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From Fold to Fold-Hopf Bifurcation in IFOC Induction Motor: a Computational Algorithm

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Abstract Analytical determination of the bifurcation thresholds is of important interest for practical electrical machine design and analysis control. This paper presents mathematical investigation of the qualitative behavior of indirect filed oriented control induction motor based on bifurcation theory. In this context, steady-state responses analysis of the motor model is discussed and an analytical study of generic bifurcations was made. Particular attention is paid to the codimension two bifurcation namely Fold-Hopf bifurcation. The paper introduces some elementary mechanisms of transit from Fold to Fold-Hopf parameter singularity, to derive some analytical rigorous existence conditions and to develop an algorithm for Fold-Hopf bifurcation detection. Some numerical results of equilibrium properties and bifurcation diagrams are then performed to outline our methodology.

Keywords: *IFOC Induction Motor, steady-state responses, bifurcation, Fold, Hopf, Fold-Hopf*

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1. Introduction

Applied mathematics is an interdisciplinary subject, which provides a broad qualitative and quantitative background for use in electrical engineering field. In this study we focus on aspects of theoretical and numerical analysis which is related to the bifurcation theory applied to the electrical motors.

The hallmark of the nonlinear behavior is the bifurcation, which is an abrupt qualitative change in the phase space motion which is refers to the small variations of one or more of system control parameters. The analysis of the behavior of dynamical systems can be carried out globally through bifurcation analysis, which describes the range of parameter values at which qualitative propriety changes occurs. Crossing a bifurcation point, existence and uniqueness of solutions is not guaranteed and a change in stability and/or order and/or the number of solutions occurs. In former studies [1,2] three types of generic codimension one bifurcations of periodic solutions of nonlinear ODE were described: tangent, period doubling and Hopf bifurcation. If an equilibrium point satisfies two bifurcation conditions, we call the singularity as codimension two bifurcations. A typical case of codimension-2 bifurcation is the Fold-Hopf bifurcation (ZH) which is a transversal intersection of fold and hopf bifurcation curves at a bifurcation value. Aiming to study the existence of such particular bifurcation point in induction motor submitted to an Indirect Field-oriented control, analytical analysis are used to put into evidence the mathematical condition of their existence. Such parametric singularities can be caused by the parameter fluctuations namely the errors in the estimate of the rotor time constant which changes widely with temperature [3].

Field-oriented controllers FOC, frequently used as nonlinear controllers for induction machines, performs asymptotic linearization and decoupling [4]. Stability of FOC is generally investigated regarding errors in the estimate of the rotor resistance. It has been previously shown that the speed control of induction motors through under field-oriented control (IFOC) is globally asymptotically stable for any constant load torque.

An analysis of parameter plane singularities (saddle-node and Hopf bifurcations) in IFOC drives with respect to the rotor time constant variation provides a guideline for setting properly the motor parameters in order to avoid such bifurcations [5,6].

In the recent year, the qualitative approach has become increasingly useful tool in the analysis of nonlinear power systems. Aiming to understand the bifurcation mechanism and their associated responses, one should identify phase plane singularities (equilibrium, limit cycles, attraction basins) and parameter plane singularities (bifurcations, chaos) [7]. In former studies [8,9,10], it has been shown the occurrence of either codimension one bifurcation such as saddle node bifurcation and Hopf bifurcation and codimension two such as Bogdanov-Takens or zero-Hopf bifurcation in IFOC induction motors. The cancelation of sustained oscillations which are, in general undesirable was the purpose of other studies which proposed the 'oscillation killer' in order to adjust the system and control parameters so that one can get rid of limit cycles [11]. Other results [12] permit to promote efficiency or improve dynamic characteristics of drives. An adequate combination between analytical and numerical tools may provide a deep comprehension of some nontrivial dynamical behavior related to bifurcation phenomena in a self-sustained oscillator [13].

The robustness margins for IFOC of induction motors can be deduced from the analysis of the bifurcation structures identified in parameter plane [14]. Since the self-sustained oscillations in IFOC for induction motors may be due to the appearance of a Hopf bifurcation [15], an exhaustive study of the bifurcation structures is mainly dedicated to preserve the local stability of the desired equilibrium point.

