] Let A and B be nonempty subsets of a metric space (X,d). A cyclic map $T: A\cup B \to A\cup B$ is said to be a cyclic contraction if there exists $k\in(0,1)$ such that $d(Tx,Ty)\le kd(x,y)$ for all $x\in A$ and $y\in B$.

Definition 1.11[4] Let A and B be nonempty subsets of a metric space (X,d). A cyclic map $T: A\cup B \to A\cup B$ is called a Kannan type cyclic contraction if there exists $k\in\left(0,\frac{1}{2}\right)$ such that $d(Tx,Ty)\le k\left[d(Tx,x)+d(Ty,y)\right]$ for all $x\in A$ and $y\in B$.

In [4] Karapinar et al has been shown that Kannan type cyclic contraction and cyclic contraction are independent of each other.

Definition 1.12 [4] Let A and B be nonempty subsets of a metric space (X,d). A cyclic map $T: A\cup B \to A\cup B$ is called a Chatterjee type cyclic contraction if there exist $k \in (0, 1/2)$ such that $d(Tx,Ty)\le k\max\left[d(x,y),d(Tx,x),d(Ty,y)\right]$ for all $x\in A$ and $y\in B$.

Definition 1.13 [18] Let A and B be nonempty subsets of a dislocated metric space (X,d). A cyclic map $T: A\cup B \to A\cup B$ is called a d-cyclic contraction if there exists $k\in(0,1)$ such that $d(Tx,Ty)\le kd(x,y)$ for all $x\in A$ and $y\in B$.

Example 1.14 Let $X=[0,1]$ and $d(x,y)=|x-y|+3|x|+3|y|$. Then, (X,d) is a dislocated metric space, but not a metric space. Let $A=B=[0,1]$ and define $T: A\cup B \to A\cup B$ by $Tx=\frac{1}{3}$ for $x=1$ and $Tx=1$ for $x\in[0,1)$. Then T is a d-cyclic contraction in the dislocated metric space (X,d). We note that in the usual metric $d(x,y)=|x-y|$ the self map T is not cyclical contraction because for $x=\frac{11}{12}$ and $y=1$ the cyclic contraction fails.

Hence the class of d- cyclical contraction in dislocated metric space is larger than the class of cyclical contraction in usual metric.

2. Main Results

Theorem 2.1 Let A and B be nonempty subsets of a complete dislocated quasi-metric space (X,d). Let $T: A\cup B \to A\cup B$ be a cyclic mapping that satisfies the condition

$$d(Tx,Ty)\le k\max\{d(x,y)d(x,Tx)d(y,Ty)\} \qquad (1)$$

for all $x\in A$ and $y\in B$ and $0\le k<1$.

Then, T has a unique fixed point in $A\cap B$.

Proof. Taking a point $x\in A$ (fix) and using contractive condition of theorem, we have

$$\begin{aligned} d\left(T^2x,Tx\right) &= d\left(T(Tx),Tx\right) \\ &\le k\max\left\{d(Tx,x),d\left(Tx,T^2x\right),d(x,Tx)\right\} \\ &\le k\max\left\{d(Tx,x),d(x,Tx)\right\} \end{aligned}$$

In the same way we have,

$$\begin{aligned} d\left(Tx,T^2x,\right) &= d\left(Tx,T(Tx)\right) \\ &\le k\max\left\{d(x,Tx),d(x,Tx),d\left(Tx,T^2x\right)\right\} \\ &\le kd(x,Tx) \\ &\le k\max\left\{d(Tx,x),d(x,Tx)\right\} \end{aligned}$$

If we put $\lambda=\max\{d(x,Tx),d(Tx,x)\}$, then from two inequalities above we have,

$$d\left(T^2x,Tx\right)\le k\lambda \qquad (2)$$

$$d\left(Tx,T^2x\right)\le k\lambda \qquad (3)$$

Using (2) and (3) we get, $d\left(T^3x,T^2x\right)\le k^2\lambda$ and $d\left(T^2x,T^3x\right)\le k^2\lambda$.

Inductively, using this process for all $n\in N$ we have $d\left(T^{n+1}x,T^nx\right)\le k^n\lambda$ and $d\left(T^nx,T^{n+1}x\right)\le k^n\lambda$ for all $n\in N$.

