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Modelling, Simulation and Analysis of Police Containment of Crowd Using Stochastic Differential Equations with Delays

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Abstract In this paper, we develop new models of crowd containment involving cordons of police agents (officers) surrounding a crowd of protesters with ultimate goal to restrict their movements. Our model employs stochastic delay differential equations (SDDEs) to simulate the scenarios in which protesters clash with police in a rank line formation. We investigate the solutions of the propose models and the stability of their solutions too. Our results show that a strategy that is integrative produces better optimal solutions and the performance of our strategy can be evaluated.

Keywords: *Modelling, simulation, predictive analysis, national police, protesters, crowd formation, crowd control and stability*

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1. Introduction

Modelling and simulation represent two important tools that are nowadays used for prediction and control of real life phenomena. Recently, these techniques have been introduced in police training and education. The earliest documented use of police method and tactics for management of large demonstrations occurred in 1986 when the Hamburg police cordoned and detained more than a thousand protesters for up to twenty hours; in order to maintain such a dynamic form of control, large number of police are necessary [1,2,3]. Another variation of this model was presented in [4] where a number of behavioral based algorithms were developed to enable a group of physical and non-holonomically constrained mobile robot to arrange themselves in a line or circle formation. In the same work, a behavior based mechanism of control relaying on a small group of police agents forcing the march of a group of protesters towards a given location /position and then stop or detain them was proposed. This model was first used in France and UK in 2010. Further Helbing et al., [5] proposed, using nonlinear attractor dynamic to model the control architecture that enable a group of robots to move in triangular formations while avoiding obstacles robots to navigate an obstacle while maintaining a possible formation which could be a line or column. Most recently, agent-based model has been explored by several researchers. Takaheshi et al., [6] and Joyce and Wain [7] have proposed a method of formation control using spectral graph theory. They show how to animate a marching band in transitioning formations, tactical formation in a soccer game and as artistic formation in a large crowd of individual. Nowadays, in many countries, police is seeking optimal ways to manage large crowd of protester which could be armed or not. Generally speaking, model of the police riot control are discrete, but in some situations the continuous time models are used and discretization techniques are to be applied for simulation purposes. The agent-based model to simulate scenarios related to riot control has been explored in many studies and particularly in non-combat police operations. The first authors in the USA who developed a training simulator used to train squad leader for non-combat peacekeeping missions were Hebing [5] and Helbing et al. [8]. Such a simulator provides the trainee with a simulated urban environment wherein they must interact in real time with a crowd of locals that respond to the trainee's actions. In their recent paper, McKenzie et al., [9] have developed an interactive crowd simulation for real time tactical training in scenarios involving crowd of non-combatant protestors. These simulations where designed for commercial gaming purposes rather than urban terrain and physical behaviors of the crowd [10]. Other group of researchers have focused on psychological studies as basic cognitive model for the crowd behavior (see for example Balch and Arkin, [4]). The behavior based model involved a set of elementary, locally reactive control behaviors for each agent, where each behavior has specific low level tasks, such as goal seeking, collision avoidance, and formation maintenance. Reynolds [11] and Raynolds and Flocks, [12] presented a model that combines three distinct behaviors of separation for collisions. We will combine all knowledge from previous study and develop a new model in the next section.

2. Modelling Framework

2.1. Model Justifications

An appropriate modeling of the crowd threat is a necessary and sufficient condition to understand the police reaction to the dynamics of the threat protestors pose. Here, we are interested in modeling two interconnected phases of the lifecycle events: the crowd of protesters, who may include both violent, non-violent elements and the police reactions when protesters actually use violence or misbehave. We simplify the model by considering only the most relevant parameters in these activities. The previous results are based on some very strict assumptions that will incorporated and used exhaustively in the approach developed in this work [8,13,14,15].

