Numerical Solution of Power-law Fluid Flow through Eccentric Annular Geometry

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Abstract Cuttings transport modeling in inclined and horizontal wellbores is complicated due to the eccentricity of the annulus. Development of a model for cuttings transport requires a deep understanding of the drilling mud flow behavior in the eccentric annular geometry. In this paper, bipolar coordinates system is used to solve for the eccentric annular geometry due to irregular shape of the boundaries. Finite difference method is used to obtain the velocity profile of Power-law non-Newtonian fluids through eccentric annuli. The discretized dimensionless Equation of flow using the finite difference method is solved iteratively using Point Successive Over Relaxation (S.O.R.) method. The results for Newtonian eccentric annular flow with 0.0001 eccentricity are in agreement with the Newtonian concentric annular flow. The Power-law eccentric annular flow results with flow index of 1.0 are verified with Newtonian fluid eccentric annular flow results. The parametric effects of flow index, Pipe/hole radius ratio, and eccentricity are investigated. We expect the development of a new model for flow in eccentric annular geometries to be an important new tool for application in oil - well drilling and production.

Keywords: bipolar coordinate, eccentric annulus, Power-law fluid, Point Successive over Relaxation

1. Introduction

Annular flow analysis is important in drilling and production in deviated and horizontal wells. In drilling, the annular velocity is one of two major variables in the process of cleaning solids (drill cuttings) from the wellbore. By maintaining the annular velocity above a certain rate (speed) in conjunction with the rheological properties of the drilling fluid, the wellbore can be kept clean of the drill cutting to prevent them from settling and causing drilling problems. Many investigators assumed the annulus to be "almost concentric," however, the drill pipe is usually positioned eccentrically in the wellbore where a drill pipe has strong tendency to offset toward the low side because of gravitational effect. Eccentric annular flows are more complicated, and have been modeled under limiting assumptions [1]. This paper describes a numerical solution of fully developed laminar flow of Power-law non-Newtonian fluid in an eccentric annulus using bipolar coordinates transformation. The resulting model will be used in modeling of cuttings transport during drilling operation.

2. Background

The main function of drilling fluid is to carry the drilled particles "cuttings" generated by the drill bit to the surface through the annulus. Most drilling fluids exhibit non-Newtonian properties [2,3]. Non-Newtonian behavior is characterized by nonlinear relationship between the shear stress and shear strain. Most drilling fluids are pseudoplastic as their shear thinning nature is very desirable in drilling.

The practical complications of flow of non-Newtonian fluid through eccentric annulus are: the eccentric geometry of the annulus (Figure 1); and the non linearity of the equations of motion of non-Newtonian fluids due to the non linear relationship between shear stress and shear rate.
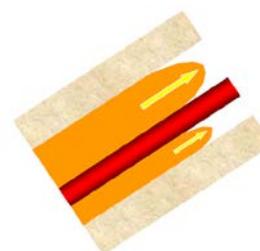

Figure 1. Annular velocity Profile for a non-Newtonian Fluid with Eccentric Drillpipe [4]

Therefore, it is difficult to find exact analytical solutions, numerical methods are usually used to solve this problem. A number of studies have focused on the problem of flow of non-Newtonian fluids flow through eccentric annulus [9,10,13]. Two alternative approaches have been used in the previous studies. The first one uses the bi-polar coordinates to describe the eccentric annular geometry; the second one treats an eccentric annulus as a slot of variable height. Using a bipolar coordinate system and Green's function, Heyda [5] presented analytical solutions for Newtonian fluid flow in an eccentric annulus in the form of an infinite series. He showed that the

velocity profile of Newtonian fluid in laminar flow regime would differ dramatically from that of an eccentric annulus. Redberger and Charles [6] reported their numerical evaluations of the velocity profile and the volumetric flowrate for the Newtonian eccentric annular flow and concluded that the fluid velocity is higher in the enlarged region than in the reduced region of an eccentric annulus and the volumetric flowrate increases with increase of the offset of the inner tube for a given pressure gradient. Their velocity profile agreed with Heyda analytical solution.

