Solving Some Definite Integrals Using Parseval's Theorem

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Abstract This article takes advantage of the mathematical software Maple for the auxiliary tool to study six types of definite integrals. The infinite series forms of these definite integrals can be obtained by using Parseval's theorem. In addition, we propose some examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions using Maple.

Keywords: definite integrals, infinite series forms, Parseval's theorem, Maple

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1. Introduction

In calculus and engineering mathematics courses, we learnt many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, and so on. In this paper, we study the following six types of definite integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int_0^{2\pi} \left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^2(r\cos x) + \cos^2(r\sin x)} \right]^2 dx \tag{1}$$

$$\int_0^{2\pi} \left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^2(r\cos x) + \cos^2(r\sin x)} \right]^2 dx \tag{2}$$

$$\int_{0}^{2\pi} \frac{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)}{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)} dx$$
(3)

$$\int_{0}^{2\pi} \left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} \right]^{2} dx$$
(4)

$$\int_0^{2\pi} \left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^2(r\cos x) + \sin^2(r\sin x)} \right]^2 dx$$
(5)

$$\int_{0}^{2\pi} \frac{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} dx$$
(6)

where r is a real number. We can obtain the infinite series forms of these definite integrals by using Parseval's

theorem; these are the major results of this paper (i.e., Theorems 1 and Theorems 2). The study of related integral problems can refer to [1-26]. On the other hand, we provide some definite integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce a notation and a definition and some formulas used in this article.

2.1. Notation

Let z = a + ib be a complex number, where $i = \sqrt{-1}$, a, b are real numbers. We denote a the real part of z by Re(z), and b the imaginary part of z by Im(z).

2.2. Definition

Suppose f(x) is a continuous function defined on $[0,2\pi]$, the Fourier series expansion of f(x) is $\frac{1}{2}a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$, where

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx \text{ and } a_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos kx dx ,$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin kx dx \text{ for all positive integers } k .$$

2.3. Formulas

2.3.1. Euler's Formula

 $e^{ix} = \cos x + i \sin x$, where x is any real number.

2.3.2. DeMoivre's Formula

 $(\cos x + i \sin x)^n = \cos nx + i \sin nx$, where *n* is any integer, and *x* is any real number.

2.3.3. ([27])

 $\sinh(p+iq) = \sinh p \cos q + i \cosh p \sin q$ where p, q are real numbers.

2.3.4. ([27])

 $\cosh(p+iq) = \cosh p \cos q + i \sinh p \sin q$

where p, q are real numbers.

2.3.5. Taylor Series Expansion of Hyperbolic Tangent Function ([28])

$$\tanh(z) = \sum_{n=1}^{\infty} \frac{2^{2n} (2^{2n} - 1) B_{2n}}{(2n)!} z^{2n-1}, \text{ where } z$$

is a complex number, $|z| < \frac{\pi}{2}$ and B_n are Bernoulli numbers for all positive integers n.

2.3.6. Taylor Series Expansion of Hyperbolic Cotangent Function ([28])

complex number, $0 < |z| < \pi$.

Next, we introduce an important theorem used in this study.

2.4. Parseval's Theorem ([29])

If f(x) is a continuous function defined on $[0,2\pi]$, and $f(0) = f(2\pi)$. Suppose the Fourier series expansion of f(x) is

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad , \quad \text{then}$$

$$\frac{1}{\pi}\int_0^{2\pi} f^2(x)dx = \frac{{a_0}^2}{2} + \sum_{n=1}^\infty ({a_n}^2 + {b_n}^2) \,.$$

Before deriving the first major result of this paper, we need a lemma.

2.5. Lemma 1

Suppose p, q are real numbers with $\sinh^2 p + \cos^2 q \neq 0$. Then

$$\tanh(p+iq) = \frac{\sinh p \cosh p + i \sin q \cos q}{\sinh^2 p + \cos^2 q}$$
(7)

$$\frac{\sinh^2 p \cosh^2 p + \sin^2 q \cos^2 q}{(\sinh^2 p + \cos^2 q)^2} = \frac{\sinh^2 p + \sin^2 q}{\sinh^2 p + \cos^2 q}.$$
 (8)