2.2. Concepts and Specifications

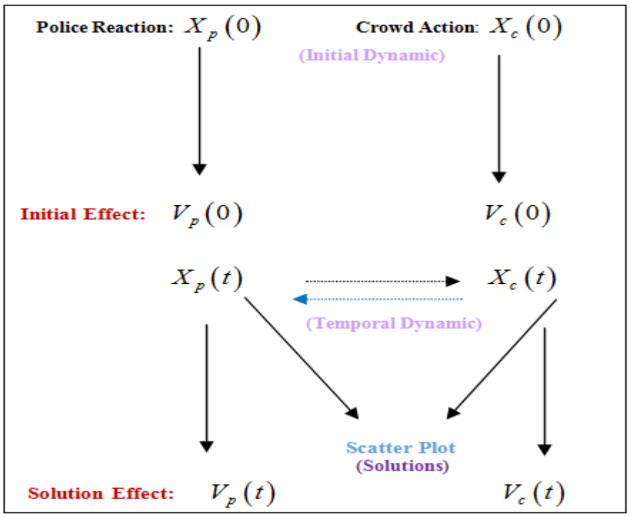

Exhibit 1. Initial and temporal conceptual model of police and crowd dynamics

The model we proposed in this work is conceptual in nature and should not be judged based on the ability to make rigorous prediction regarding crowd behavior in a real word situation. But the purpose here is more to gain insights into the dynamics of a crowd containment scenario by specifying the variables involved and the degrees to which they might influence the resulting behavior and also to understand the modelling in the study and development of police crowd control techniques. We present two groups of models of police containment of large crowds. The first model is deterministic in which the behavior of protestors is predictable, but the second model is stochastic and the behaviors of protesters are unpredictable. Clearly, in the second model the protester are much noisier, and the noise level is correlated with the number of risky actions as well as timely taken by the crowd of protesters. Furthermore, the police reactions might be proportionate or disproportionate to protester actions depending on the type of recommendations the police personnels might receive from their supervisors.

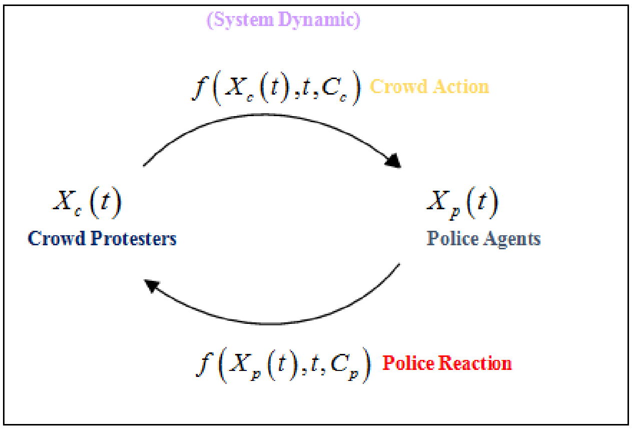

Exhibit 2. The dual newly developed Crowd action – Police reaction is presented in a dynamical framework. Each function $f(.)$ is modelled with a differential equation

2.3. Modelling and Simulation

The first model we present is a Deterministic Delay Differential Equation (DDDEs) in continuous time.

A. Model-1. Deterministic Delay Differential Equations (DDDEs)

The model of the police-crowd dynamic can be represented with a system of DDDEs as follows:

$$\begin{cases} \dot{X}_p(t) = aX_p(t)\left(1 - bX_p\left(t-\tau_p\right)\right) + C_p \\ \dot{X}_c(t) = aX_c(t)\left(1 - bX_c\left(t-\tau_c\right)\right) + C_c \end{cases} \quad (1)$$

The parameters a, b, delay τ and constant terms C_c and C_p are set in advance. The indexes c and p stand respectively for crowd and police. Also in this setup, the input is X_t and the output $f(X_t) = DX_t$ is representing the state variable (population of crowd and police) and V_t the solution to the system (effects of crowd actions and police reactions). We use Matlab 2.4 to carry out the simulations in this work and obtain the following results.

Each function f is modelled with a differential equation. It can be seen that over time the crowd is forced to specific location or direction providing stable solution to the problem.