An outline of the paper is organized as follows. The governing equations of the IFOC induction motor and a general reminder will be described in section 2. The detailed analytical analysis of fold and zero-hopf bifurcations will be proposed in section 3, together with the numerical algorithm of ZH-bifurcation detection. Some numerical simulations will be given in section 4 and the conclusion in the section 5.

2. System Equation Formulations

2.1. Mathematical Model of IFOC Motor

Mathematically, the governing equations used for the modeling of indirect field-oriented control of induction motor can be described as [9] by the following set of four nonlinear autonomous differential equations:

$$\dot{x}_1 = -c_1.x_1 + (k.c_1 / u_2^0).x_2.x_4 + c_2.u_2^0 \tag{1}$$

$$\dot{x}_2 = -c_1.x_2 + (k.c_1 / u_2^0).x_2.x_4 + c_2.x_4 \tag{2}$$

$$\dot{x}_3 = -c_3.x_3 - c_4.(c_5(x_1.x_4 - x_2.u_2^0) - T_L) \tag{3}$$

$$\dot{x}_4 = (k_i - k_p.c_3).x_3 - k_p.c_4.(c_5(x_1.x_4 - x_2.u_2^0) - T_L) \tag{4}$$

For this set of equations, x_1, x_2, x_3 and x_4 denote the variable states. Preciously, x_1 and x_2 are the direct and the quadratic component of the rotor flux, respectively. x_3 being the difference between reference and the real mechanical speed. The last variable x_4 present's the quadratic component of the stator current results from the outer loop PI. The parameter bifurcation $k = \tau_r / \tau_e$ is the ratio of the rotor time constant τ_r to its estimate τ_e and u_2^0 is a design parameter. k_p and k_i are the proportional and the integral PI controller gains, respectively. We can also define the following constants c_1, c_2, c_3 and c_4, where:

$c_1 = \tau_r^{-1} = L_r.R_r^{-1}$ is the rotor flux time constant.

$c_2 = L_m.\tau_r^{-1}$, L_m is the mutual inductance

$c_3 = f_c.\tau_r^{-1}$, f_c is the friction constant.

$c_4 = n_p.J^{-1}$, where J is the moment of inertia and n_p is the pole pair number.

Finally, let $\Gamma = T_L + \frac{c_3}{c_4}\omega_{ref}$ be a constant where T_L denotes the load torque.

2.2. Analytical Steady- State Solution

In numerical methods or bifurcation problems, the phase plane analysis is an important technique for studying the behavior of nonlinear systems. Since, a two-parameter plane can be considered as made up of sheets as mentioned in previous paper [2], each one being associated with a well defined behaviour such as a fixed point, or an equilibrium or a periodic orbit.

With change in temperature or frequency there will be change in the values of the motor parameters. As the variation of these parameters increases, so will their effects on system dynamic behavior. In particular, the rotor time constant which may vary considerably over the operational range of the rotor resistance which changes widely with temperature. So, we choose in this study $k = \tau_r / \tau_e$ as the bifurcation parameter. This parameter is defined as the degree of tuning, i.e., if $k = 1$ the system is considered to be tuned, otherwise it is said to be detuned [9].

The purpose of the present work is to give an analytical investigation of the influence of small variation of the parmeter k on the dynamical behavior of motor model. A closed form analytical solution is developed for the tuned case characteristic of the motor ($k = 1$). In addition to this tuned phase, a considerable simplification of equation model (1)-(4) may be achieved in the magnetization phase of IFOC obtained by putting $T_L = 0$ and $\omega_{ref} = 0$. It is easy to verify that the equilibrium point $x_0^e = (x_1^e, x_2^e, x_3^e, x_4^e)^t = (0, \frac{c_2}{c_1}u_2^c, 0, 0)^t$ is reached while the machine is in the standstill conditions.

