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Thermal Radiation Effect on a Porous Media under Optically Thick Approximation

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Abstract The purpose of this paper is to investigate thermal radiation effect on a porous media under optically thick approximation using Newton scheme method from Taylor series implemented numerically on MATLAB. The velocity profiles and temperature profile are studied for different physical parameters like, Porous term P, Radiation F and thermal Grashof number Ga. The results shows that Porous parameter increases with increasing velocity, while the trend reverses with thermal radiation and thermal Grashof number under optically thick approximation. The flow rate increases asymmetrically due to conduction.

Keywords: *porous media, recurrence relation, radiation, flow rate*

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1. Introduction

Fluid flow through a porous channel has been studied theoretically and experimentally by numerous authors due to its wide applications in various fields such as diffusion technology, transpiration cooling, hemodialysis processes, flow control in nuclear reactors, etc. In a pioneering work, Berman [1] studied an exact solution of the Navier-Stokes equations that describes the steady two-dimensional flow of an incompressible viscous fluid along a channel with parallel rigid porous walls, the flow being driven by uniform suction or injection at the walls. henceforth, many authors such as Terrill [2], Makinde [3], Robinson [4] and Brady [5] have extended and reconsidered the problem under various physical conditions. Minkowycz and Cheng [6], Cheng and Minkowycz [7], and Badr and Pop [8] were based on Darcy's law which states that the volume-averaged velocity is proportion to the pressure gradient. Kaviany [9] used the line integral method to study the heat transfer from a semi-infinite flat plate embedded in a fluid-saturated porous medium. Jang and Shiang [10] studied the mixed convection along a vertical adiabatic surface embedded in a porous medium. Few studies of convective boundary-layer flows in porous media using the Darcy-Brinkman equation model are considered for the momentum equation, for example, Hsu and Cheng [11], Rees and Vafai [12], Nazar et al. [13,14], Ishak et al. [15], and Harris et al [16].

Sreekanth *et al*. [17] examined the effect of permeability variation on free convective flow past a vertical porous wall in a porous medium when the permeability varies with respect to time. Subsequently, Singh *et al*. [18] studied hydro magnetic free convective and mass transfer flow of a viscous stratified fluid considering the variation in permeability with direction. The problem of unsteady oscillatory laminar free convection flow of an electrically conducting fluid through a porous medium along a porous hot vertical plate with time dependent suction in the presence of heat source or sink was analyzed by Kumar *et al.* [19] while Acharya *et al*. [20] discussed the magnetic field effects on the free convention and mass transfer flow through a porous medium with constant suction and constant heat flux. An experimental and analytic investigation was carried out by Fand et al. [21] to study the free convection heat transfer from a horizontal cylinder embedded in porous medium consisting of randomly packed glass spheres and the medium is saturated by water or silicon oil. Murthy and Singh [22] have modified the viscous dissipation effect on the flow of an incompressible fluid in a saturated porous medium. They applied the Forchheimer-Darcy model for momentum equation. Neild [23] suggested a modified formula to model viscous dissipation in non-Darcy porous medium. He concluded that modelling of viscous dissipation is related with local drag modelling. Recently Taiwo and Ogunlaran [24] studied Numerical solution of fourth order linear ordinary differential equations by cubic spline collocation tau method.

Makinde [25] studied Hermite-Pad´e approximation approach to nonlinear wall driven steady flow in a tube, Recently Ibrahim *et al* [27] presented and analysed

Radiative Effect on MHD Fluid Flow in a Vertical Channel under Optically Thick Approximation.

This research work is an improved work of Ibrahim [28] to include porous term. Hence it is proposed to study thermal radiation effect on porous media under optically thick approximation.