Proof

$$\tanh(p + iq)$$

$$= \frac{\sinh(p + iq)}{\cosh(p + iq)}$$

$$= \frac{\sinh p \cos q + i \cosh p \sin q}{\cosh p \cos q + i \sinh p \sin q}$$
(By Formulas 2.3.3 and 2.3.4)

$$= \frac{(\sinh p \cos q + i \cosh p \sin q)(\cosh p \cos q - i \sinh p \sin q)}{\cosh^2 p \cos^2 q + \sinh^2 p \sin^2 q}$$
$$= \frac{\sinh p \cosh p + i \sin q \cos q}{\sinh^2 p + \cos^2 q}.$$

And

$$\frac{\sinh^2 p \cosh^2 p + \sin^2 q \cos^2 q}{(\sinh^2 p + \cos^2 q)^2}$$
$$= \frac{\sinh^2 p (1 + \sinh^2 p) + (1 - \cos^2 q) \cos^2 q}{(\sinh^2 p + \cos^2 q)^2}$$

$$=\frac{\sinh^2 p + \sin^2 q}{\sinh^2 p + \cos^2 q}$$

In the following, we find the infinite series forms of the definite integrals (1), (2) and (3).

2.6. Theorem 1

Suppose r is a real number with $|r| < \pi / 2$. Then the definite integrals

$$\int_{0}^{2\pi} \left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)} \right]^{2} dx$$

$$= \pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n}(2^{2n}-1)^{2}(B_{2n})^{2}}{[(2n)!]^{2}} r^{4n-2}.$$
(9)
$$\int_{0}^{2\pi} \left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)} \right]^{2} dx$$

$$= \pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n}(2^{2n}-1)^{2}(B_{2n})^{2}}{[(2n)!]^{2}} r^{4n-2}.$$
(10)
$$\int_{0}^{2\pi} \frac{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)}{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)} dx$$

$$= 2\pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n}(2^{2n}-1)^{2}(B_{2n})^{2}}{[(2n)!]^{2}} r^{4n-2}.$$
(11)

Proof Because

$$\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^2(r\cos x) + \cos^2(r\sin x)}$$

= Re[tanh(re^{ix})] (ByEq.(7))
= Re $\left[\sum_{n=1}^{\infty} \frac{2^{2n}(2^{2n}-1)B_{2n}}{(2n)!} (re^{ix})^{2n-1}\right]$

(By Formula 2.3.5)

$$= \operatorname{Re}\left[\sum_{n=1}^{\infty} \frac{2^{2n} (2^{2n} - 1)B_{2n}}{(2n)!} r^{2n-1} e^{i(2n-1)x}\right]$$

(By DeMoivre's formula)

$$=\sum_{n=1}^{\infty} \frac{2^{2n} (2^{2n} - 1) B_{2n}}{(2n)!} r^{2n-1} \cos(2n-1) x$$

(By Euler's formula) (12)

By Parseval's theorem, we obtain

$$\int_0^{2\pi} \left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^2(r\cos x) + \cos^2(r\sin x)} \right]^2 dx$$
$$= \pi \cdot \sum_{n=1}^\infty \frac{2^{4n} (2^{2n} - 1)^2 (B_{2n})^2}{[(2n)!]^2} r^{4n-2}.$$

Similarly, because

$$\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^2(r\cos x) + \cos^2(r\sin x)}$$

= Im[tanh(re^{ix})] (By:Eq.(7))
= Im $\left[\sum_{n=1}^{\infty} \frac{2^{2n}(2^{2n}-1)B_{2n}}{(2n)!} (re^{ix})^{2n-1}\right]$

(By Formula 2.3.5)

$$=\sum_{n=1}^{\infty} \frac{2^{2n} (2^{2n} - 1) B_{2n}}{(2n)!} r^{2n-1} \sin(2n-1) x$$
(13)

Also using Parseval's theorem, we have

$$\int_{0}^{2\pi} \left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)} \right]^{2} dx$$
$$= \pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n} (2^{2n} - 1)^{2} (B_{2n})^{2}}{[(2n)!]^{2}} r^{4n-2}.$$

On the other hand, from the summation of Eq. (9) and (10) and using Eq. (8), we obtain

$$\int_{0}^{2\pi} \frac{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)}{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)} dx$$
$$= 2\pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n} (2^{2n} - 1)^{2} (B_{2n})^{2}}{[(2n)!]^{2}} r^{4n-2}.$$

Before deriving the second major result of this study, we also need a lemma.