In previous studies (see [15,16,17]) the equilibrium solutions of the system of equations (1)-(4) have been determined under general conditions. More importantly, it has been proven that an equilibrium point is parameterized in term of two dimensionless parameters (r and $\hat{r}$) and it has the following analytical expression:

$$x_r^e = \begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \\ x_4^e \end{bmatrix} = \begin{bmatrix} \frac{c_2 u_2^c}{c_1} \frac{1-k}{1+k^2 r^2} r \\ \frac{c_2 u_2^c}{c_1} \frac{1+kr^2}{1+k^2 r^2} \\ 0 \\ u_2^e r \end{bmatrix} \tag{5}$$

Analytically, r presents any real solution of the following 3rd order polynomial equation:

$$kr^3 - \hat{r}k^2 r^2 + kr - \hat{r} = 0 \tag{6}$$

In fact, r is a mathematically term related to physical properties of the system loading. This term is defined by the following expression:

$$kr^3 - \hat{r}k^2 r^2 + kr - \hat{r} = 0 \tag{7}$$

where β is a constant denoted by $\beta = \frac{c_2 c_4 c_5 u_2^c}{c_1}$.

On the other hand, the term r is related to the quadrature axis component of the stator current. This parameter is defined by the following simple expression:

$$r = \frac{x_4^e}{u_2^c} \quad (8)$$

In the previous paper [8], it has been proven that for any numerical parameter k which verifies the following property $k \le 3$, this polynomial has a unique real solution. When $k > 3$, it has been demonstrated also that the number of equilibrium points varies under the following two criteria:

$$|\hat{r}| \ge \hat{r}_a = kr_a \frac{1+r_a^2}{1+k^2 r_a^2} \quad (9)$$

$$|\hat{r}| \le \hat{r}_b = kr_b \frac{1+r_b^2}{1+k^2 r_b^2} \quad (10)$$

The expresssions of r_a and r_b are defined by, respectively:

$$r_a = \frac{\sqrt{2}}{2k}\sqrt{k^2 - 3 + \sqrt{(k^2-9)(k^2-1)}} \quad (11)$$

$$r_b = \frac{\sqrt{2}}{2k}\sqrt{k^2 - 3 - \sqrt{(k^2-9)(k^2-1)}} \quad (12)$$

When $k > 3$, the IFOC equations (1)-(4) possess three, two or one equilibrium point according to the fact that criteria (9) and (10) are strictly verified or not. If either (9) and (10) are both verified with equality, then the system has 2 equilibrium points. In any other case the system has a unique equilibrium point.

2.3. Mathematic Reminders in Bifurcation Theory

A composite picture of the phase space behaviour can be gained by studying the 4th order autonomous ODEs (1-4) describing the IFOC induction motor (Equilibrium points, limit cycles, chaotic orbits). Each phasingularity involves four eigenvalues describing its stability. Generally, a local bifurcation at an equilibrium happens when some eigenvalues of the parametrized linear approximating differential equation cross some critical values such us the origin or the imaginary axis. Self-sustained oscillations in IFOC of induction motors can be originated by a codimension one bifurcation namely the Hopf bifurcation (H). Such kind of bifurcation can be computed from differential system (1)-(4), when a pair of complex conjugate eigenvalues among the eigenvalues set of the associate linearized system change from negative to positive real parts or vice versa.

Therefore the Hopf bifurcation results from the transversal crossing of the imaginary axis by a pair of complex conjugate eigenvalues. Such bifurcation is said to be supercritical if the periodic branch is initially stable and subcritical if the periodic branch is initially unstable.

The singular curves of the parameter plane corresponding to codimension-1 bifurcations may contain singular points of higher codimension [18,19]. The simplest one located on a Hopf curve has the codimension-2 singular point, a Fold-Hopf (ZH). This bifurcation, also called the Zero-Hopf (ZH), is a codimension 2 bifurcation of an equilibrium point at which the critical equilibrium lies at a tangential intersection of curves of fold bifurcation curve and Andronov-Hopf bifurcation curve within the two parameter family.

Definition:

Consider an autonomous system of ordinary differential equations (ODEs),

$$\dot{x} = f(x,\mu),\ x \in \mathbb{R}^n \quad (13)$$

where x denotes the state vector, $\mu = (\mu_1, \mu_2) \in \mathbb{R}^2$ is a 2-dimensional parameters vector and f is smooth.