Snyder and Goldstein [7] presented an analysis of fully developed laminar flow of Newtonian fluid in an eccentric annulus. An exact solution for the velocity distribution was presented in terms of infinite series summation. From this solution expressions were obtained for local shear stresses on the inner and outer surfaces of the annulus, friction factors based on the inner and outer surfaces, and the overall friction factor. Curves of these parameters were presented covering a range of eccentricity values and radius ratio values. Mitsuishi and Aoyagi [8] extended the bipolar coordinate method to the eccentric annular flow of non-Newtonian fluids using the Sutterby model and similar conclusions as those obtained by Redberger and Charles were reached. They validated the calculated velocities and the volumetric flowrates with the experimental data. Guckes [9] presented a systematic procedure for calculating the volumetric flowrate by using the bipolar coordinate approach and developed a series of dimensionless curves for the laminar eccentric annular flow of power-law fluids. However, after a lengthy treatment, Tosun [10] developed expressions for the volumetric flow-rate of Newtonian fluids through an eccentric annulus. Using the expressions for volumetric flow-rate in a concentric annuli developed by Bird et al. [11], he presented the ratio of the volumetric flow rates in eccentric and concentric annuli in terms of the diameter ratio and the eccentricity, for a given pressure gradient. Their results showed that this ratio increases as the diameter ratio and the eccentricity increases. Haciislamoglu and Langlinais [12] used an iterative finite difference method to model Bingham fluids, but they did not provide information on computing times and numerical stability or code portability. However, limitations on the mapping used means that the methodology cannot be extended to handle boreholes with cuttings beds [1].

Examining the analytical solution that approximate an eccentric annulus with equivalent slot [13,14,15,16,17], it is can be realized that bipolar coordinates system should be used to obtain an exact solution for the eccentric annular geometry rather than approximate slot solution.

3. Methodology

The fluid flow in an annulus is idealized as steady, isothermal, fully developed laminar flow through a straight annulus consisting of an outer cylindrical casing (R_0) and an inner cylinder (R_i) which is offset (i.e. eccentric). The geometry of the system is depicted in Figure 2.

The flow domain under consideration is not easy to describe in any of the classical coordinate system due to the irregular shape of the boundaries. Therefore a bipolar coordinate system is suggested.
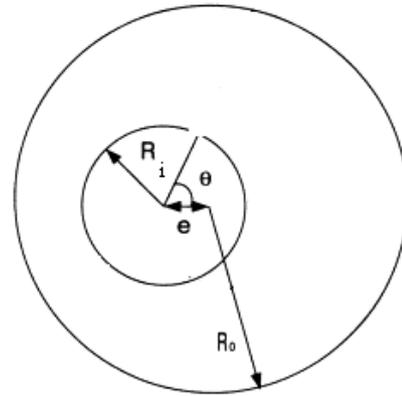

Figure 2. Physical domain in cylindrical coordinate

In bipolar coordinates system, the two cylindrical boundaries of the annulus coincide with two surfaces having constant values of η (η_i and η_o , which can be expressed in terms of the annulus radius ratio k and the dimensionless eccentricity ξ. The other coordinate (ξ) represents a set of eccentric cylinders whose centers lie on the y-axis and which intersect orthogonally the boundaries of the annulus (Figure 3) .
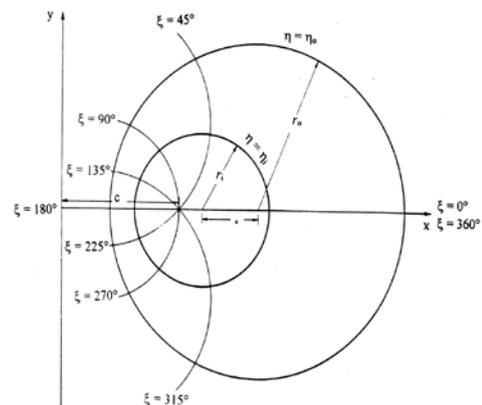