2.7. Lemma 2

Suppose p, q are real numbers with $\sinh^2 p + \sin^2 q \neq 0$. Then

$$\operatorname{coth}(p+iq) = \frac{\sinh p \cosh p - i \sin q \cos q}{\sinh^2 p + \sin^2 q}$$
(14)

$$\frac{\sinh^2 p \cosh^2 p + \sin^2 q \cos^2 q}{(\sinh^2 p + \sin^2 q)^2} = \frac{\sinh^2 p + \cos^2 q}{\sinh^2 p + \sin^2 q}$$
(15)

Proof

 $\operatorname{coth}(p+iq)$

$$= \frac{\cosh(p+iq)}{\sinh(p+iq)} c = \frac{\cosh p \cos q + i \sinh p \sin q}{\sinh p \cos q + i \cosh p \sin q}$$

$$= \frac{(\cosh p \cos q + i \sinh p \sin q)(\sinh p \cos q - i \cosh p \sin q)}{\sinh^2 p \cos^2 q + \cosh^2 p \sin^2 q}$$
$$= \frac{\sinh p \cosh p - i \sin q \cos q}{\sinh^2 p + \sin^2 q}.$$

And

$$\frac{\sinh^2 p \cosh^2 p + \sin^2 q \cos^2 q}{(\sinh^2 p + \sin^2 q)^2}$$

$$= \frac{\sinh^2 p (1 + \sinh^2 p) + \sin^2 q (1 - \sin^2 q)}{(\sinh^2 p + \sin^2 q)^2}$$
$$= \frac{\sinh^2 p + \cos^2 q}{\sinh^2 p + \sin^2 q}.$$

In the following, we determine the infinite series forms of the definite integrals (4), (5) and (6).

2.8. Theorem 2

Suppose r is a real number with $0 < |r| < \pi$. Then the definite integrals

$$\int_{0}^{2\pi} \left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} \right]^{2} dx$$

$$= \pi \cdot \left[\left(\frac{1}{r} + \frac{r}{3} \right)^{2} + \sum_{n=2}^{\infty} \frac{2^{4n} (B_{2n})^{2}}{\left[(2n)! \right]^{2}} r^{4n-2} \right]. \quad (16)$$

$$\int_{0}^{2\pi} \left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} \right]^{2} dx$$

$$= \pi \cdot \left[\left(\frac{1}{r} - \frac{r}{3} \right)^{2} + \sum_{n=2}^{\infty} \frac{2^{4n} (B_{2n})^{2}}{\left[(2n)! \right]^{2}} r^{4n-2} \right]. \quad (17)$$

$$\int_{0}^{2\pi} \frac{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} dx$$

$$= 2\pi \left(\frac{1}{r^2} + \frac{r^2}{9}\right) + 2\pi \cdot \sum_{n=2}^{\infty} \frac{2^{4n} (B_{2n})^2}{\left[(2n)!\right]^2} r^{4n-2} .$$
(18)

Proof Because

$$\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^2(r\cos x) + \sin^2(r\sin x)}$$

= Re[coth(re^{ix})] (By Eq. (14))
= Re $\left[\frac{1}{re^{ix}} + \sum_{n=1}^{\infty} \frac{2^{2n}B_{2n}}{(2n)!} (re^{ix})^{2n-1}\right]$

(By Formula 2.3.6)

$$= \left(\frac{1}{r} + \frac{r}{3}\right) \cos x + \sum_{n=2}^{\infty} \frac{2^{2n} B_{2n}}{(2n)!} r^{2n-1} \cos(2n-1)x$$
(19)

Using Parseval's theorem, we have

$$\int_{0}^{2\pi} \left[\frac{\sinh(r\cos x)\cosh(r\cos x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} \right]^{2} dx$$
$$= \pi \cdot \left[\left(\frac{1}{r} + \frac{r}{3} \right)^{2} + \sum_{n=2}^{\infty} \frac{2^{4n}(B_{2n})^{2}}{[(2n)!]^{2}} r^{4n-2} \right].$$

Similarly, because

$$\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^2(r\cos x) + \sin^2(r\sin x)}$$

= - Im[coth(re^{ix})]
= - Im $\left[\frac{1}{re^{ix}} + \sum_{n=1}^{\infty} \frac{2^{2n}B_{2n}}{(2n)!} (re^{ix})^{2n-1}\right]$