Suppose that at μ_0 the system has equilibrium x^*. In the n-dimensional case with $n \ge 3$, the Jacobian matrix J_f has:

1. a simple zero eigenvalue $\lambda_1 = 0$ and a simple pair of purely imaginary eigenvalues $\lambda_{1,2} = \pm iq$, as well as
2. n_s eigenvalues with $\mathrm{Re}(\lambda_j) < 0$, and
3. n_u eigenvalues with $\mathrm{Re}(\lambda_j) > 0$, with $3 + n_s + n_u = n$.

3. Analytical Bifurcation Analysis

Local phase portrait near singularities can be very complicated. This phase-plane picture is developed by determining the form of the solution of the 4th order autonomous ODEs (1)-(4) describing the IFOC induction motor near each of its singularities (equilibrium points EP, limit cycles LC). Each one of these solutions involves four eigenvalues describing its stability. Evaluating the expression of Jacobian J_F at the equilibrium point (5), we get the following form:

$$J_F = \begin{bmatrix} -c_1 & -c_1 kr & 0 & c_2\delta_1 \\ c_1 kr & -c_1 & 0 & c_2\delta_1 kr \\ \frac{c_1}{c_2}\beta & -\frac{c_1}{c_2}\beta r & c_3 & -\beta\delta_2 \\ \frac{c_1}{c_2}\beta k_p & -\frac{c_1}{c_2}\beta r k_p & k_i - k_p c_3 & -\beta\delta_2 k_p \end{bmatrix} \quad (14)$$

where $\gamma_1 = \frac{(1-k)}{1+k^2 r^2}$ and $\gamma_2 = \frac{(1+kr)}{1+k^2 r^2}$.

Let P_λ define the characteristic equation of J_F:

$$P_\lambda = \det(J_F - \lambda . I_4) \quad (15)$$

λ being the eigenvalue parameter and I_4 denotes a unit matrix of order 4. It is easy to verify that P_λ is a 4th degree polynomial function:

$$P_\lambda = \sum_{i=0}^{4} \omega_i \lambda^i \quad (16)$$

The coefficients $\omega_i (i = 1,...,4)$ are defined as follows:

$$\omega_0 = c_1^2 \beta k_i k \sigma_0 \quad (17)$$

$$\omega_1 = c_1^2 c_3 (1 + k^2 r^2) + c_1^2 \beta k_p k \sigma_0 + c_1 \beta k_i \sigma_1 \quad (18)$$

$$\omega_2 = c_1^2 (1 + k^2 r^2) + c_1 \beta k_p \sigma_1 + \beta \delta_2 k_i + 2 c_1 c_3 \quad (19)$$

$$\omega_3 = \beta \delta_2 k_p + 2c_1 + c_3 \quad (20)$$

$$\omega_4 = 1 \quad (21)$$

where σ_0 and σ_1 have the following expressions:

$$\sigma_0 = \frac{k^2 r^4 + (3 - k^2) r^2 + 1}{1 + k^2 r^2} \quad (22)$$

$$\sigma_1 = \frac{k(3-k) r^2 + k + 1}{1 + k^2 r^2} \quad (23)$$

We next give the appropriate analytical conditions characterising the occurence of fold and fold-hopf bifurcations.

3.1. Analytical Condition of Fold Detection

A Fold (F) bifurcation of equilibrium point, called also Saddle-Node or a limit point (LP) is a codimension one bifurcation which occurs when a single eigenvalue of the characteristic polynomial is equal to zero.

To check whether $\lambda = 0$ is a solution of $P_\lambda = 0$, simply verify that the coefficient ω_0 is equal to zero. Thus, announced the following theorem using the equations (16) and (21):

Theorem 1:

If there exists a real k ($k > 0$) and a real r, solution of (6), for which the following condition is satisfied:

$$k^2 r^4 + (3 - k^2) r^2 + 1 = 0$$

Then, the Jacobian of system (1)-(4) presents a single zero eigenvalue and fold bifurcation can occur.

3.2. Analytical Condition of Fold-Hopf Bifurcation (ZH)

In the general case, the necessary analytical condition for the existence of a Zero-Hopf bifurcation (ZH) for the system (1)-(4), at the equilibrium point (5), is that the equation $P_\lambda = 0$ has a simple zero eigenvalue $\lambda_1 = 0$ and one pair of purely imaginary roots $\lambda_{2,3}$.