Figure 3. Eccentric Annulus in Bipolar Coordinates [18]

The relationships needed to transform from Cartesian coordinates to bipolar coordinates are given by Speigel [19].

$$x = \frac{a \sinh \eta}{\cosh \eta - \cos \xi} \qquad (1)$$

$$y = \frac{a \sin \eta}{\cosh \eta - \cos \xi} \qquad (2)$$

Where a is the pole of the bipolar coordinate system on the x-axis (a>0), which is expressed as

$$a = r_i \sinh \eta_i = r_o \sinh \eta_o$$

To achieve the scope of this paper, the work done was categorized into two sections

I. Development of model for Newtonian fluid flow in eccentric annulus.

II. Extend the developed model for non-Newtonian eccentric annular flow.

3.1 Newtonian Eccentric Annular Flow

For axial, laminar, steady flow of incompressible fluid, the z-direction momentum equation in bipolar coordinate system can be expressed in terms of velocity as:

$$\left[\frac{\partial}{\partial \xi}\left(\frac{\partial v}{\partial \xi}\right)+\frac{\partial}{\partial \eta}\left(\frac{\partial v}{\partial \eta}\right)\right]=\frac{r_i^2 \sinh^2 \eta_i}{\mu\left(\cosh\eta-\cos\xi\right)^2}\frac{dP}{dz} \quad (3)$$

Here μ denotes the Newtonian viscosity and v the velocity

According to Guckes [9], the outer and inner walls of an eccentric annulus can be expressed respectively in a bipolar coordinate system as:

$$\eta_0 = \cosh^{-1}\left[\frac{r^*\left(1-e^{*2}\right)+\left(1+e^{*2}\right)}{2e^*}\right] \quad (4)$$

$$\eta_i = \cosh^{-1}\left[\frac{r^*\left(1+e^{*2}\right)+\left(1-e^{*2}\right)}{2r^*e^*}\right] \quad (5)$$

Where e^* is the non-dimensional eccentricity and r^* is the radius ratio given by:

$$r^* = \frac{r_i}{r_o}, \qquad e^* = \frac{e}{\left(r_o - r_i\right)} \quad (6)$$

Equation (3) is made dimensionless by introducing dimensionless velocity

$$v^* = -\frac{v}{\dfrac{r_i^2}{\mu}\dfrac{dp}{dz}} \quad (7)$$

After the dimensionless analysis, equation (3) becomes

$$\left[\frac{\partial}{\partial \xi}\left(\frac{\partial v^*}{\partial \xi}\right)+\frac{\partial}{\partial \eta}\left(\frac{\partial v^*}{\partial \eta}\right)\right]=-\frac{\sinh^2 \eta_i}{\left(\cosh\eta-\cos\xi\right)^2} \quad (8)$$

This is so-called Poisson's equation in mathematics. In an eccentric annulus, the axis that goes through the center points of the inner and outer pipes is the line-of-symmetry which divides the velocity profile into two identical parts. Therefore we shall solve for the velocity profile only in one half of the eccentric annulus [20].

To build a system of linear equations involving the unknowns, we apply the above differential equation at the (i,j) grid point and approximate the second partial derivatives using central difference, setting

$$\left(\frac{\partial^2 v}{\partial \eta^2}\right)_{i,j} \cong \frac{\left(v_{i+1,j}-2v_{i,j}+v_{i-1,j}\right)}{\left(\Delta\eta^2\right)} \quad (9)$$

$$\left(\frac{\partial^2 v}{\partial \xi^2}\right)_{i,j} \cong \frac{\left(v_{i,j+1}-2v_{i,j}+v_{i,j-1}\right)}{\left(\Delta\xi^2\right)} \quad (10)$$

These approximations allow us to convert the partial differential equation to the finite difference equation choosing mesh spacing $\Delta\eta, \Delta\xi$ on a rectangular domain

$$\frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{\Delta\eta^2}+$$
$$\frac{v_{i,j+1}-2v_{i,j}+v_{i,j-1}}{\Delta\xi^2}=G_{i,j} \quad (11)$$