2. Formulation of the Problem

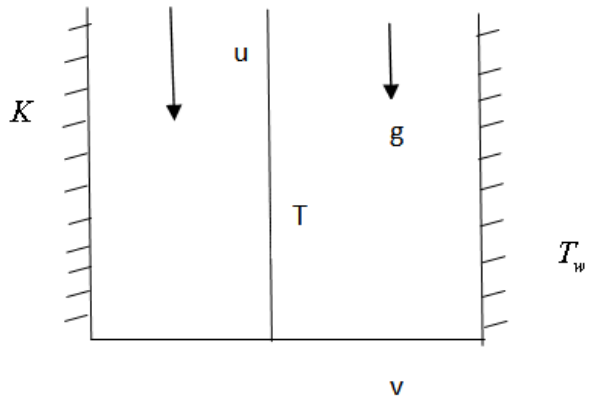

Figure 1. Geometry of the problem

The geometry of the problem shows thermal radiation effect on a porous media under optically thick approximation formulated, analysed and solved numerically. The x - axis is taken along the plate in the vertically upward direction and also the y -axis is taken normal to the plate. That the flow is fully developed, the velocity and temperature fields are symmetrical about the central line of the channel in a porous media. The temperature of the walls is the same and is maintained at a constant temperature. The viscosity, the thermal conductivity and specific heat are independent of temperature and the essential influence of the variation in density is included in the body force term. Steady flow equations are momemtum and energy equation.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{v'u}{K}+g\beta\left(T_w-T\right)+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (1)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho_0 C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho_0 C_p}\frac{\partial q_r}{\partial y} \quad (2)$$

where u and v are the velocity of the fluid, T is the fluid temperature, β is the thermal expansion of the fluid ρ_0 is the fluid density, C_p is the specific heat capacity, v is the viscosity of the fluid, k is the thermal conductivity and the temperature of the wall T_w, K permeability of the porous medium parameter and v' the kinematic viscosity.

In this research work the mathematical formulation has Porous term which is not included in the work of Ibrahim (1997) where α the thermal diffusivity and q_r is the radiative, using the Rosseland differential approximation.

$$\frac{\partial q_r}{\partial y}=-\frac{16}{3\alpha}\sigma T^3\frac{\partial T}{\partial y} \quad (3)$$

The boundary conditions are;

$$\left.\begin{array}{lll} T=0 & u=0 & y=b \\ T''=0 & u''=0 & y=-b \\ & & as\ y\to\infty \end{array}\right\} \quad (4)$$

On introducing the following non-dimensional quantities

$$\left.\begin{array}{l} y=bY,\ u=\dfrac{\alpha U}{b},\ T=-\tau\phi,\ P=\dfrac{b^2}{k} \\ F=bL\tau^3,\ G_a=\dfrac{g\beta b^3\tau}{v\alpha} \end{array}\right\} \quad (5)$$

Substituting the non-dimensional quantities of equation (5) into (1) to (2), leads to

$$\frac{d^2U}{dY^2}-PU=G_a\phi-\gamma \quad (6)$$

$$U=F\phi^3\frac{\partial\phi}{\partial Y}-\frac{\partial^2\phi}{\partial Y^2} \quad (7)$$

Where $\gamma=-\frac{b^3}{v\alpha}\left(\frac{1}{\rho_0}\frac{\partial p}{\partial x}\right)$

Equation (6) and (7) leads to

$$\frac{\partial^4\phi}{\partial Y^4}-F\phi^3\frac{\partial^3\phi}{\partial Y^3}-\left(6F\phi^2+M\right)\frac{\partial^2\phi}{\partial Y^2}-\left(6F\phi-MF\phi^3\right)\frac{\partial\phi}{\partial Y}+G_a\phi=\gamma \quad (8)$$

$$\phi(1)=0=\phi''(-1),\ U(1)=0=U''(-1) \quad (9)$$

3. Solution to the Problem

To solve equations (8), subjected to the boundary conditions of (9), the solutions are obtained for temperature and velocity flow. using the Newton scheme method from Taylor series to solve the fourth order non linear problem.

$$G+\Delta\phi\frac{\partial G}{\partial\phi_n}+\Delta\phi'\frac{\partial G}{\partial\phi_n'}+\Delta\phi''\frac{\partial G}{\partial\phi_n''}+\Delta\phi'''\frac{\partial G}{\partial\phi_n'''}+\Delta\phi^{iv}\frac{\partial G}{\partial\phi_n^{iv}}=0 \quad (10)$$