Let's solve the following four degrees equation:

$$P_\lambda = \omega_4 + \omega_3 \lambda^3 + \omega_2 \lambda^2 + \omega_1 \lambda + \omega_0 = 0 \quad (24)$$

Let $\omega_0 = 0$, the condition of fold bifurcation occurence.

Then the equation (24) becomes:

$$P_\lambda = \lambda.(\lambda^3 + \omega_3 \lambda^2 + \omega_2 \lambda^1 + \omega_1) = \lambda.Q_\lambda \quad (25)$$

Let's consider the following third degree polynomial function

$$Q_\lambda = a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0 \quad (26)$$

Were $a_3 = 1$, $a_2 = \omega_3$, $a_1 = \omega_2$ and $a_0 = \omega_1$ are the real coefficients.

Examing the zero of 3-dimensional polynomial Q_λ.

Let the roots of Q_λ be denoted μ_1, μ_2 and μ_3, then the factored form of the polynomial is given by:

$$Q_\lambda = a_3.(\lambda - \mu_1).(\lambda - \mu_2).(\lambda - \mu_3) \quad (27)$$

Expanding the plynomial factors, we easily obtain the following relations:

$$\begin{cases} \mu_1 + \mu_2 + \mu_3 = -\omega_3 \\ \mu_1.\mu_2 + \mu_1.\mu_3 + \mu_2.\mu_3 = \dfrac{a_1}{a_3} = \omega_3 \\ \mu_1.\mu_2.\mu_3 = -\dfrac{a_0}{a_3} = -\omega_1 \end{cases} \quad (28)$$

The general form of the discriminant formula \$\delta\$ of a n-dimensional polynomial is given by:

$$\delta = a_n^{2n-2}.\prod_{i<j} (\mu_i - \mu_j)^2 \quad (29)$$

Where $\mu_{i,j} (1 \le i < j \le n)$ are the roots of the polynomial.

For $n = 3$, the discriminant of Q_λ is defined as follows:

$$\delta = a_3^4.(\mu_1 - \mu_2)^2.(\mu_1 - \mu_3)^3.(\mu_2 - \mu_3)^2 \quad (30)$$

The discriminant δ can be expressed in term of the coefficients $\mu_i (i = 1:3)$ as follows:

$$\delta = a_2^2 a_1^2 + 18 a_3 a_2 a_1 a_0 - 27 a_3^2 a_0^2 - 4 a_3 a_1^3 - 4 a_2^3 a_0 \quad (31)$$

Finally,

$$\delta = \omega_3^2 + 18 \omega_3 \omega_2 \omega_1 - 27 \omega_1^2 - 4 \omega_2^3 - 4 \omega_3^3 a_1 \quad (32)$$

When does a real polynomial have a pair of complex conjugate roots?

3.2.1. Root Types

Computation of the purely imaginary eigenvalues based on elementary properties of cubic polynomials, we report on three cases of root types:

Case 1: μ_1, μ_2 and μ_3 are distinct real roots.

In which case, δ can be written as:

$$\delta = (a_3^2.(\mu_1 - \mu_2).(\mu_1 - \mu_3).(\mu_2 - \mu_3))^2$$

It is easy to verify that the discriminant δ is positive.

Case 2: Existence of two equal real roots.

We can find three possible double roots.

$$\mu_1 - \mu_2 = 0, \mu_1 - \mu_3 = 0, \mu_2 - \mu_3 = 0$$

Since the discriminant is zero:

$$\delta = (a_3^2.(\mu_1 - \mu_2).(\mu_1 - \mu_3).(\mu_2 - \mu_3))^2 = 0$$

Case 3: Two complex conjugate roots.

By considering the symmetric properties, let (μ_1, μ_2) denote the two complexe conjugate roots and μ_3 denotes the real root of Q_λ.

Roots of the polynomial can be written in the following forms:

$$\begin{cases} \mu_1 = r + iq \\ \mu_2 = r - iq \\ \mu_3 = s \end{cases} \quad (33)$$

Then the discriminant δ can be calculated as:

$$\delta = a_3^4.(\mu_1 - \mu_2)^2.(\mu_1 - \mu_3)^2.(\mu_2 - \mu_3)^2$$

then, $\delta = -\left[2qa_3^2((r-s)^2 + q^2)\right]^2$.