Let $n,m\in\mathbb{N}$ with $m>n$, using the triangular inequality, we obtain:

$$\begin{aligned} d\left(T^mx,T^nx\right) &\le d\left(T^mx,T^{m-1}x\right)+d\left(T^{m-1}x,T^{m-2}x\right) \\ &\quad +\ldots.+d\left(T^{n+1}x,T^nx\right) \\ &\le k^n\lambda+k^{n+1}\lambda+\ldots.+k^{m-1}\lambda \\ &=\left[k^n+k^{n+1}+\ldots.+k^{m-1}\right]\lambda \\ &\le \frac{k^n}{1-k}\lambda \end{aligned}$$

Since $0\le k<1$, $k^n\to 0$ as $n\to\infty$, we get $d\left(T^mx,T^nx\right)\to 0$. Thus (T^nx) is a Cauchy sequence.

Since (X,d) is complete, we have (T^nx) dq-converges to some $z\in X$. We note, that $\left(T^{2n}x\right)$ is a sequence in A and $\left(T^{2n-1}x\right)$ is a sequence in B in a way that both sequences tend to same limit z.

Since A and B are closed have that $z\in A\cap B$. Hence $A\cap B\neq\varnothing$.

We claim that $Tz=z$.

Considering the condition (1) we have:

$$\begin{aligned} &d\left(T^{2n}x,Tz\right) \\ &= d\left(TT^{2n-1}x,Tz\right) \\ &\le k\max\left\{d\left(T^{2n-1}x,z\right),d\left(T^{2n-1}x,T^{2n}x\right),d(z,Tz)\right\} \end{aligned}$$

Taking limit as $n \to \infty$ in above inequality, we have

$$d(z,Tz) \le kd(z,Tz) < d(z,Tz)$$

This implies that $d(z,Tz) = 0$ since $0 \le k < 1$.

Similarly considering (1) have,

$$d\left(Tz,T^{2n}x\right)$$
$$= d\left(Tz,TT^{2n-1}x\right)$$
$$\le k\max\left\{d\left(z,T^{2n-1}x\right),d(z,Tz),d\left(T^{2n-1}x,T^{2n}x\right)\right\}$$

Taking limit as $n \to \infty$ and since $0 \le k < 1$, we obtain $d(Tz,z) = 0$.

Hence $d(z,Tz) = d(Tz,z) = 0 \Rightarrow Tz = z$ and z is a fixed point of T.

We shall prove that z is the unique fixed point of T. Clearly from (1) if u and v be fixed points of T we have $d(u,u) = 0, d(v,v) = 0$.

Then we have,

$$\begin{aligned} d(u,v) &= d(Tu,Tv) \\ &\le k\max\{d(u,v),d(u,u),d(v,v)\} \\ &= kd(u,v) \end{aligned}$$

Since $0 \le k < 1$ this implies $u = v$. Hence the proof is completed.

For following theorem we denotes with S the class of those real functions $\beta : [0,\infty) \to [0,1)$ that satisfy the condition $\beta(t_n) \to 1$ implies $t_n \to 0$. Examples of those functions exist in the corresponding literature. Using this class of functions we give this definition in the framework of dislocated metric spaces.

Definition. Let A and B be nonempty subsets of a dislocated quasi-metric space (X,d). A cyclic map $T : A \cup B \to A \cup B$ is called a Geraghty type dq-cyclic contraction if there exists $\beta \in S$ such that $d(Tx,Ty) \le \beta(d(x,y))d(x,y)$ for all $x \in A$ and $y \in B$.

Theorem 2.2 Let A and B be nonempty closed subsets of a dislocated metric space (X,d) and $T : A \cup B \to A \cup B$ be a cyclic mapping that satisfies the Geraghty type condition:

$$d(Tx,Ty) \le \beta(d(x,y))d(x,y) \tag{4}$$

for all $x \in A$ and $y \in B$ where $\beta \in S$.

Then T has a unique fixed point in $A \cap B$.

Proof. Fix a point $x \in A$. If $T^n x = T^{n+1}x$ for some $n \in N$, then $T^{n+1}x = T^{n+2}x$ and so $(T^n x)$ converges to some $z \in X$. Suppose $T^n x \ne T^{n+1}x$. Using condition (4) we have:

$$\begin{aligned} d\left(T^2x,Tx\right) &\le \beta(d(Tx,x))d(Tx,x) \\ &< d(Tx,x) \end{aligned}$$

Also we have

$$\begin{aligned} d\left(T^3x,T^2x\right) &\le \beta\left(d\left(T^2x,Tx\right)\right)d\left(T^2x,Tx\right) \\ &< d\left(T^2x,Tx\right) \end{aligned}$$