B. Model-2. Trigonometric Deterministic Delay Differential Equations (TDDDEs)

We propose and use the deterministic delay differential equation to model the dynamic of police crowd containment dynamic in continuous time.

$$\begin{cases} \dot{X}_c(t) = a\sin\left(X\left(t-\tau_c\right)\right) + C_c \\ \dot{X}_p(t) = a\sin\left(X\left(t-\tau_p\right)\right) + C_p \end{cases} \quad (2)$$

The computational solution to equation (1) is presented bellow

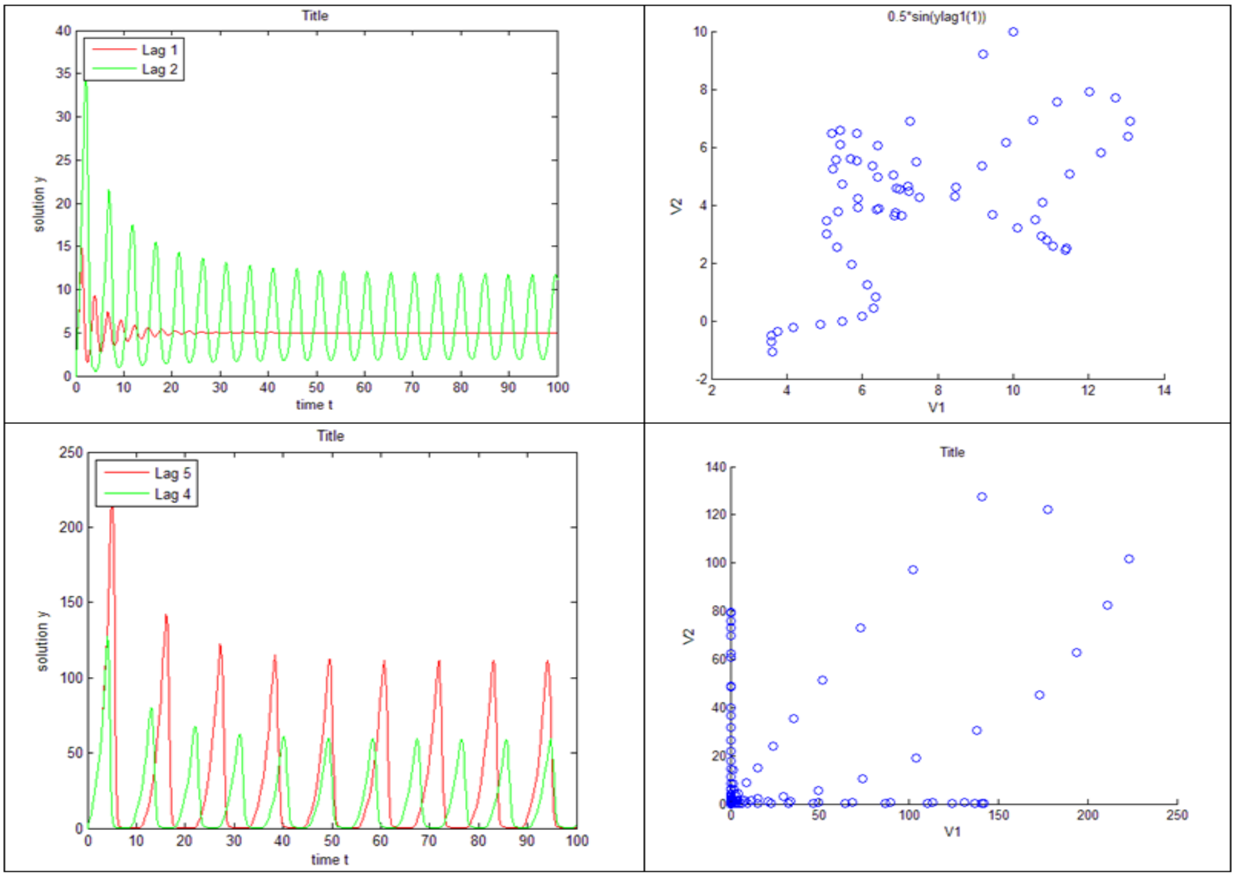

Figure 1. Model 1, **t**he dual (Crowd action – Police reaction) dynamic is presented in a dynamical framework using Deterministic Delay Differential Equations (DDDEs)