Where

$$G_{i,j} = \frac{\sinh^2 \eta_i}{\left(\cosh\eta-\cos\xi\right)^2}$$

Equation (11) can be rearranged to yield

$$\Delta\xi^2 v_{i+1,j}-2(\Delta\eta^2+\Delta\xi^2)v_{i,j}+\Delta\xi^2 v_{i-1,j}$$
$$+\Delta\eta^2 v_{i,j+1}+\Delta\eta^2 v_{i,j-1}=\Delta\eta^2\Delta\xi^2 G_{i,j} \quad (12)$$

For a uniform grid, the final formulation of the finite-difference equation is:

$$D_{i,j}v_{i,j}+C_{i,j}v_{i+1,j}+A_{i,j}v_{i-1,j}+F_{i,j}v_{i,j+1}+E_{i,j}v_{i,j-1}=B_{i,j} \quad (13)$$

Where A, C, E, F, D, and B are given as:

$$A_{i,j} = C_{i,j} = \Delta\xi^2$$

$$E_{i,j} = F_{i,j} = \Delta\eta^2$$

$$D_{i,j} = -(C_{i,j}+A_{i,j}+F_{i,j}+E_{i,j})$$

And

$$B_{i,j} = \Delta\eta^2\Delta\xi^2 G_{i,j}$$

These C, A, F, E, and D expressions refer to the coefficients associated with any set of left, right, top, bottom and central mesh points as shown in Figure 4. Because of this point configuration, equation (12) is called a five-point formula
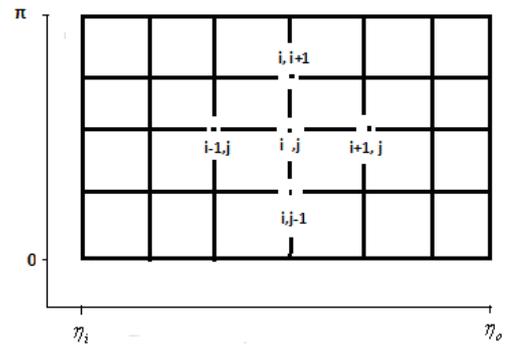

Figure 4. Eccentric annulus in transformed coordinates

The boundary conditions of the region are

v=0 at $\eta = \eta_i$ (at the pipe wall)

v=0 at $\eta = \eta_0$ (at the hole wall)

$$\frac{\partial v}{\partial \xi} = 0 \text{ at } \xi = 0 and \pi$$

The problem is then to find values of the dependent variable V at internal locations. The matrix form of Equation (13) is pentadiagonal (most entries are zero, and the five diagonals contain all the nonzero entries). The basic idea when storing sparse matrices is to store only the

non-zero entries as opposed to storing all entries. Fig. (5) demonstrated the final matrix form after applying boundary conditions. For a 2D-grid of mxn interior points, **V** has m x n grid points, but the coefficient matrix (A) has (m x n) x (m x n) entries. Hence, this yields linear system of equations as
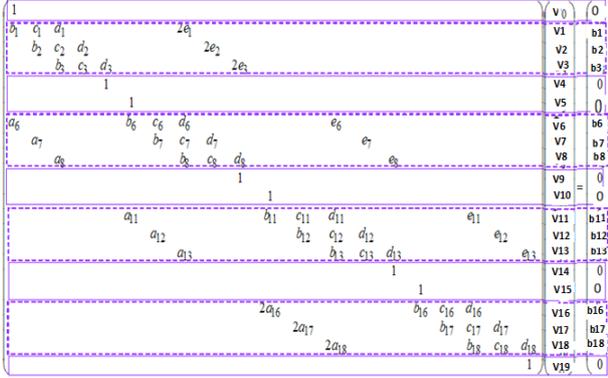
$$A v = b \qquad (14)$$