(By Formula 2.3.6)

$$= \left(\frac{1}{r} - \frac{r}{3}\right) \sin x$$

- $\sum_{n=2}^{\infty} \frac{2^{2n} B_{2n}}{(2n)!} r^{2n-1} \sin(2n-1)x$ (20)

Also by Parseval's theorem, we obtain

$$\int_{0}^{2\pi} \left[\frac{\sin(r\sin x)\cos(r\sin x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} \right]^{2} dx$$
$$= \pi \cdot \left[\left(\frac{1}{r} - \frac{r}{3} \right)^{2} + \sum_{n=2}^{\infty} \frac{2^{4n} (B_{2n})^{2}}{[(2n)!]^{2}} r^{4n-2} \right]$$

In addition, from the summation of Eq. (16) and (17) and using (15), we have

$$\int_{0}^{2\pi} \frac{\sinh^{2}(r\cos x) + \cos^{2}(r\sin x)}{\sinh^{2}(r\cos x) + \sin^{2}(r\sin x)} dx$$
$$= 2\pi \left(\frac{1}{r^{2}} + \frac{r^{2}}{9}\right) + 2\pi \cdot \sum_{n=2}^{\infty} \frac{2^{4n} (B_{2n})^{2}}{\left[(2n)!\right]^{2}} r^{4n-2}$$

3. Examples

In the following, for the definite integrals in this study, we provide some examples and use Theorems 1 and 2 to determine their infinite series forms. On the other hand, we employ Maple to calculate the approximations of these definite integrals and their solutions for verifying our answers.

3.1. Example 1

Taking r = 1/3 into Eq. (9), we obtain the definite integral

$$\int_{0}^{2\pi} \left[\frac{\sinh(1/3 \cdot \cos x) \cosh(1/3 \cdot \cos x)}{\sinh^{2}(1/3 \cdot \cos x) + \cos^{2}(1/3 \cdot \sin x)} \right]^{2} dx$$
$$= \pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n} (2^{2n} - 1)^{2} (B_{2n})^{2}}{[(2n)!]^{2}} \left(\frac{1}{3}\right)^{4n-2}.$$
 (21)

Next, we use Maple to verify the correctness of Eq. (21). >evalf(int((sinh(1/3*cos(x))*cosh(1/3*cos(x)))^2/((sinh(1/3*cos(x))))^2+(cos(1/3*sin(x))))^2)^2,x=0..2*Pi),18);

0.349545626476568261 >evalf(Pi*sum(2^(4*n)*(2^(2*n)-1)^2*(bernoulli(2*n))^2/((2*n)!)^2*(1/3)^(4*n-

2),n=1..infinity),18);

0.349545626476568260

Similarly, if $r = 1/\sqrt{2}$ in Eq. (10), we have

$$\int_{0}^{2\pi} \left[\frac{\sin(1/\sqrt{2} \cdot \sin x) \cos(1/\sqrt{2} \cdot \sin x)}{\sinh^{2}(1/\sqrt{2} \cdot \cos x) + \cos^{2}(1/\sqrt{2} \cdot \sin x)} \right]^{2} dx$$

$$= \pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n} (2^{2n} - 1)^{2} (B_{2n})^{2} (-1)}{\sum_{n=1}^{4n-2} (B_{2n})^{2} (-1)^{2n}} = 0$$

 $\sum_{n=1}^{n} [(2n)!]^2 \left(\sqrt{2}\right)$ (22) >evalf(int((sin(1/sqrt(2)*sin(x))*cos(1/sqrt(2)*sin(x)))^2/((sinh(1/sqrt(2)*cos(x)))^2+(cos(1/sqrt(2)*sin(x)))^2)^2/2,

x=0..2*Pi),18);

1.61624943295020547

 $> evalf(Pi*sum(2^{(4*n)*(2^{(2*n)})})$

 $1)^{2}(bernoulli(2*n))^{2}/((2*n)!)^{2}*(1/sqrt(2))^{(4*n-1)}$

2),n=1..infinity),18);