Thus from (8) we have

$$\frac{\partial G}{\partial\phi_n}=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3PF\phi_n^2\right)\phi_n'+G_a \quad (11)$$

$$\frac{\partial G}{\partial\phi_n'}=-\left(6F\phi_n-PF\phi_n^3\right) \quad (12)$$

$$\frac{\partial G}{\partial\phi_n''}=-\left(6F\phi_n^2+P\right) \quad (13)$$

$$\frac{\partial G}{\partial\phi_n'''}=-F\phi_n^3 \quad (14)$$

$$\frac{\partial G}{\partial\phi_n^{iv}}=1 \quad (15)$$

Substituting equations (11) to (15) into equation (10) we obtain

$$\begin{aligned}&\Delta\phi_n\left(-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3PF\phi^2\right)\phi_n'+G_a\right)\\&+\Delta\phi_n'\left(-\left(6F\phi_n-PF\phi_n^3\right)\right)+\Delta\phi_n''\left(-\left(6F\phi_n^2+P\right)\right)\\&+\Delta\phi_n'''\left(-F\phi_n^3\right)+\Delta\phi_n^{iv}(1)=0\end{aligned}\tag{16}$$

Expanding equations (16) where

$$\begin{aligned}&\Delta\phi_n=\phi_{n+1}-\phi_n,\ \ \Delta\phi_n'=\phi_{n+1}'-\phi_n',\ \ \Delta\phi_n''=\phi_{n+1}''-\phi_n'',\\&\Delta\phi_n'''=\phi_{n+1}'''-\phi_n''',\ \ \Delta\phi_n^{iv}=\phi_{n+1}^{iv}-\phi_n^{iv}\end{aligned}\tag{17}$$

$$\begin{aligned}&\left(\phi_{n+1}-\phi_n\right)\begin{pmatrix}-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''\\-\left(6F-3PF\phi^2\right)\phi_n'+G_a\end{pmatrix}\\&+\left(\phi_{n+1}'-\phi_n'\right)\left(-\left(6F\phi_n-PF\phi_n^3\right)\right)\\&+\left(\phi_{n+1}''-\phi_n''\right)\left(-\left(6F\phi_n^2+P\right)\right)\\&+\left(\phi_{n+1}'''-\phi_n'''\right)\left(-F\phi_n^3\right)+\left(\phi_{n+1}^{iv}-\phi_n^{iv}\right)(1)=0\end{aligned}$$

$$\begin{aligned}&\Rightarrow-3F\phi_n{}^2\phi_n'''\phi_{n+1}-12F\phi_n\phi_n''\phi_{n+1}-6F\phi_n'\phi_{n+1}+3PF\phi_n^2\phi_n'\phi_{n+1}\\&-G_a\phi_{n+1}+3F\phi_n{}^2\phi_n'''\phi_n+12F\phi_n\phi_n''\phi_n+6F\phi_n'\phi_n+3PF\phi_n^2\phi_n'\phi_n\\&+G_a\phi_n-6F\phi_n\phi_{n+1}'-PF\phi_n^3\phi_{n+1}'+6F\phi_n\phi_n'+PF\phi_n^3\phi_n'\\&-6F\phi_n\phi_{n+1}''+P\phi_{n+1}''+6F\phi_n^2\phi_n''-P\phi_n''\\&-F\phi_n^3\phi_{n+1}'''+F\phi_n^3\phi_n'''+\phi_{n+1}^{iv}-\phi_n^{iv}=0\end{aligned}\tag{18}$$

Collecting the terms involving

$$\phi_{n+1},\ \ \phi_{n+1}',\ \phi_{n+1}'',\ \phi_{n+1}'''\ and\ \ \phi_{n+1}^{iv}$$

On the L.H.S and the terms involving ϕ_n, ϕ_n', ϕ_n'', ϕ_n''' neglecting ϕ_n^{iv}

On the R.H.S gives

$$\begin{aligned}&\phi_{n+1}^{iv}-\left(F\phi_n^3\right)\phi_{n+1}'''-\left(6F\phi_n^2-P\right)\phi_{n+1}''\\&-\left(6F\phi_n+PF\phi^3\right)\phi_{n+1}'-\begin{pmatrix}3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''\\+\left(6F-3PF\phi^2\right)\phi_n'+G_a\end{pmatrix}\phi_{n+1}\\&=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3PF\phi^2\right)\phi_n'\end{aligned}\tag{19}$$

Where ϕ_n is our initial solution and ϕ_{n+1} is the assumed solution.

