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Simulation and Optimal Decision Making the Design of Technical Systems (2. The Decision with a Criterion Priority)

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Abstract The paper presents a methodology for modeling and choice of optimum parameters of technical systems with a priority (characteristic) of criterion. It is further development of works of authors in which the problem of decision-making was solved with equivalent criteria. The model is created as a vector problem of mathematical programming. Criteria (characteristics) are formed in the conditions of definiteness (functional dependence of each characteristic and restrictions on parameters is known) and in the conditions of uncertainty (there is no sufficient information on functional dependence of each characteristic on parameters). In the constructed mathematical model of technical system criteria in the conditions of uncertainty will be transformed to definiteness conditions. We have submitted the theory and methods of vector optimization. The vector problem is solved on the basis of normalization of criteria and the principle of the guaranteed result at the set priority. The methodology of research, modeling and the system choice of optimum parameters at design of technical systems is illustrated on a numerical example of model of technical system, in the form of a vector problem of nonlinear programming with five criteria and the set priority of one of them.

Keywords: *modeling technical systems, vector optimization, optimum decision-making, the decision with a criterion priority*

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1. Introduction

The choice of optimum parameters of technical system depending on its functional characteristics is the main objective of the computer-aided engineering system (CAD). One of the directions of design is connected with creation of mathematical models of technical systems. On the basis of the developed models we conduct research, modeling and we choose optimum parameters of technical systems. Such direction of researches considerably reduces terms of design and increases quality of the created technical systems. Therefore to a problem of mathematical modeling of technical systems as much attention is paid to a component of system of the automated design as in Russia [1-13], and abroad in theoretical [15,17,18] and applied aspects [16,19,20,21].

Functioning of technical object, system is defined by some set of the characteristics which are functionally dependent on parameters of system. Improvement of one of these characteristics leads another to deterioration. There is a problem of determination of such parameters which would improve all functional characteristics of technical system at the same time. These problems are solved now, both at technological (experimental) level, and at the mathematical (model) level. The model in this case can be created in the form of a vector problem of mathematical programming in which the vector criterion defines characteristics of technical system [3,5,9,10,11,12,13].

For the solution of a vector task we use the methods based on normalization of criteria and the principle of the guaranteed result [4]. Further we used these methods when modeling technical systems [3,5,7,13]. We use methods at the solution of vector problems with equivalent criteria [3,8,13] and to the set priority of criterion [3,10]. If functional dependence of each characteristic and restrictions on parameters is known, we formulate mathematical model of technical system in the conditions of definiteness [6,7,13]. If functional dependence of each characteristic and restrictions on parameters isn't known, we formulate mathematical model of technical system in the conditions of uncertainty [7,13]. Works are directed to the solution of these problems in the conditions of definiteness and uncertainty of set [6,13]. In real life we have to investigate all set of possible parameters of technical system (Pareto's great number) and to choose the most preferred (optimum) solution. This work is in total directed on the solution of these problems.

The purpose of this work consists in creation of methodology of creation of mathematical model of technical system in the form of a vector problem of mathematical programming. Solutions of a vector problem in the conditions of definiteness and uncertainty in total.

Researches, modeling and the system choice of optimum parameters at design of technical systems from all admissible set of parameters (Pareto's great number). In the simulation, we use decision methods at equivalent criteria and with the set criterion priority.

For realization of a goal in work it is presented: creation of model of technical system in the form of a vector problem of mathematical programming; the methodology of creation of mathematical model of technical system conditions of definiteness and uncertainty in total is shown;

decision-making realization (i.e. the choice of optimum parameters of technical system) at equivalent criteria;

realization of decision-making at the set criterion priority, *i.e. the choice of any optimum point from Pareto's great number*; we have developed the software for the solution of vector tasks with equivalent criteria and with the set criterion priority.

The methodology of modeling is illustrated on a numerical example of model of the technical system, in the form of a vector problem of nonlinear programming realized in Matlab [14] system. The methodology has system character and can be used as for technical and economic problems, [11,12].

2. Statement of a Problem. Methodology of Modeling of Technical Systems in the Conditions of Definiteness and Uncertainty

The problem of a choice of optimum parameters of technical systems according to functional characteristics arises during the studying, the analysis and design of technical systems and is connected with quality production.

The problem includes the solution of the following tasks:

Creation of mathematical model which defines interrelation of each functional characteristic from parameters of technical system i.e. is formed of the vector problem of mathematical programming;

Choice of methods of the decision: we suggest using the methods based on normalization of criteria and the principle of the guaranteed result with equivalent criteria and with the set criterion priority;

The software which realizes these methods is developed. Statement of a problem is executed according to [3].

2.1. Creation of Mathematical Model of Technical System

The technical system which functioning depends on $\boldsymbol{N}$ - a set of design data is considered: $X=\{x_1\ x_2\ \ldots\ x_N\}$, N - number of parameters, each of which lies in the set limits

$$x_j^{\min} \le x_j \le x_j^{\max},\ j=\overline{1,N},\ or\ X^{min} \le X \le X^{max} \quad (1)$$

где $x_j^{\min}$, $x_j^{\max}$, $\forall j\in \boldsymbol{N}$ - lower and top limits of change of a vector of parameters of technical system.