Inductively in general we have $d\left(T^{n+1}x,T^n x\right) \le d\left(T^n x,T^{n-1}x\right) \le .... \le d(Tx,x)$. Thus the sequence $d\left(T^{n+1}x,T^n x\right)$ is decreasing and bounded from below, thus it converges to some $z \ge 0$ If we suppose that $z > 0$, then from (4) have

$$\frac{d\left(T^{n+1}x,T^n x\right)}{d\left(T^n x,T^{n-1}x\right)} \le \beta\left(d\left(T^n x,T^{n-1}x\right)\right) < 1$$

Taking limit as $n \to \infty$, we get

$$\lim_{n\to\infty} \beta\left(d\left(T^{n-1}x,T^n x\right)\right) = 1$$

By property of β follows that $\lim\limits_{n\to\infty} d\left(T^{n-1}x,T^n x\right) = 0$. In a similar way we obtain $\lim\limits_{n\to\infty} d\left(T^n x,T^{n-1}x\right) = 0$ So our supposition fail from this contradiction. Hence $z = 0$. To proceed further we define $r = \sup_{x,y\in X}\{\beta(d(x,y))\}$ then $\beta\left(d\left(T^n x,T^{n-1}x\right)\right) \le r < 1$ for all $n \in N$. Using the main condition of theorem we obtain:

$$\begin{aligned} d\left(T^2x,Tx\right) &\le \beta(d(Tx,x))d(Tx,x) \\ &\le rd(Tx,x) \end{aligned}$$

Similarly,

$$\begin{aligned} d\left(T^3x,T^2x\right) &\le \beta\left(d\left(T^2x,Tx\right)\right)d\left(T^2x,Tx\right) \\ &< rd\left(T^2x,Tx\right) \end{aligned}$$

As a result get $d\left(T^3x,T^2x\right) \le r^2 d(Tx,x)$.

Thus in general we get $d\left(T^{n+1}x,T^n x\right) \le r^n d(Tx,x)$ for $n,m \in \mathbb{N}$ with $m > n$, using the triangular inequality, we obtain:

$$\begin{aligned} d\left(T^m x,T^n x\right) &\le d\left(T^m x,T^{m-1}x\right) + d\left(T^{m-1}x,T^{m-2}x\right) \\ &\quad + ... + d\left(T^{n+1}x,T^n x\right) \\ &\le r^n d(Tx,x) + r^{n+1}d(Tx,x) \\ &\quad + ... + r^{m-1}d(Tx,x) \\ &= \left[r^n + r^{n+1} + ... + r^{m-1}\right]d(Tx,x) \\ &\le \frac{r^n}{1-r}d(Tx,x) \end{aligned}$$

Since $0 \le r < 1$, $r^n \to 0$ as $n \to \infty$, we get $d\left(T^m x,T^n x\right) \to 0$. This proves that $(T^n x)$ is a Cauchy sequence. Since (X,d) is complete, we have $(T^n x)$ dq-converges to some $z \in X$. Note that $\left(T^{2n}x\right)$ is a sequence in A and $\left(T^{2n-1}x\right)$ is a sequence in B in a way that both sequences tend to same limit $z \in A \cap B$.

Considering the condition (4) we have:

$$d(z.Tz) \le d\left(z,T^{2n}x\right) + d\left(T^{2n}x,Tz\right)$$
$$\le d\left(z,T^{2n}x\right) + \beta\left(d\left(T^{2n-1}x,z\right)\right)d\left(T^{2n-1}x,z\right)$$

Taking limit as $n \to \infty$ in above inequality, we have $d(Tz,z)=0$ and in similar have $d(z,Tz)=0$ as a result $Tz=z$.

Uniqueness: Let u and v be two fixed points of T. Then:

$$d(u,v) = d(Tu,Tv)$$
$$\le \beta\left(d(u,v)\right)d(u,v)$$
$$< d(u,v)$$
$$d(v,u) = d(Tv,Tu)$$
$$\le \beta\left(d(v,u)\right)d(v,u)$$
$$< d(v,u)$$

from those inequalities we get $d(u,v)=d(v,u)=0$ and also, by property d_2 have $u=v$.

Example 3.3 Let $X=[0,1]$ and $T: X \to X$ be given as $Tx=\frac{x}{7}$. Let $A=B=[0,1]$. Define the function $d: X^2 \to [0,\infty)$ by $d(x,y)=\max\{x,y\}$. The function $\beta:[0,\infty)\to[0,1)$ defined as $\beta(t)=\frac{e^{-t}}{t+1}$, for $t>0$ and $0 \le \beta(0) < 1$. We note that d is a dislocated metric on X and the map T is cyclic on X and $\beta \in S$.