Let

$$\phi_{N,\,n+1}(y)=\sum_{i=0}^{7}a_iy^i$$

Considering N = 7

$$\begin{aligned}\phi_{n+1}(y)&=a_0+a_1y+a_2y^2+a_3y^3\\&\quad+a_4y^4+a_5y^5+a_6y^6+a_7y^7\end{aligned}$$

$$\begin{aligned}\phi_{n+1}'(y)&=a_1+2a_2y+3a_3y^2+4a_4y^3\\&\quad+5a_5y^4+6a_6y^5+7a_7y^6\end{aligned}$$

$$\begin{aligned}\phi_{n+1}''(y)&=2a_2+6a_3y+12a_4y^2\\&\quad+20a_5y^3+30a_6y^4+42a_7y^5\end{aligned}$$

$$\phi_{n+1}'''(y)=6a_3+24a_4y+60a_5y^2+120a_6y^3+210a_7y^4$$

$$\phi_{n+1}^{iv}(y)=24a_4+120a_5y+360a_6y^2+840a_7y^3\tag{20}$$

Putting (20) into (19)

$$\begin{aligned}&24a_4+120a_5y+360a_6y^2+840a_7y^3-\left(F\phi_n^3\right)\\&\left(6a_3+24a_4y+60a_5y^2+120a_6y^3+210a_7y^4\right)\\&-\left(6F\phi_n^2-P\right)\begin{pmatrix}2a_2+6a_3y\\+12a_4y^2+20a_5y^3\\+30a_6y^4+42a_7y^5\end{pmatrix}\\&-\left(6F\phi_n+PF\phi_n^3\right)\begin{pmatrix}a_1+2a_2y\\+3a_3y^2+4a_4y^3\\+5a_5y^4+6a_6y^5+7a_7y^6\end{pmatrix}\\&-\begin{pmatrix}3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''\\+\left(6F-3PF\phi^2\right)\phi_n'+G_a\end{pmatrix}\begin{pmatrix}a_0+a_1y+a_2y^2\\+a_3y^3+a_4y^4\\+a_5y^5+a_6y^6+a_7y^7\end{pmatrix}\\&=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n''-\left(6F-3PF\phi^2\right)\phi_n'\end{aligned}\tag{21}$$

Simplifying by collecting terms in a_0, a_1, a_2, a_3, a_4, a_5, a_6 *and* a_7 we obtain.

$$-a_0\begin{pmatrix}3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''\\+\left(6F-3PF\phi^2\right)\phi_n'+G_a\end{pmatrix}$$

$$-a_1\left\{\begin{matrix}\left(6F\phi_n+P\phi^3\right)\\-y\begin{pmatrix}3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''\\+\left(6F-3PF\phi^2\right)\phi_n'+G_a\end{pmatrix}\end{matrix}\right\}$$

$$-a_2\left\{\begin{matrix}\left(12F\phi_n{}^2-2P\right)\\+\left(12F\phi_n-2P\phi_n{}^2\right)y+\\y^2\begin{pmatrix}3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''\\+\left(6F-3PF\phi^2\right)\phi_n'+G_a\end{pmatrix}\end{matrix}\right\}$$

$$-a_3\left\{\begin{matrix}6F\phi_n{}^3+\left(36F\phi_n{}^2-6P\right)y\\+\left(18F\phi_n+3P\phi_n{}^3\right)y^2\\+y^3\begin{pmatrix}3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n''\\+\left(6F-3PF\phi^2\right)\phi_n'+G_a\end{pmatrix}\end{matrix}\right\}$$

$$-a_4\begin{Bmatrix} 24yF\phi_n{}^3-24 \\ +y^2\left(72F\phi_n{}^2-12P\right) \\ +y^3\left(24F\phi_n+4P\phi_n{}^3\right) \\ +y^4\begin{pmatrix} 3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n'' \\ +\left(6F-3PF\phi^2\right)\phi_n'+G_a \end{pmatrix} \end{Bmatrix}$$

$$-a_5\begin{Bmatrix} -120y+60y^2\left(F\phi_n{}^3\right) \\ +y^3\left(120F\phi_n{}^2-20P\right) \\ +y^4\left(30F\phi_n+5P\phi_n{}^3\right) \\ +y^5\begin{pmatrix} 3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n'' \\ +\left(6F-3PF\phi^2\right)\phi_n'+G_a \end{pmatrix} \end{Bmatrix}$$