1.61624943295020547 Finally, let r = 3/4 in Eq. (11), then

$$\int_{0}^{2\pi} \frac{\sinh^{2}(3/4 \cdot \cos x) + \sin^{2}(3/4 \cdot \sin x)}{\sinh^{2}(3/4 \cdot \cos x) + \cos^{2}(3/4 \cdot \sin x)} dx$$
$$= 2\pi \cdot \sum_{n=1}^{\infty} \frac{2^{4n} (2^{2n} - 1)^{2} (B_{2n})^{2}}{[(2n)!]^{2}} \left(\frac{3}{4}\right)^{4n-2}.$$
 (23)

>evalf(int(((sinh(3/4*cos(x)))^2+(sin(3/4*sin(x)))^2)/((sinh(3/4*cos(x)))^2+(cos(3/4*sin(x)))^2),x=0..2*Pi),18);

3.66517840220898049

>evalf(2*Pi*sum(2^(4*n)*(2^(2*n)-

1)^2*(bernoulli(2*n))^2/((2*n)!)^2*(3/4)^(4*n-

2),n=1..infinity),18);

3.66517840220898048

3.2. Example 2

Let r = 3 in Eq. (16), we obtain the definite integral

$$\int_{0}^{2\pi} \left[\frac{\sinh(3\cos x)\cosh(3\cos x)}{\sinh^{2}(3\cos x) + \sin^{2}(3\sin x)} \right]^{2} dx$$
$$= \pi \cdot \left[\frac{16}{9} + \sum_{n=2}^{\infty} \frac{2^{4n} (B_{2n})^{2}}{[(2n)!]^{2}} 3^{4n-2} \right].$$
(24)

>evalf(int((sinh(3*cos(x))*cosh(3*cos(x)))^2/((sinh(3* cos(x)))^2+(sin(3*sin(x)))^2)^2,x=0..2*Pi),18);

11.5167959003610174

>evalf(Pi*(16/9+sum(2^(4*n)*(bernoulli(2*n))^2/((2*n)))^2*3^(4*n-2),n=2..infinity)),18);

11.5167959003610174

In addition, if taking $r = \sqrt{5}$ into Eq. (17), then

$$\int_{0}^{2\pi} \left[\frac{\sin(\sqrt{5}\sin x)\cos(\sqrt{5}\sin x)}{\sinh^{2}(\sqrt{5}\cos x) + \sin^{2}(\sqrt{5}\sin x)} \right]^{2} dx$$
$$= \pi \cdot \left[\frac{4}{45} + \sum_{n=2}^{\infty} \frac{2^{4n}(B_{2n})^{2}}{[(2n)!]^{2}} (\sqrt{5})^{4n-2} \right].$$
(25)

>evalf(int((sin(sqrt(5)*sin(x))*cos(sqrt(5)*sin(x)))^2/((sinh(sqrt(5)*cos(x)))^2+(sin(sqrt(5)*sin(x)))^2)^2,x=0..2* Pi),18);

0.531916497721471181

>evalf(Pi*(4/45+sum(2^(4*n)*(bernoulli(2*n))^2/((2*n)!)^2*(sqrt(5))^(4*n-2),n=2..infinity)),18);

0.531916497721471182

On the other hand, let r = 13/6 in Eq. (18), then

$$\int_{0}^{2\pi} \frac{\sinh^{2}(13/6 \cdot \cos x) + \cos^{2}(13/6 \cdot \sin x)}{\sinh^{2}(13/6 \cdot \cos x) + \sin^{2}(13/6 \cdot \sin x)} dx$$
$$= 2\pi \left(\frac{36}{169} + \frac{169}{324}\right) + 2\pi \cdot \sum_{n=2}^{\infty} \frac{2^{4n}(B_{2n})^{2}}{\left[(2n)!\right]^{2}} \left(\frac{13}{6}\right)^{4n-2}.$$
(26)

>evalf(int(((sinh(13/6*cos(x)))^2+(cos(13/6*sin(x)))^2)/((sinh(13/6*cos(x)))^2+(sin(13/6*sin(x)))^2),x=0..2*Pi), 18);

5.01918539817249445

>evalf(2*Pi*(36/169+169/324)+2*Pi*sum(2^(4*n)*(be rnoulli(2*n))^2/((2*n)!)^2*(13/6)^(4*n-

2),n=2..infinity),18);

5.01918539817249446

4. Conclusion

In this paper, we use Parseval's theorem to determine some types of definite integrals. In fact, the applications of this theorem are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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