Considering all cases and general cases if $x \le y$, for all $x,y \in X$ we have, $\forall y \in [0,1]$

$$d(Tx,Ty) = d\left(\frac{x}{7},\frac{y}{7}\right)$$
$$= \max\left\{\frac{x}{7},\frac{y}{7}\right\}$$
$$= \frac{y}{7}$$
$$< \frac{1}{7} < \frac{1}{2e} \le \frac{ye^{-y}}{y+1}$$
$$= \beta(y)y = \beta\left(d(x,y)\right)d(x,y)$$

Clearly all conditions of theorem 3.2 are satisfied and 0 is the unique fixed point of T.

For the following theorems and corollaries we consider the set G_3 of all continuous functions $g:[0,\infty)^3 \to [0,\infty)$ [some examples for these functions see in 10] with the following properties:

a). g is non-decreasing in respect to each variable.

b). $g(t,t,t) \le t$, for $t \in [0,\infty)$.

Theorem 2.4 Let A and B be nonempty closed subsets of a dislocated quasi-metric space (X,d) and $T: A\cup B \to A \cup B$ be a cyclic mapping that satisfies the following condition:

$$d(Tx,Ty) \le cg\left[d(x,y), d(x,Tx), d(y,Ty)\right] \qquad (5)$$

for all $x \in A$ and $y \in B$, and $0 < c < 1$, where $g \in G_3$.

Then T has a unique fixed point in $A \cap B$.

Proof. Let x be a fixed point in X. By condition (5) and properties of g we have:

$$d\left(T^2x,Tx\right) \le cg\left[d(Tx,x), d\left(Tx,T^2x\right), d(x,Tx)\right]$$
$$\le cd(Tx,x)$$

Similarly we have

$$d\left(T^3x,T^2x\right) \le cg\left[d\left(T^2x,Tx\right), d\left(T^3x,T^2x\right), d\left(Tx,T^2x\right)\right]$$
$$\le cd\left(T^2x,Tx\right)$$

Generally from the above inequalities have:

$$d\left(T^{n+1}x,T^nx\right) \le cd\left(T^nx,T^{n-1}x\right) \le ..... \le c^n d(Tx,x)$$

for $n \in N$.

Since $0 \le c < 1$ we obtain for $n \to \infty$ that $d\left(T^{n+1}x,T^nx\right) \to 0$. In the same way we can show that $d\left(T^nx,T^{n+1}x\right) \to 0$.

Easily as in the above theorems we can show that the sequence $(T^n x)$ is a Cauchy sequence in complete dislocated metric space (X,d). So there exists $z \in X$ such that $(T^n x)$ dislocated quasi converges to z. Note that, $\left(T^{2n}x\right)$ is a sequence in A and $\left(T^{2n-1}x\right)$ is a sequence in B in a way that both sequences tend to same limit $z \in A \cap B$. For proving that z is a fixed point of T we use again the contractive condition (5),

$$d(z.Tz)$$
$$\le d\left(z,T^{2n}x\right) + d\left(T^{2n}x,Tz\right)$$
$$\le d\left(z,T^{2n}x\right)$$
$$+ cg\left[d\left(T^{2n-1}x,z\right), d\left(T^{2n-1}x,T^{2n}x\right), d(z,Tz)\right]$$

In this inequality passing in limit as $n \to \infty$ and since g is non decreasing and continuous we get, $d(z,Tz) \le cd(z,Tz)$ and since $0 \le c < 1$ we obtain $d(z,Tz)=0$. Again from (5) get $d(Tz,z)=0$. As a result $z=Tz$.

Uniqueness Let suppose that u and v are two fixed points of T where $Tu=u$ and $Tv=v$.