$$-a_6\begin{Bmatrix} -360y^2+120y^3F\phi_n{}^3 \\ +y^4\left(180F\phi_n{}^2-30P\right) \\ +y^5\left(36F\phi_n+6P\phi_n^3\right) \\ +y^6\begin{pmatrix} 3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n'' \\ +\left(6F-3PF\phi^2\right)\phi_n'+G_a \end{pmatrix} \end{Bmatrix}$$

$$-a_7\begin{Bmatrix} -840y^3+210y^4F\phi_n{}^3 \\ +y^5\left(252F\phi_n{}^2-42P\right) \\ +y^6\left(42F\phi_n+7P\phi_n^3\right) \\ +y^7\begin{pmatrix} 3F\phi_n{}^2\phi_n'''+12F\phi_n\phi_n'' \\ +\left(6F-3PF\phi^2\right)\phi_n'+G_a \end{pmatrix} \end{Bmatrix}$$

$$=-3F\phi_n{}^2\phi_n'''-12F\phi_n\phi_n'' -\left(6F-3PF\phi^2\right)\phi_n'+\tau_2(y) \tag{22}$$

We now collate equations (22) at point $y=y_i$, where

$$y_i=a+\frac{(b-a)i}{N},\ i=1,2,.....,N-1$$

In the problem under consideration, N=7, a =-1 and b=+1.

The boundary conditions are

$$\phi(1)=0=\phi''(-1),\ U(1)=0=U''(-1)$$

Thus, we obtain five equations from (22).

Let ϕ_n be our initial solution

$$\phi_n=\frac{\gamma}{G_a}\left[1-\frac{b_2^2\cosh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_1^2\cosh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$

$$\phi_n'=\frac{\gamma}{G_a}\left[-\frac{b_1b_2^2\sinh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_2b_1^2\sinh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$

$$\phi_n''=\frac{\gamma}{G_a}\left[-\frac{b_1^2b_2^2\cosh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_2^2b_1^2\cosh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$

$$\phi_n'''=\frac{\gamma}{G_a}\left[-\frac{b_1^3b_2^2\sinh b_1y}{b_2^2-b_1^2\cosh b_1}+\frac{b_2^3b_1^2\sinh b_2y}{b_2^2-b_1^2\cosh b_2}\right]$$

We now substitute for $\phi_n,\phi_n',\phi_n'',\phi_n'''$ into (22) and then solve the equation simultaneously to obtain the unknowns $a_0,a_1,a_2,a_3,a_4,a_5,a_6,a_7$ *and* τ_2 using Gaussian elimination method with partial pivoting using MATLAB software to obtain the unknown constants

But

$$\phi_7(y)=\sum_{1=0}^{7}a_iy^i+\tau_2e^y$$

$$\phi_7(y)=a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5 +a_6y^6+a_7y^7+\tau_2e^y$$

$$\phi_7(1)=a_0+a_1+a_2+a_3+a_4+a_5+a_6+a_7 +\tau_2e^1=0$$

$$\phi_7(-1)=a_0-a_1+a_2-a_3+a_4-a_5+a_6-a_7+\tau_2e^{-1}=0$$

$$\phi_7''(1)=2a_2+6a_3+12a_4+20a_5+30a_6+42a_7 +\tau_2e^1=0$$

$$\phi_7''(-1)=2a_2-6a_3+12a_4-20a_5 +30a_6-42a_7+\tau_2e^{-1}=0$$

$$\begin{pmatrix} -2.6086 & -0.3825 & -15.6317 & -8.1498 & 28.2323 & -88.7253 & 174.9258 & -101.0555 & 0 \\ -22.6083 & -1.3056 & -8.4833 & -5.5555 & 23.9565 & -42.1823 & 41.0855 & -46.0445 & 0 \\ -22.1387 & 1.2238 & 13.0562 & -25.0038 & 30.6335 & -90.3884 & 203.2226 & -101.0067 & 0 \\ -10.5000 & -8.6557 & -5.6772 & -5.5870 & 8.4516 & 17.2080 & 17.1338 & 21.1998 & 0 \\ -1.5393 & -12.9986 & 0.9959 & 0.2601 & 15.0763 & 37.5997 & 133.5557 & 200.9933 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2.7183 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0.3679 \\ 0 & 0 & 2 & 6 & 12 & 20 & 30 & 42 & 2.7183 \\ 0 & 0 & 2 & -6 & 12 & -20 & 30 & -42 & 0.3679 \end{pmatrix} \times \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} 0.6700 \\ 0.3333 \\ -103.2648 \\ -0.3333 \\ -0.6667 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{23}$$

$$a_0 = -1.0111, a_1 = 10.3337,\ a_2 = -1.1178,$$
$$a_3 = -2.3322, a_4 = -1.1053, a_5 = 1.9222,$$
$$a_6 = 1.1202, a_7 = 1.6690, \tau_2 = -1.0242$$