In this case, the discriminant δ is negative. We report all results on root types in the following Lemma:

Lemma 1:

If $\delta > 0$, then Q_λ has three real roots.

If $\delta = 0$, then Q_λ has a double root.

If $\delta < 0$, then Q_λ has has a pair of complex conjugate roots.

3.2.2. Computation of the Purely Imaginary Eigenvalues

In the case of $\delta < 0$, Q_λ has one pair of complex conjugate roots. The Hopf bifurcation point can be detected when only one pair of the roots of Q_λ have a zero real parts. This condition can be deduced generally from the generalized Routh-Hurwitz criterion applied to the characteristic polynomial (26) [9]:

λ^3	1	ω_2
λ^2	ω_3	ω_1
λ^1	$\gamma = \frac{\omega_2.\omega_3 - \omega_1}{\omega_3}$	--
λ^0	--	--

if $\gamma = \frac{\omega_2.\omega_3 - \omega_1}{\omega_3} = 0$ then one pair of purely imaginary roots can be detected.

Proof:

Let the purely imaginary roots of Q_λ be denoted $\mu_1 = iq_0$ and $\mu_2 = -iq_0$, were q_0 is a non-zero real numbers. Each of the two roots (μ_1, μ_2) satisfies the given relation:

$$Q_\lambda(\mu_1) = Q_\lambda(\mu_2) = 0 \quad (34)$$

As a result,

$$Q_\lambda(iq_0) = -iq_0^3 - \omega_3 q_0^2 + i\omega_2 q_0 + 1 = 0 \quad (35)$$

and

$$Q_\lambda(-iq_0) = iq_0^3 - \omega_3 q_0^2 - i\omega_2 q_0 + 1 = 0 \quad (36)$$

Subtraction of the equation (35) from the equation (36) gives:

$$2iq_0^3 - 2iq_0\omega_2 = 0 \quad (37)$$

Then

$$q_0^2 = \omega_2 \quad (38)$$

Addition of two equations (35) and (36) gives:

$$2\omega_1 - 2\omega_3 q_0^2 = 0 \quad (39)$$

Then

$$q_0^2 = \frac{\omega_1}{\omega_3} \quad (40)$$

The equality between two expressions (38) and (40) gives: $\frac{\omega_1}{\omega_3} = \omega_2$, we then obtain $\omega_2.\omega_3 - \omega_1 = 0$.

The particular roots of Q_λ are:

1. ω_2 is real positive number.
2. Two purely imaginary roots $\mu_1 = \bar{\mu}_2 = i\sqrt{\omega_2}$
3. One real root $\mu_3 = -\omega_3$.

3.2.3. The conditions of Fold-Hopf Bifurcation

Now, our main results, with respect to the above conditions, are announced in the following theorem.

Theorem:

If there exists a real k ($k > 0$), r and μ for which the following conditions are satisfied:

(ZH. 1) Occurrence test: $\delta < 0$,

(ZH. 2) One zero root for: $k^2r^4 + (3 - k^2)r^2 + 1 = 0$,

(ZH. 3) One pair of purely imaginary roots for:

$$\left(\beta\delta_2 k_p + 2c_1 + c_3\right).\begin{pmatrix} c_1^2(1 + k^2r^2) + c_1\beta k_p\sigma_1 \\ +\beta\delta_2 k_i + 2c_1c_3 \end{pmatrix}$$
$$-\left(c_1^2c_3(1 + k^2r^2) + c_1^2\beta k_p k\sigma_0 + c_1\beta k_i\sigma_1\right) = 0$$

The Jacobian of system (1)-(4) presents one zero eigenvalue in addition of one pair of purely imaginary eigenvalues and a Fold-Hopf bifurcation can be detected.

3.3. Fold-Hopf Bifurcation Detection Algorithm

Let's now define the following numerical algorithm of ZH detection based on the announced theorem and in light of the obtained results discussed above. The proposed algorithm includes 7 steps detailed as follows:

Step 1: Set appropriate initial parameter values starting out from the tuned case $k = 1$ and the equilibrium point $(x_1^e, x_2^e, x_3^e, x_4^e) = (0, \frac{c_2}{c_1}u_2^c, 0, 0)$.