From condition of theorem we have,

$$d(Tu,Tv) \le cg\left[d(u,v),d(u,Tu),d(v,Tv)\right] \qquad (6)$$

If we replace $v = u$ in (6) then we obtain,

$$\begin{aligned} d(u,u) &= d(Tu,Tu) \\ &\le cg\left[d(u,u),d(u,Tu),d(u,Tu)\right] \\ &= cg\left[d(u,u),d(u,u),d(u,u)\right] \\ &\le cd(u,u) \end{aligned}$$

Thus from $d(u,u) \le cd(u,u)$ and since $0 \le c < 1$, we get $d(u,u) = 0$. Similarly we have that $d(v,v) = 0$. Therefore using condition (5) we have:

$$\begin{aligned} d(u,v) &= d(Tu,Tv) \\ &\le cg\left[d(u,v),d(u,u),d(v,v)\right] \\ &\le cd(u,v) \end{aligned}$$

And also,

$$\begin{aligned} d(v,u) &= d(Tv,Tu) \\ &\le cg\left[d(v,u),d(v,v),d(u,u)\right] \\ &\le cd(v,u) \end{aligned}$$

So from this inequality we have $d(u,v) = d(v,u) = 0$ and property d_2 implies $v = u$. Hence fixed point is unique.

Example 2.5 Let $X = [-1,1]$ and $T : X \to X$ be given as $Tx = \frac{-x}{4}$. Let $A = [-1,0]$ and $B = [0,1]$. Define the function $d : X^2 \to [0,\infty)$ by $d(x,y) = |x-y| + |x| + |y|$. We note that d is a dislocated quasi-metric on X and the map T is cyclic on X.

If we consider from G_3 the function $g(t_1,t_2,t_3) = \max\{t_1,t_2,t_3\}$ we see:

$$d(x,y) = |x-y| + |x|$$

$$d(Tx,Ty) = \left|\frac{-x}{4} - \frac{-y}{4}\right| + \left|\frac{-x}{4}\right| = \frac{1}{4}|x-y| + \frac{1}{4}|x|$$

Then clearly have,

$$\begin{aligned} d(Tx,Ty) &= \left|\frac{-x}{4} - \frac{-y}{4}\right| + \left|\frac{-x}{4}\right| \\ &= \frac{1}{4}\left(|x-y| + |x|\right) \\ &= \frac{1}{4}d(x,y) \\ &\le cg\left[d(x,y),d(x,Tx),d(y,Ty)\right] \end{aligned}$$

So for constant $\frac{1}{4} \le c < 1$ the map T satisfies the condition (5) of theorem 3.4 and 0 is the unique fixed point of T.

From general character of theorem 3.4 we can give many corollaries as follows using functions

$$g(t_1,t_2,t_3) = \left[\max\left\{t^p{}_1,t^p{}_2,t^p{}_3\right\}\right]^{\frac{1}{p}},\ p > 0$$

$$g(t_1,t_2,t_3) = \left[\max\{t_1t_2,t_2t_3,t_1t_3\}\right]^{\frac{1}{2}},\ p > 0$$

$$g(t_1,t_2,t_3) = \max\{t_1+t_2,t_2+t_3,t_1+t_3\}.$$

Corollary 2.6 Let A and B be nonempty closed subsets of a dislocated quasi-metric space (X,d) and $T : A \cup B \to A \cup B$ be a cyclic mapping that satisfies the following condition:

$$d^p(Tx,Ty) \le c\max\left\{d^p(x,y),d^p(x,Tx),d^p(y,Ty)\right\}$$

for all $x \in A$ and $y \in B$, and $0 < c < 1$.

Then T has a unique fixed point in $A \cap B$.

Corollary 2.7 Let A and B be nonempty closed subsets of a dislocated quasi-metric space (X,d) and $T : A \cup B \to A \cup B$ be a cyclic mapping that satisfies the following condition:

$$d^2(Tx,Ty) \le c\max\left\{\begin{aligned} &d(x,y)d(x,Tx), \\ &d(x,y)d(y,Ty), \\ &d(x,Tx)d(y,Ty) \end{aligned}\right\}$$

for all $x \in A$ and $y \in B$, and $0 < c < 1$.

Then T has a unique fixed point in $A \cap B$.

Corollary 2.8 Let A and B be nonempty closed subsets of a dislocated quasi-metric space (X,d) and $T : A \cup B \to A \cup B$ be a cyclic mapping that satisfies the following condition:

$$d(Tx,Ty) \le c\max\left\{\begin{aligned} &d(x,y)+d(x,Tx), \\ &d(x,y)+d(y,Ty), \\ &d(x,Tx)+d(y,Ty) \end{aligned}\right\}$$

for all $x \in A$ and $y \in B$, and $0 < c < 1$.

Then T has a unique fixed point in $A \cap B$.