Determination of the Temperature profile, to obtain the solution for the temperature

$$\phi_7(y) = \sum_{i=0}^{7} a_i y^i + \tau_2 e^y$$

in the interval $-1 \le y \le 1$ since the various values of a_i and τ_2 are known from the MatLab Program

Determination of the Velocity profile

The velocity distribution of the flow can be obtained from equation (6)

$$\frac{d^2U}{dY^2} - PU = G_a\phi - \gamma$$

$$U = -\left(\frac{(G_a\phi+\gamma)e^{-\sqrt{P}}}{P\left(e^{2\sqrt{P}} + e^{-2\sqrt{P}}\right)}\right)e^{y\sqrt{P}} + \left(\frac{(G_a\phi+\gamma)e^{-\sqrt{P}}}{P\left(e^{2\sqrt{P}} + e^{-2\sqrt{P}}\right)}\right)e^{-y\sqrt{P}} - \frac{1}{P}(G_a\phi+\gamma) \tag{24}$$

$$U = -\left(\frac{\left(G_a\begin{pmatrix} a_0 + a_1y + a_2y^2 \\ +a_3y^3 + a_4y^4 + a_5y^5 \\ +a_6y^6 + a_7y^7 + \tau_2e^y \end{pmatrix} + \gamma\right)e^{-\sqrt{P}}}{P\left(e^{2\sqrt{P}} + e^{-2\sqrt{P}}\right)}\right)e^{y\sqrt{P}}$$
$$+\left(\frac{\left(G_a\begin{pmatrix} a_0 + a_1y + a_2y^2 \\ +a_3y^3 + a_4y^4 + a_5y^5 \\ +a_6y^6 + a_7y^7 + \tau_2e^y \end{pmatrix} + \gamma\right)e^{-\sqrt{P}}}{P\left(e^{2\sqrt{P}} + e^{-2\sqrt{P}}\right)}\right)e^{-y\sqrt{P}}$$
$$-\frac{1}{P}\left(G_a\begin{pmatrix} a_0 + a_1y + a_2y^2 \\ +a_3y^3 + a_4y^4 + a_5y^5 \\ +a_6y^6 + a_7y^7 + \tau_2e^y \end{pmatrix} + \gamma\right) \tag{25}$$

We have our assumed solution to be

$$\phi(y) = a_0 + a_1y + a_2y^2 + a_3y^3 + a_4y^4 + a_5y^5 + a_6y^6 + a_7y^7 + \tau_2e^y$$

$$\phi'(y) = a_1 + 2a_2y + 3a_3y^2 + 4a_4y^3 + 5a_5y^4 + 6a_6y^5 + 7a_7y^6 + \tau_2e^y$$

$$\phi''(y) = 2a_2 + 6a_3y + 12a_4y^2 + 20a_5y^3 + 30a_6y^4 + 42a_7y^5 + \tau_2e^y$$

Substituting the expression $\phi(y)$, $\phi'(y)$ *and* $\phi''(y)$ into (6) gives

$$U = F\begin{pmatrix} a_0 + a_1y + a_2y^2 + a_3y^3 \\ +a_4y^4 + a_5y^5 + a_6y^6 \\ +a_7y^7 + \tau_2e^y \end{pmatrix}^3 \begin{pmatrix} a_1 + 2a_2y + 3a_3y^2 + 4a_4y^3 \\ +5a_5y^4 + 6a_6y^5 + 7a_7y^6 + \tau_2e^y \end{pmatrix} - \begin{pmatrix} 2a_2 + 6a_3y + 12a_4y^2 \\ +20a_5y^3 + 30a_6y^4 + 42a_7y^5 + \tau_2e^y \end{pmatrix} \tag{26}$$

Evaluating equation (6) in the interval $-1 \le y \le 1$ will enable us to determine the effect of different radiation parameter.