Figure 5. Structure of pentadiagonal matrix coefficients A

In this paper point SOR method (Fig.6) is used to solve Equation (9). It is an iterative scheme that uses a relaxation parameter ω and is a generalization of the Gauss-Seidel method in the special case ω = 1.
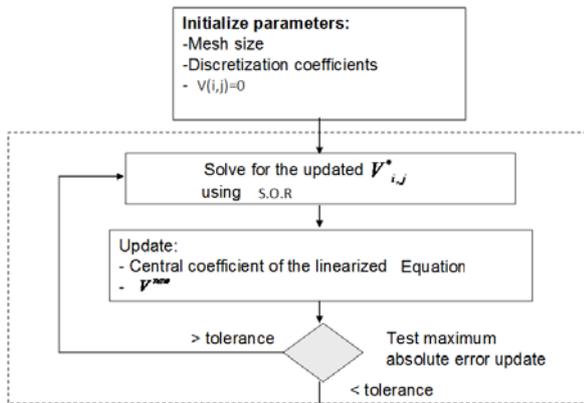

Figure 6. Flow chart of S.O.R. method

A FORTRAN computer program was written to obtain the velocity profile for Newtonian eccentric annular flow. To verify our numerical model, the results of Newtonian eccentric annular flow with 0.0001 eccentricity are compared with the concentric annular flow results. Results are shown in Figure 9. It shows that this program can be used for modeling Newtonian eccentric annular flow.

3.2 Power-law Eccentric Annular Flow

For axial, laminar, steady flow of incompressible fluid, the z-direction momentum equation in Cartesian coordinate system is:

$$\frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial x}\right) = -\frac{dP}{dz} \qquad (15)$$

For Power-law model, the relationship between the shear stress and shear rate is

$$\tau = K \gamma^n \qquad (16)$$

Where K denotes the fluid consistency index

Then the governing equation of motion for Power-law fluids in eccentric annulus can be expressed in terms of velocity as:

$$\frac{\partial}{\partial x}\left(K \left(\frac{\partial v}{\partial x}\right)^n\right) + \frac{\partial}{\partial y}\left(K \left(\frac{\partial v}{\partial y}\right)^n\right) = -\frac{dP}{dz} \qquad (17)$$

The z-direction momentum equation in bipolar coordinate system

$$\frac{\partial}{\partial \eta}\left(\left(\frac{\partial v}{\partial \eta}\right)^n\right) + \frac{\partial}{\partial \xi}\left(\left(\frac{\partial v}{\partial \xi}\right)^n\right) = -\frac{r_i^2 \sinh^2 \eta_i}{K\left(\cosh \eta - \cos \xi\right)^2}\frac{dP}{dz} \qquad (18)$$

The dimensionless form of Equation (18) is

$$\frac{\partial}{\partial \eta}\left(\left(\frac{\partial v^*}{\partial \eta}\right)^n\right) + \frac{\partial}{\partial \xi}\left(\left(\frac{\partial v^*}{\partial \xi}\right)^n\right) = \frac{\sinh^2 \eta_i}{\left(\cosh \eta - \cos \xi\right)^2} \qquad (19)$$

Where v^* is the dimensionless velocity given as:

$$v^* = -\frac{v}{\left(\dfrac{r_i^2}{K}\dfrac{dp}{dz}\right)^{\frac{1}{n}}} \qquad (20)$$

Equation (19) can be written as:

$$\frac{\partial}{\partial \eta}\left(\left(\frac{\partial v^*}{\partial \eta}\right)^{n-1}\left(\frac{\partial v^*}{\partial \eta}\right)\right) + \frac{\partial}{\partial \xi}\left(\left(\frac{\partial v^*}{\partial \xi}\right)^{n-1}\left(\frac{\partial v^*}{\partial \xi}\right)\right) =$$

$$= \frac{\sinh^2 \eta_i}{\left(\cosh \eta - \cos \xi\right)^2} \qquad (21)$$

After discretizing, Equation (21) will be:

$$\left(\frac{v^*_{i+1,j} - v^*_{i-1,j}}{2\Delta\eta}\right)^{n-1}\left[\frac{\left[\frac{v^*_{i+1,j} - v^*_{i,j}}{\Delta\eta}\right] + \left[\frac{v^*_{i,j} - v^*_{i-1,j}}{\Delta\eta}\right]}{\Delta\eta}\right] +$$

$$\left(\frac{v^*_{i,j+1} - v^*_{i,j-1}}{2\Delta\xi}\right)^{n-1}\left[\frac{\left[\frac{v^*_{i,j+1} - v^*_{i,j}}{\Delta\xi}\right] + \left[\frac{v^*_{i,j} - v^*_{i,j-1}}{\Delta\xi}\right]}{\Delta\xi}\right] =$$

$$= \frac{\sinh^2 \eta_i}{\left(\cosh \eta - \cos \xi\right)^2} \qquad (22)$$

After grouping the terms of Equation (22), the discretized differential equation constants will be:

$$\left(\frac{v^*_{i+1,j} - v^*_{i-1,j}}{2\Delta\eta}\right)^{n-1}\left[v^*_{i+1,j} + v^*_{i-1,j}\right] +$$

$$\frac{\Delta\eta^2}{\Delta\xi^2}\left(\frac{v^*_{i,j+1} - v^*_{i,j-1}}{2\Delta\xi}\right)^{n-1}\left[v^*_{i,j+1} + v^*_{i,j-1}\right] -$$

$$2\left(\left(\frac{v^*_{i+1,j} - v^*_{i-1,j}}{2\Delta\eta}\right)^{n-1} + \frac{\Delta\eta^2}{\Delta\xi^2}\left(\frac{v^*_{i,j+1} - v^*_{i,j-1}}{2\Delta\xi}\right)^{n-1}\right)v^*_{i,j} = \qquad (23)$$

$$= \frac{\Delta\eta^2 \sinh^2 \eta_i}{\left(\cosh \eta - \cos \xi\right)^2}$$

Equation (23) is strongly nonlinear, the direct solutions are not numerically possible, but exact solutions can be obtained iteratively.

4. Results and Discussion

The eccentricity of an annular geometry has a strong influence on the laminar flow velocity distribution. This is seen in Figure 7, where the dimensionless axial velocity profiles of Newtonian fluid, are plotted versus x direction (21) for 0.625 Pipe/hole radius ratio and at 0.6 dimensionless eccentricity. Due to the eccentricity, the velocity in the narrowest part of the annulus $\left(\xi = 0^o \right)$ is reduced because the resistance to flow is increased as the gap between two pipes decreases. While at the widest gap $\left(\xi = 180^o \right)$ the fluid reaches its maximum velocity.
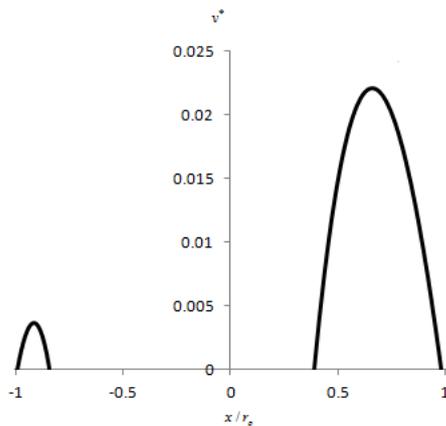

Figure 7. Velocity distribution of Newtonian fluid flow in the widest and narrowest gabs of eccentric annulus