Step 2: Identify the coefficients ω_i of the polynomials expressions P_λ.

Step 3: Computing the discriminant δ of Q_λ, testing if $\delta < 0$.

Step 4: verify the condition ω_0 =0.

Step 5: Verify the condition $\omega_2\omega_3 - \omega_1 = 0$.

Step 6: The Zero-Hopf bifurcation is detected with eigenvalues $\lambda_1 = \bar{\lambda}_2 = i.\sqrt{\omega_2}$, $\lambda_3 = 0$ and λ_4 is a non zero real number.

Step 7: Update parameter, change k by $k+h$ where h is an appropriate small number. Go to step 2.

4. Numerical Simulations

The Zero-Hopf bifurcation mechanism's generation from an equilibrium point, is illustrated by the graph of Figure 1. For the parameters $k=4$, $k_i=2$, k_p and $T_L=2.5$ an equilibrium point (EP) is identified.

The EP vector is $(x_{10}^*, x_{20}^*, x_{30}^*, x_{40}^*)$ = (2.426, -1.76, 0.089, 0.495), solution of the differential system (2) for the initial conditions set $(x_{10}, x_{20}, x_{30}, x_{40})$ =(0,0.5,0, 0).

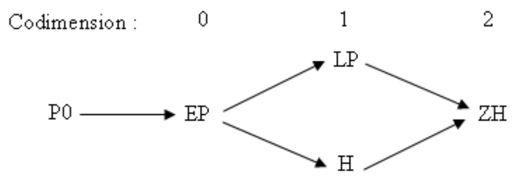

Figure 1. Equilibrium Point: bifurcation process

Starting from this located initial equilibrium, a continuation method permits to obtain the evolution of the direct compound flux x_1 versus the values of bifurcation parameter k (see Figure 2). One Hopf bifurcation is obtained in such curve. This singularity has one pair of purely imaginary eigenvalues and the following coordinates in phase space: (3.694, -0.723, 0, and 0.211) for k = 2.0135. The corresponding eigenvalues are (-1.61, -0.946, i2.936, - i2.936).

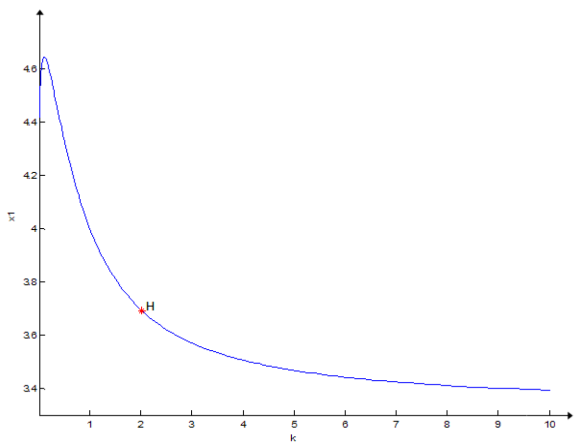

Figure 2. Bifurcation diagram

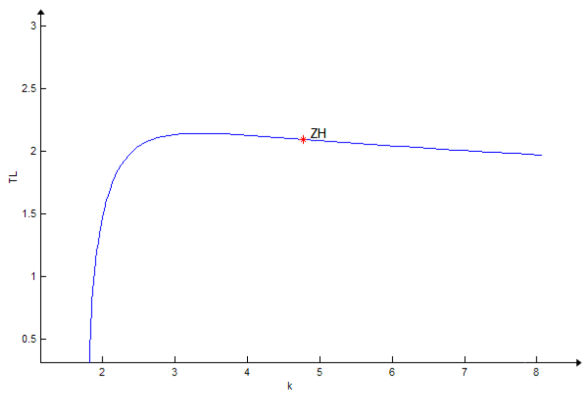

Figure 3. Hopf bifurcation curve

The continuation of the Hopf point (H) leads to trace the Hopf bifurcation curve shown in Figure 3. Such curve includes a 2-codimension Zero-Hopf bifurcation point in (2.2573, -1.5733, 0, 0.2318) for the parameters k = 4.7792 and T_L = 2.0965. The corresponding eigenvalues of this singularity are λ = (2.3386, i2.8593, - i2.8593, 0).