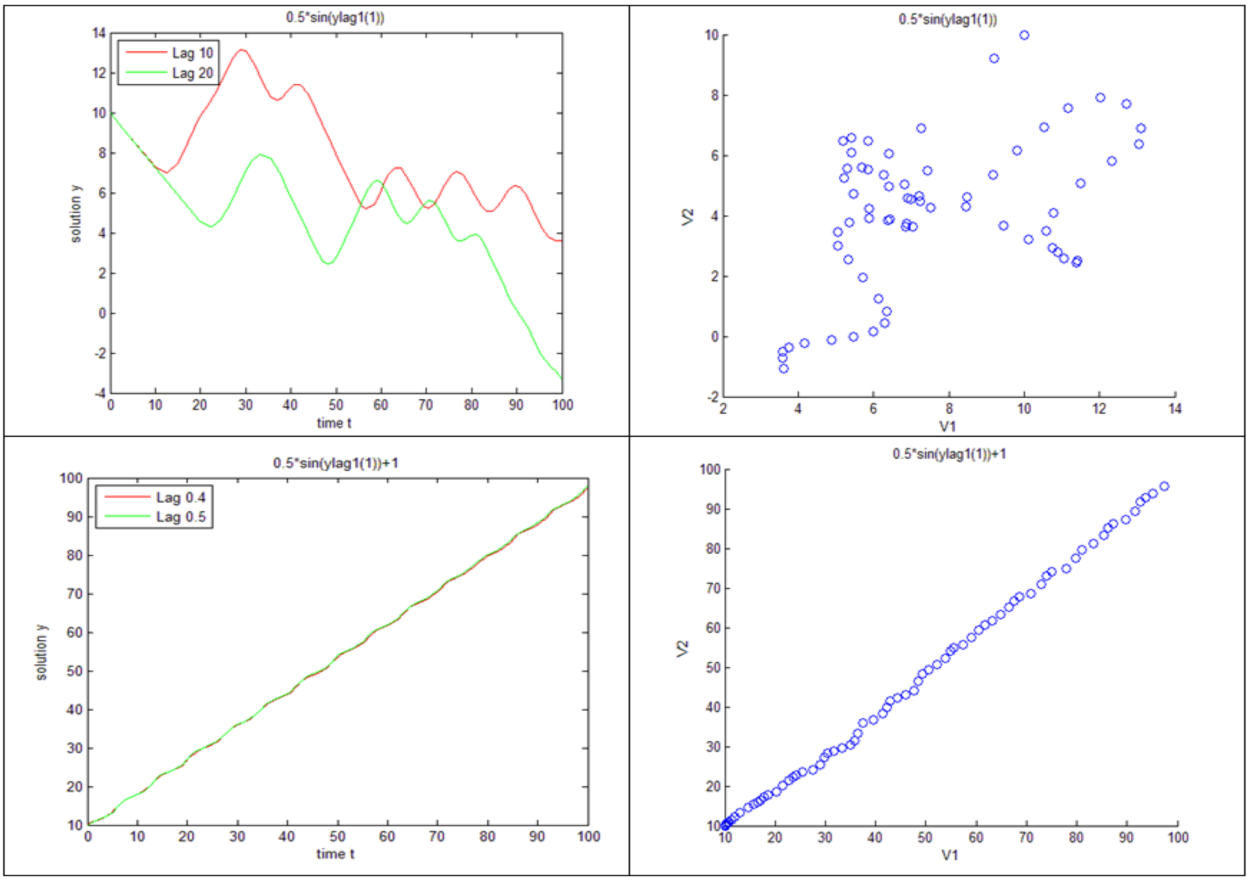

Figure 2. Model 2, the dual (Crowd action – Police reaction) is presented in a dynamical framework using Trigonometric Deterministic Delay Differential Equations (TDDDEs)

The Police agents are chasing the crowd of protesters and forcing them to move toward a specific direction or location where they will be able to control their behaviours. Such location is the only stable point of the system.