To verify the proposed model, firstly, the velocities at widest gap of an eccentric annulus with 0.0001 eccentricity were plotted with the velocity values at narrowest (Figure 8), it was found that they were identical that means eccentric annulus with small eccentricity resembles the concentric annular geometry. Secondly, the results obtained for Newtonian fluid flow through eccentric annulus with 0.0001 eccentricity were compared with the results of Newtonian fluid flow in concentric annulus (e=0.0). As seen in Figure 9, the results of eccentric annulus with 0.0001 eccentricity are in agreement with the concentric annulus (e=0.0) solution.
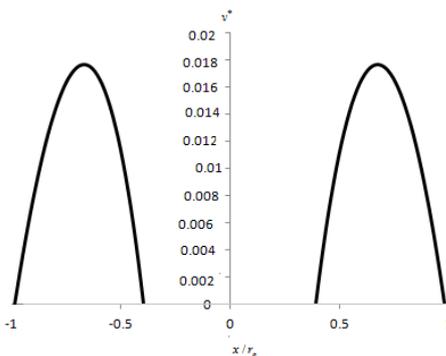

Figure 8. Relationship between velocity profiles in the widest and narrowest gap of annulus with 0.0001 dimensionless eccentricity
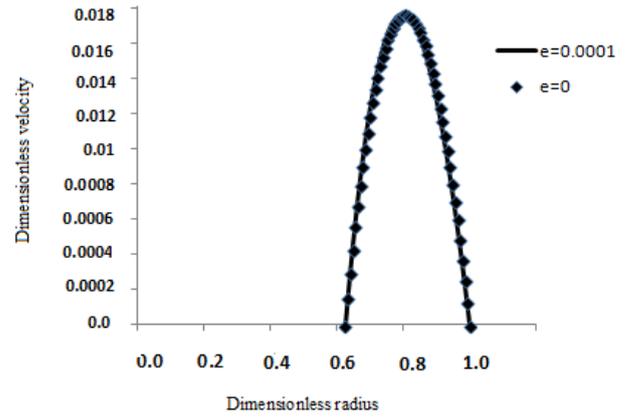

Figure 9. Velocity profiles of Newtonian fluid flow in concentric annulus and eccentric annulus with small eccentricity

The investigated model was extended to be used for the non-Newtonian fluid flow through eccentric annular geometry. To validate non-Newtonian fluid model, the results of Power-law flow through eccentric annulus with 0.0001eccentricity are compared with the experimental data by Nouri and Whitelaw (1994) for Power-law fluid flow in concentric annulus.

The agreement between the experimental data and the proposed model are clearly satisfactory (Figure 10).
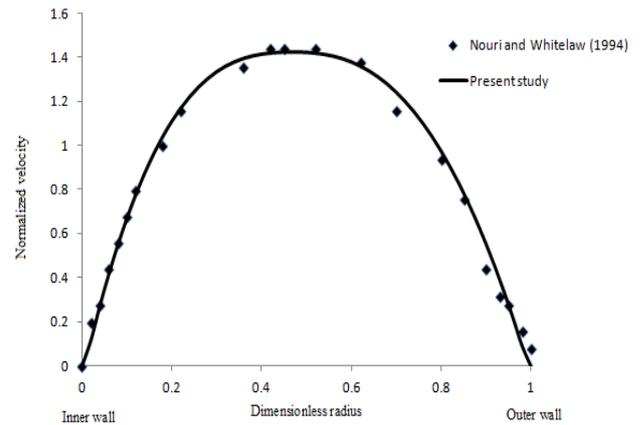

Figure 10. Comparision between proposed model normalized velocity data and Nouri and Whitelaw (1994) normalized velocity data

The parametric effect of eccentricity and pipe/hole radius ratio on the laminar flow velocity distribution was investigated. The eccentricity of an annular geometry has a strong influence on the laminar flow velocity distribution. This is seen in Figure 11, where the dimensionless axial velocity profiles of Power-law non-Newtonian fluid, are presented for 0.625 Pipe/hole radius ratio and at four different dimensionless eccentricities (0.05, 0.2, 0.5, and 0.6). For zero eccentricity, the velocity profiles on both sides of the inner cylinder are identical. With the increase of the eccentricity, the difference between the width of the wide and narrow gaps increases. The velocity increases in the wide and decreases in the narrow gap with increase of eccentricity.