Determination of the Non-dimensional flow rate

The non-dimensional flow rate through the channel per unit width is given by

$$2\int_0^1 \frac{U}{\gamma}dy = \frac{2}{\gamma}\int_0^1 [F\begin{pmatrix} a_0 + a_1y + a_2y^2 + a_3y^3 \\ +a_4y^4 + a_5y^5 + a_6y^6 \\ +a_7y^7 + \tau_2e^y \end{pmatrix}^3 \begin{pmatrix} a_1 + 2a_2y + 3a_3y^2 \\ +4a_4y^3 + 5a_5y^4 \\ +6a_6y^5 + 7a_7y^6 + \tau_2e^y \end{pmatrix} - \begin{pmatrix} 2a_2 + 6a_3y + 12a_4y^2 \\ +20a_5y^3 + 30a_6y^4 \\ +42a_7y^5 + \tau_2e^y \end{pmatrix}]dy \tag{27}$$
$$= 5.3334\ \textit{per unit}$$

Using the equation (6) and evaluating the integral will enable us to determine the flow rate.

Determination of Heat Transfer coefficient

Thus heat transfer coefficient due to thermal conduction is given by

$$h = -\left(\frac{d\phi}{dy}\right)$$

$$h = -a_1 - 2a_2y - 3a_3y^2 - 4a_4y^3 - 5a_5y^4 - 6a_6y^5 - 7a_7y^6 - \tau_2e^y$$

$$h = -10.3337 + 2(1.1178)y + 3(2.3322)y^2 + 4(1.1053)y^3 - 5(1.9222)y^4 - 6(1.1202)y^5 - 7(1.6690)y^6 + (\ 1.0242)e^y \tag{28}$$

4. Results and Discussion

The problem of thermal radiation effect on a porous media under optically thick approximation formulated, analysed and solved numerically. In order to point out the effects of physical parameters namely; thermal Grashof number Ga, radiation parameter F, Porous term P. on the flow patterns, the computation of the flow fields are carried out.The values of velocity and temperature are obtained for the physical parameters as mention. The velocity profiles has been studied and presented in Figure 2 to Figure 4 The effect of velocity for different values of Porous parameter (P = 2, 5, 10, 20) is presented in Figure 2. The trend shows that the velocity increases with increasing Magnetic parameter. The effect of velocity for different values of radiation (F = 2, 5, 10) is also presented in Figure 3. It is then observed that the velocity decreases with increasing values of radiation. The effect of velocity for different values of thermal Grashof number (Ga= 1, 2, 3) is also presented in Figure 4. It is then observed that the velocity decreases with increasing values of thermal Grashof number