Further as common applications of fixed point theorems we are giving some corollaries for cyclic maps for integral type contraction. (taking $A = B = X$)

Corollary 2.9 Let (X,d) be a complete dislocated quasi-metric space and $T : X \to X$ be a mapping such that for any $x,y \in X$,

$$\int_0^{d(Tx,Ty)} \rho(t)\,dt \le r\int_0^{\beta(d(x,y))d(x,y)} \rho(t)\,dt$$

where the function $\phi \in \Phi$, the constant $r \in [0,1)$ and $\rho : [0,\infty) \to [0,\infty)$ is Lesbegue-integrable mapping satisfying $\int_0^{\varepsilon} \rho(t)\,dt > 0$ for $\varepsilon > 0$. Then, T has a unique fixed point in X.

Remark 2.10 Our Theorem 3.4 generalizes and unifies results for Kannan type cyclic contraction, Chatterjea cyclic contraction, C cyclical contraction, Zamfirescu contraction and some existing results in dislocated-metric spaces [2,3,10,11,15]. Statements of many theorems and results can be obtained by taking $A = B = X$.

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References

[1] A. A Eldered and P Veeramani (2006) Convergence and existence for Best proximity Points. J. Math Analysis and Applications, 323, 1001-1006.

[2] C. T. Aage and J. N. Salunke. The results on fixed points in dislocated and dislocated quasi-metric space. *Appl. Math. Sci.*, 2(59): 2941-2948, 2008.

[3] C. T. Aage and J. N. Salunke Some results of fixed point theorem in dislocated quasi-metric spaces, *Bulletin of Marathwada Mathematical Society*, 9(2008), 1-5.

[4] Erdal Karapinar and Inci M Erhan (2010) Best Proximity on Different Type Contractions *Applied Mathematics and Information Science*

[5] E. Karapinar and H. K. Nashine (2013) Fixed point Theorems for Kannan type cyclic weakly contractions *Journal of Nonlinear Analysis and Optimization*, vol. 4, N0. 1, (2013), 29-35.

[6] E. Karapinar, Fixed point Ttheory for cyclic weak ϕ -contraction, Appl. Math. Lett. 24 (2011) 822-825.

[7] E. Karapinar and P. Salimi, Dislocated metric space to metric spaces with some fixed point theorems, *Fixed point theory and applications (2013).*

[8] G Petrushel (2005), Cyclic representations and Periodic points, *Studia Univ. Babes - Bloyai Math*, 50, 107-112.

[9] K. Jha and D. Panthi, A Common Fixed Point Theorem in Dislocated Metric Space, *Appl. Math. Sci*., vol. 6, 2012, no. 91, 4497-4503.

[10] K. Zoto and E. Hoxha Fixed point theorems in dislocated and dislocated quasi-metric space *Journal of Advanced Studies in Topology*; Vol. 3, No.1, 2012.

[11] K. Zoto and Hoxha, E: Fixed point theorems for cyclic contractions. Proceedings in ARSA, the second conference of advanced research in scientific areas. 2-6 december2013.

[12] P. Hitzler and A. K. Seda. Dislocated topologies. *J. Electr. Engin.*, 51(12/S):3:7, 2000.

[13] P. Hitzler. Generalized Metrics and Topology in Logic Programming Semantics. Ph.d. thesis, *National University of Ireland,University College Cork*, 2001.

[14] Pacurar M., Rus, I.A - (2010) Fixed Point Theory for φ -contractions, *Nonlinear Anlaysis*, 72, (3-4), 1181-1187.

[15] Reny George, R. Rajagopalan, S. Vinayagam;Cyclic contractions and fixed points in dislocated metric spaces. *Int. Journal of Math. Analysis, vol. 7, 2013, no.9, 403-411.*

[16] S. Karpagam and Sushma Agrawal (2010) Best Proximity Points theorems for Cyclic Meir Keeler Contraction Maps.

[17] Sh.Rezapur and M.Derafshpour and N.Shahzad - Best Proximity point of cyclic φ contractions in ordered metric spaces *Topological Methods in Nonlinear Analysis* (in press).

[18] W.A.Kirk and P.S. Srinivasan and P.Veeramani(2003) Fixed Points for mapping satsifying Cyclic contractive conditions. *Fixed Point Theory*, 4, 79-89.

[19] M. A. Geraghty, On contractive mappings, Proc. Amer. Math. Soc. 40(1973), 604-608.

[20] F. M. Zeyada, G. H. Hasan and M. A. Ahmed, A generalization of a fixed point theorem due to Hitzler and Seda in dislocated quasi-metric spaces, *Arab. J. Sci. Eng. Sec. A Sci.* 31(1) (2006) 111-114.