Figure 12 illustrates the effect of radius ratio on the velocity distribution, results obtained for three different radius ratios (k=0.5, 0.625, and 0.7) at 0.6 dimensionless eccentricity. Larger is the radius ratio, lesser is the velocity.
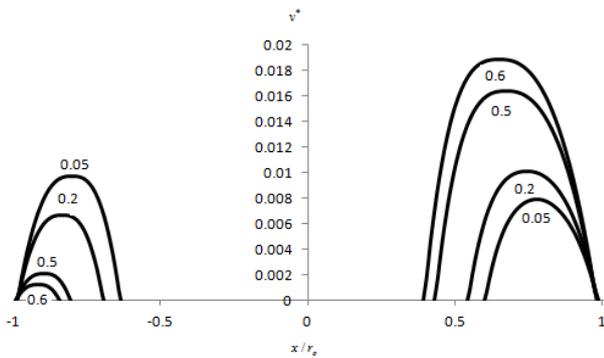
Figure 11. Effect of eccentricity on the velocity distribution of Power-law non-Newtonian fluid in the wide and narrow gap of annulus
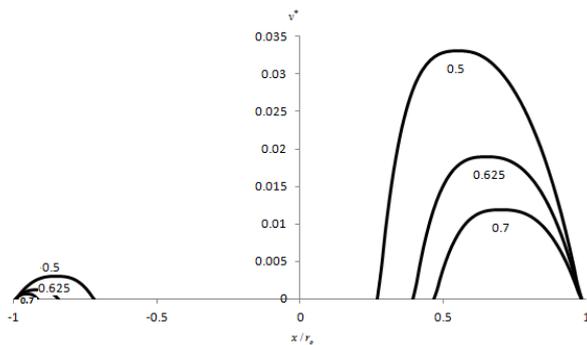

Figure 12. Effect of radius ratio on the velocity distribution of Power-law non-Newtonian fluid in the wide and narrow gap of annulus

5. Conclusion

The equation of motion in axial direction has been transformed into bipolar coordinate system. First, Newtonian model has been incorporated into the equation of motion. A numerical solution of the discretized equation of motion has been developed using Successive Over Relaxation method to obtain the velocity profile in eccentric annular geometry. Comparison between Newtonian concentric and eccentric annular flow with small eccentricity showed the validity of the numerical model. Second, the numerical model for Newtonian model has been extended to obtain the Power-law velocity profile.

The results of Power-law flow through eccentric annulus with small eccentricity were in agreement with the experimental data published in literature for Power-law fluid flow in concentric annulus. The velocity profiles have shown higher velocities in the wide gap of the annulus and lower velocities in the narrow gap of the annulus where the cuttings bed will be formed. Therefore, pipe eccentricity contributes to hole cleaning problems. It was found that the velocity increases with the increase of the flow index and decreases with the increase of the radius ratio.

Nomenclature

A, B, C, D, F, and G = Coefficients of Pentadiagonal system Equation

a= is the pole of the bipolar coordinate system on the x-axis

dP/dz = Pressure gradient, Pa/m [Psi/ft]

e= offset distance between the centers of the inner and outer pipes m [in.]

e^* =dimensionless eccentricity

h =scale factor of the bipolar coordinate system

K=Flow consistency index, Pa. s^n [lbf.s^n/ft^2]

k = Annulus aspect ratio

n = Flow behavior index, dimensionless

R_i= Drillpipe radius, m [in.]

R_o= Outer radius of annulus, m [in.]

r = Radius coordinate, m [in.]

r^*= radius ratio of the annulus

v= Axial velocity, m/s [ft/min]

v^* = Dimensionless velocity

x = the first transverse direction in the Cartesian coordinate system

Z= axial coordinate, m [ft]

Greek Symbols

η_i=constant variable used to describe bipolar coordinate system

θ = angular coordinate, (deg.)

τ = Shear stress, Pa [Psi]

ρ = fluid density, kg/m^3 [lbm/gal]

γ= Shear rate, (s^{-1})

μ = Viscosity, Pa.s [cp]

Subscript

i =outer surface of inner cylinder

o= inner surface of outer cylinder

Acknowledgment

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