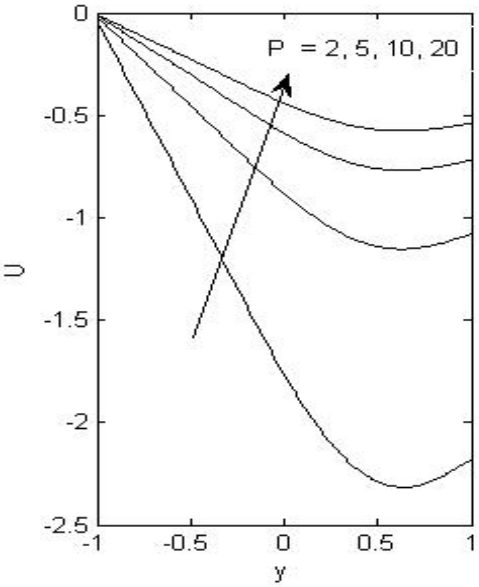

Figure 2. Velocity profiles for different values of P

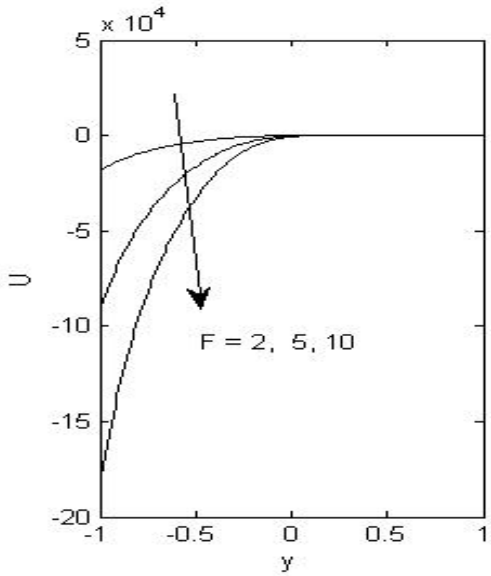

Figure 3. Velocity profiles for different values of F

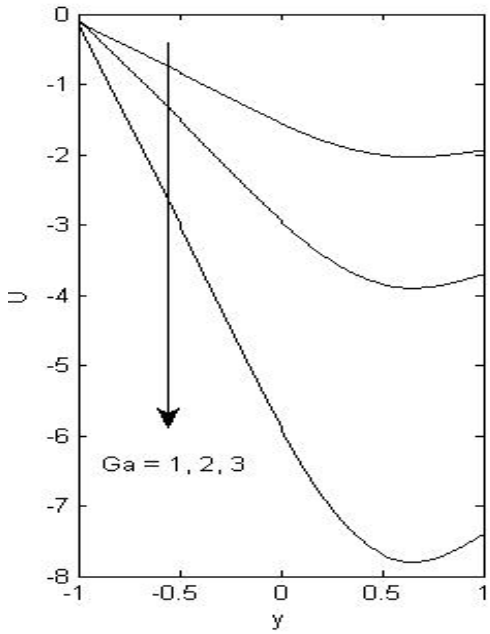

Figure 4. velocity profiles for different values of Ga

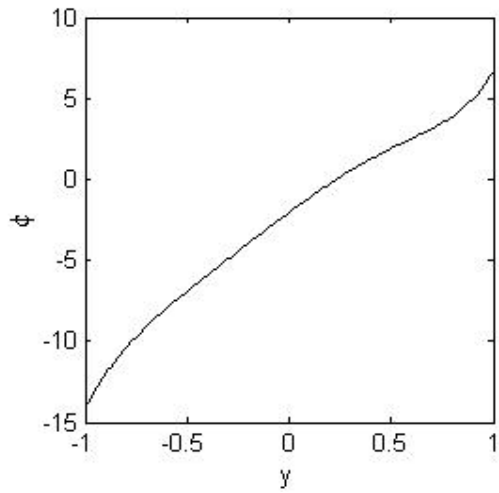

Figure 5. Trend for temperature profiles ϕ against y

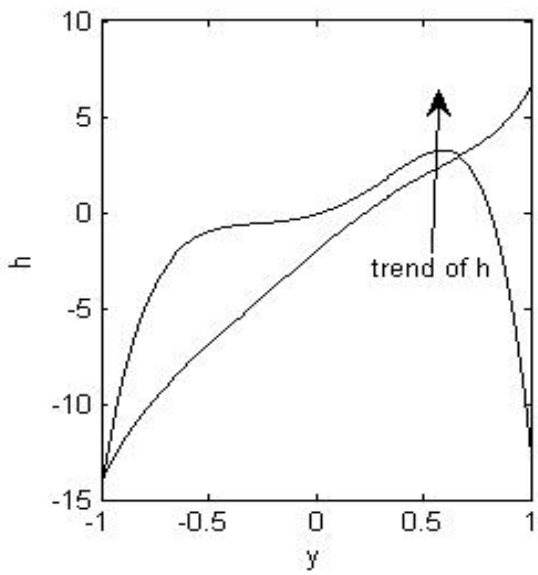

Figure 6. Trend of heat transfer coefficient due to thermal conduction h

In Figure 5 It is observed that temperature rise with increasing values of a_i *and* τ_2 while Figure 6 shows that trend of heat transfer coefficient due to thermal conduction h increases with the rate of change of ϕ at less than 5 cooling sets in.

5. Summary and Conclusion

Thermal radiation effect on a porous media under optically thick approximation has been studied. .In order to point out the effects of physical parameters namely; thermal Grashof number Ga, radiation parameter F, Porous parameter P. are presented graphically. It is observed that the velocity increases with increasing Porous parameter, while radiation and thermal Grashof number decreases with increasing values of radiation and thermal Grashof number respectively.

It also observed that temperature rise with increasing values of a_i *and* τ_2 and trend of heat transfer coefficient due to thermal conduction h increases with the rate of change of ϕ at less than 5 cooling sets in.

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