C. Model-3. Stochastic Delay Differential Equations (SDDEs)

During last few decades' stochastic differential equations have attracted lots of attentions and became one of the most active areas of stochastic differential analysis and many applied area of science. Therefore SDEs are taken as important tool in modelling and simulating real phenomena. Stochastic delay differential equations also called stochastic differential equations with delays represent good model for modelling random phenomena with delays. We use the indexes and respectively for police and crowd denominations. Following Model – I and II limitations, we add noise to the state dynamics and introduce a new model. Hypothetically, this model will capture additional important features in the system. This model is a system of Stochastic Delay Differential Equations (SDDEs) with parameters a, b, delay τ_p, τ_c and C constant term. Also $\varepsilon_p, \varepsilon_c$ are Gaussian random perturbations respectively for Police and crowd. We set:

$$\begin{cases} \dot{X}_p(t) = aX_p(t)\left(1 - bX_p\left(t - \tau_p\right)\right) + \varepsilon_p + C_p \\ \dot{X}_c(t) = aX_c(t)\left(1 - bX_c\left(t - \tau_c\right)\right) + \varepsilon_c + C_c \end{cases}. \quad (3)$$

The computational solution of equation (3) is presented below in Figure 3.

D. Model 4. Extremal Trigonometric Deterministic Delay Differential Equations (ExTDDDEs)

$$\begin{cases} \dot{X}_c(t) = a\sin\left(X\left(t - \tau_c^{extrem}\right)\right) + C_c \\ \dot{X}_p(t) = a\sin\left(X\left(t - \tau_p^{extrem}\right)\right) + C_p \end{cases}. \quad (4)$$

Now, let us look at the effect of extremal noise on the model. It can be seen that as we increase the noise in the dynamic, the system becomes less stable.