In fact, the ZH-bifurcation point in the parameters (k, T_L) -plane is a critical point at which the critical equilibrium lies at a tangential intersection of fold bifurcation curve ζ_F and Hopf bifurcation curve ζ_H within the two parameter family (see Figure 4).

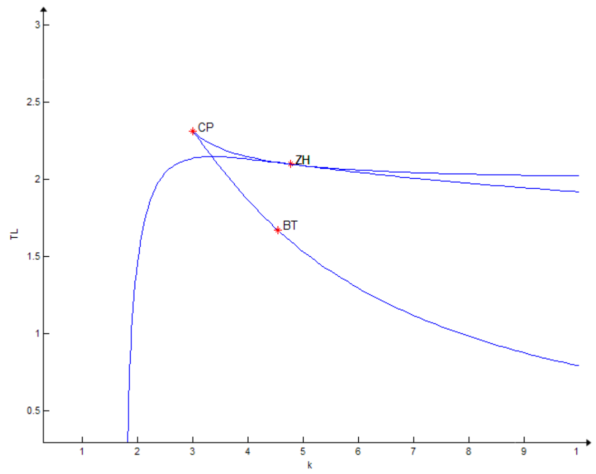

Figure 4. Fold-Hopf bifurcation detection

For the parameters values k = 3.2 and T_L =2.409, the Fold curve ζ_F includes two branches joining in a codimension two bifurcation points, namely cuspidal point (CP) having the following phase space coordinates $(x_{10}, x_{20}, x_{30}, x_{40})$ = (2.01, -1.1547, 0, 0.5774).

Besides, such curve presents a Bogdanov-Taken bifurcation (BT) in $(x_{10}, x_{20}, x_{30}, x_{40})$ = (1.062, -0.727, 0, 0.891) for the parameters values k = 4.53852 and T_L = 1.6721, with the associates eigenvalues λ = (-1.0796 + i4.2773, -1.0796- i4.2773, 0, 0).

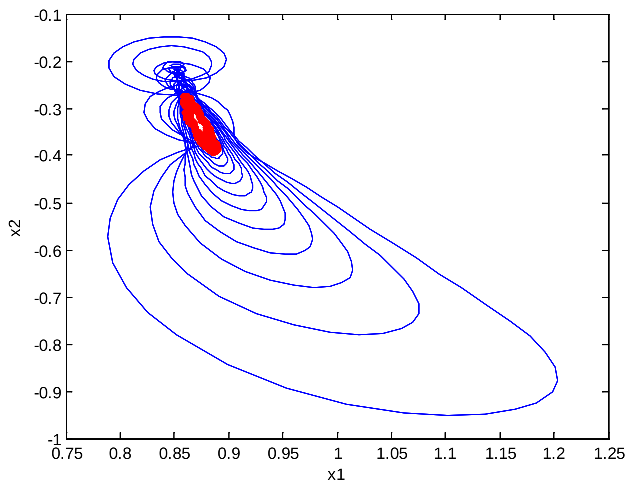

Figure 5. Phase plane trajectory at Fold-Hopf bifurcation

The process of convergence of phase trajectory in the (x_1, x_2) −plane shows a critical equilibrium at the Zero-

Hopf bifurcation point (see Figure 5). The red points present the generated limit cycle.

5. Conclusion

This paper applies bifurcation and singularity analysis to study the complex dynamic behavior of IFOC induction motor. The computed steady state is shown to be adequat for low codimension bifurcation studies. Particularely, the paper clearly proposes an analytical analysis of fold and fold-hopf (ZH) bifurcations. This mathematical approach yields the theoretical test conditions under which the existence of such bifurcation singularities is verified and that can be solved analytically. The paper also introduces a computational algorithm for the detection of fold-hopf bifurcation using developed conditions. Finally, bifurcation curves that exhibit fold (F), hopf (H) and fold-hopf (ZH) bifurcations for various parameters are traced through numerical continuation methods.

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