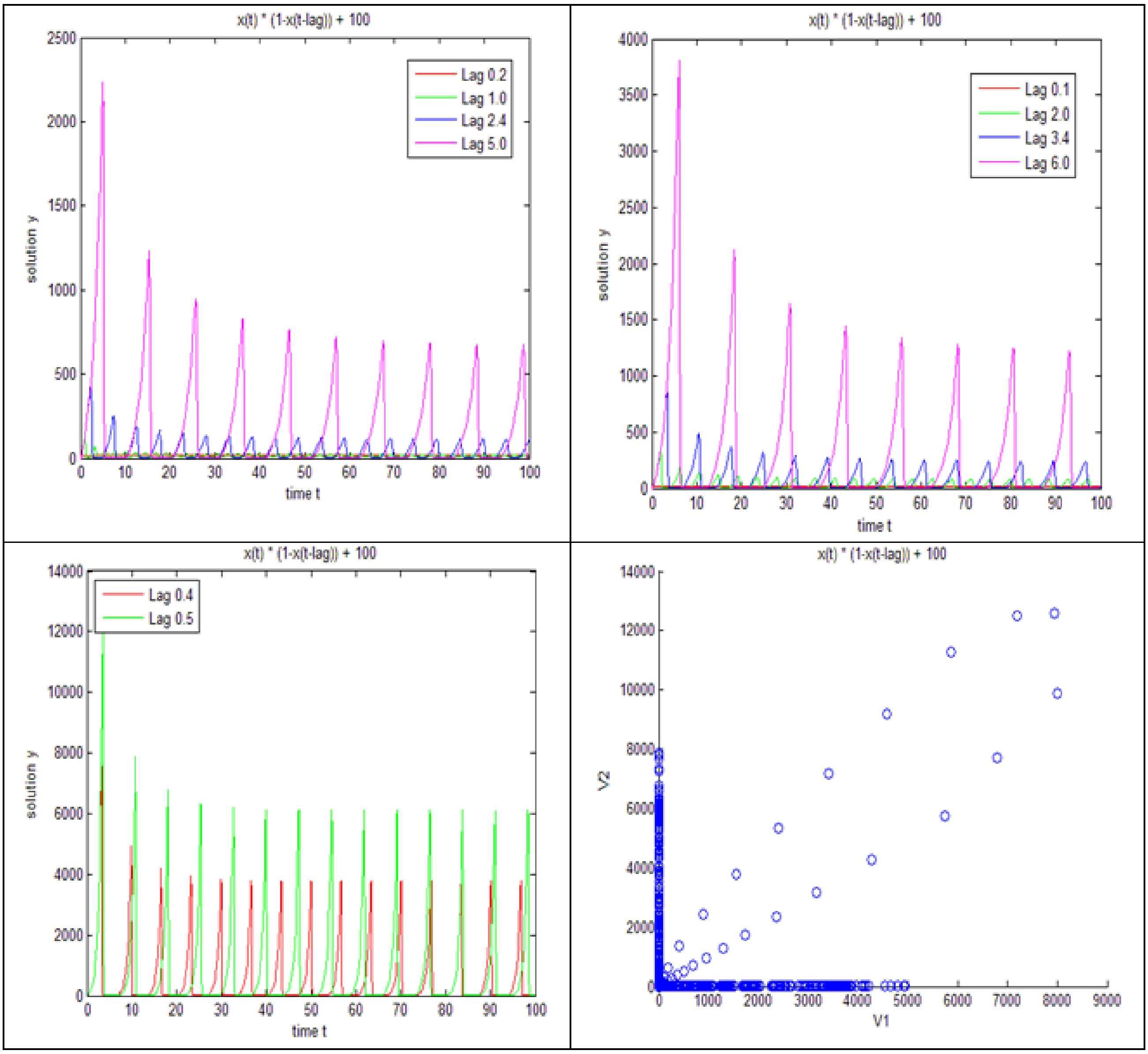

Figure 3. Model 3, the dual (Crowd action – Police reaction) is presented in a dynamical framework using Trigonometric Deterministic Delay Differential Equations (TDDDEs)

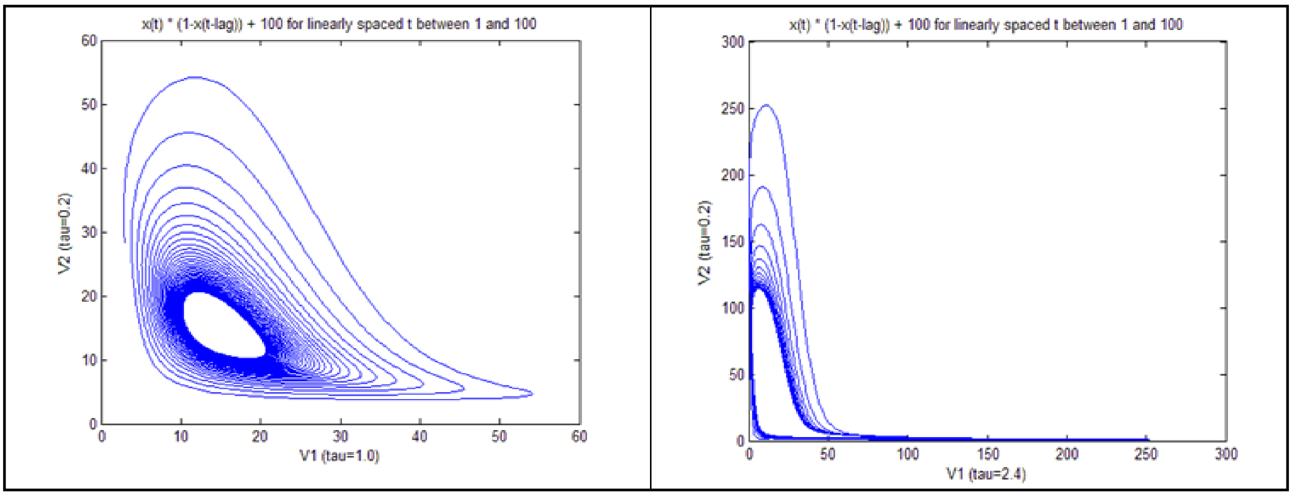

Figure 4. Model 4 - the dual (Crowd action – Police reaction) is presented in a dynamical framework using Trigonometric Deterministic Delay Differential Equations (TDDDEs)

The effect of extremal delays on the dynamics is presented in the above figures. When the delays increase drastically, both police and crowd dynamics become less uniform. The crowd of protesters are either captured or neutralized.

3. Stability Study of the Solutions

The stability of stochastic differential equations has been widely studied by mathematicians in different senses, such as stochastically stable, stochastically asymptotically stable, moment exponentially stable, and almost surely stable mean square polynomial stable. We will prove the stability of the solution using the following lemma.

3.1. Understanding the Solutions

Lemma 1. *Let* $\left(X_c(t), X_p(t)\right)$ *be a stochastically stable system (process) satisfying SDDEs, then the trivial solution* $\left(V_c(t), V_p(t)\right)$ *of such a system is also stochastically stable in the sense of Kolmogorov. Also under certain conditions, such stability can be reduced to stability in the sense of Mineev and Chernov (asymptotic stability) respectively*.

Proof: *Let* $P = P(.,.)$ *be the probability that the process* $X = X_c(t, x_0)$ *is bounded above. The solution mdel-1 (1) is* $V = V_c(t, x_0) = V_c(E_t, x_0) = \underset{\{E_t:0\le t\le\infty\}}{Sup} \left|V_c(E_t, x_0)\right|$ *so that*

$$\begin{aligned} & P\left\{X_c(t, x_0) < h, \forall t \ge 0\right\} \\ & = P\left\{V_c(E_t, x_0) < h, \forall t \ge 0\right\} \\ & = P\left\{\underset{\{E_t:0\le t\le\infty\}}{Sup} \left|V_c(E_t, x_0)\right| < h\right\} \\ & = P\left\{\underset{0\le\tau\le\infty}{Sup} \left|V_c(E_t, x_0)\right| < h\right\} \\ & = P\left\{\left|V_c(E_t, x_0)\right| < h, \forall t \ge 0\right\} = 1-\varepsilon. \end{aligned}$$

Here, we use the fact that the image of $[0,\infty)$ under E_t process is almost surely equal to $[0,\infty)$. We also recall that $E_t(.,.)$ is the expected value taken at time t; epsilon is a small number and x_0 the initial condition.

3.2. Extremal Stability

A stochastic process is extremely stable if and only if it has some points of stability only at the extreme values. In our situation we see that whenever the crowd goes to extreme (taking violent actions), it is immediately neutralized by the police.

4. Discussion

We have conducted simulation experiments on our models by setting some initial values. We produced the solution in models 1, 2 and 3 with relatively low initial values $X_p(0) = 20$ for the police and $X_c(0) = 100$ for the protesters. Although our models perform very well and produce the solution very fast, we still need to develop more appropriate behavior of the system as a whole and check the consistency of the solutions under some variations of parameters.

We have still not yet investigated the kind of improvements we may have when drastically increase the number of police agents and crowd protestors. Also the question of how the police reactions will be affected when we increase the actions of crowd of protestors over time remain not fully unswered in this research. Further the question whether we may have a coloration between the solutions and respective time delays still need to be investigated. Finally, it will be interesting to work on these limitations of the present results in the future investigations.

5. Conclusion

This paper investigates the solution to dynamic models of police - crowd containment system. For the first time, we have been able to use dynamic models to analyse and understand the effects of police reactions on crowd actions. Although the obtained results are encouraging, its scientific validation with some real experiments involving human subjects in real physical contacts the police agents and the crowd of protesters are still needed. Such experiments will help to a) collect the data and b) adjust the model to the reality. Performing all those steps will be necessary in order to realise the model's long term potential for the development of relevant computational tools capable of helping the training the police service in some more efficient crowd containment approaches. The objective of this paper was to model, simulate and analyse the police – crowd containment dynamic systems and also examine the effects of the Police reactions on crowd actions. Further directions for this research will consist of:

a) Exhaustive exploration of the stability of the solutions (effects of action – reaction) in controlling or neutralizing a huge crowd of protesters
b) Investigate new models for improving the performance by either combining the features of different modelling constraints as well as introducing time varying-type parameters.

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Conflict of Interests

None.

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