The result of functioning of technical system is defined by a set $\boldsymbol{K}$ to technical characteristics of $f_k(X)$, $k=\overline{1,K}$ which functionally depend on design data $X=\{x_j,\ j=\overline{1,N}\}$, in total they represent a vector function:

$$F(X)=\left(f_1(X)\, f_2(X) \ldots f_K(X)\right)^T. \quad (2)$$

The set of characteristics (criteria) to is subdivided into two subsets $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$: $\boldsymbol{K}=\boldsymbol{K}_1 \cup \boldsymbol{K}_2$

$\boldsymbol{K}_1$ is a subset of technical characteristics which numerical sizes it is desirable to receive as it is possible above: $f_k(X) \rightarrow \mathbf{max}$, $k=\overline{1,K_1}$.

$\boldsymbol{K}_2$ - it subsets of technical characteristics which numerical sizes it is desirable to receive as it is possible below: $f_k(X) \rightarrow \mathbf{min}$, $k=\overline{K_1+1,K}$, $\boldsymbol{K}_2 \equiv \overline{K_1+1,K}$.

Mathematical model of technical system which solves in general a problem of a choice of the optimum design decision (a choice of optimum parameters), we will present in the form of a vector problem of mathematical programming.

$$Opt\ F(X)=\{\mathbf{max}\ F_1(X)=\left\{\mathbf{max}\ f_k(X),\ k=\overline{1,K_1}\right\}, \quad (3)$$

$$\mathbf{min}\ F_2(X)=\left\{\mathbf{min}\ f_k(X),\ k=\overline{1,K_2}\right\}\} \quad (4)$$

$$G(X) \le 0, \quad (5)$$

$$x_j^{\min} \le x_j \le x_j^{\max},\ j=\overline{1,N}, \quad (6)$$

where X - a vector of operated variable (design data) from (1);

$F(X)=\{f_k(X),\ k=\overline{1,K}\}$ - criterion which everyone a component submits the characteristic of technical system (2) which is functionally depending on a vector of variables X;

in (5) $G(X)=(g_1(X)\ g_2(X)\ \ldots\ g_M(X))^{\mathrm{T}}$ – vector function of the restrictions imposed on functioning of technical system, $\boldsymbol{M}$ – a set of restrictions.

Restrictions are defined proceeding in them technological, physical and to that similar processes and can be presented by functional restrictions, for example, $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$.

It is supposed that the $f_k(X)$, $k=\overline{1,K}$ functions are differentiated and convex, $g_i(X)$, $i=\overline{1,M}$ are continuous, and (5)-(6) set of admissible points of $\boldsymbol{S}$ set by restrictions isn't empty and represents a compact:

$$S=\{X \in R^N \mid G(X) \le 0,\ X^{min} \le X \le X^{max}\} \ne \varnothing.$$

Criteria and restrictions (3)-(6) form mathematical model of technical system. It is required to find such vector of the $X^o \in \boldsymbol{S}$ parameters at which everyone a component the vector - functions $F_1(X)=\{f_k(X),\ k=\overline{1,K_1}\}$ accepts the greatest possible value, and a vector - functions $F_2(X)=\{f_k(X),\ k=\overline{1,K_2}\}$ are accepted by the minimum value.

To a substantial class of technical systems which can be presented by a vector task (3)-(6), it is possible to refer their rather large number of tasks from various branches of economy of the state: electrotechnical, aerospace, metallurgical (choice of optimal structure of material), etc. In this article for technical system are considered in a statics. But technical systems can be considered in dynamics [22], using differential-difference methods of transformation [5], conducting research for a small discrete period $\Delta t \in T$.

2.2. Creation of Mathematical Model of Technical System in the Conditions of Definiteness and Uncertainty in Total

At creation of mathematical model of technical system (3)-(6) conditions are possible: definiteness and uncertainty.

Conditions of definiteness are characterized by that functional dependence of each characteristic and restrictions on parameters of technical system [6,8,13] is known.

Conditions of uncertainty are characterized by that there is no sufficient information on functional dependence of each characteristic and restrictions from parameters [6,9,13,15].

In real life of a condition of definiteness and uncertainty are combined. The model of technical system also has to reflect these conditions. We will present model of technical system in the conditions of definiteness and uncertainty in total [13]:

$$Opt\ F(X) = \{\max F_1(X) = \{\max f_k(X), k = \overline{1, K_1^{def}}\}, \quad (7)$$

$$\max I_2(X) = \{\max \{f_k(X_i, i = \overline{1,M})\}^T, k = \overline{1, K_1^{unc}}\},$$

$$\min F_2(X) = \{\min f_k(X), k = \overline{1, K_2^{def}}\}, \quad (8)$$

$$\min I_2(X) = \{\min \{f_k(X_i, i = \overline{1,M})\}^T, k = \overline{1, K_2^{unc}}\}\},$$

at restrictions

$$f_k^{\min} \le f_k(X) \le f_k^{\max}, k = \overline{1, K}, \quad (9)$$

$$x_j^{\min} \le x_j \le x_j^{\max}, j = \overline{1, N}, \quad (10)$$

where X - a vector of operated variable (design data) equivalent (1); $F(X)=\{ F_1(X)\ F_2(X)\ I_1(X), I_2(X)\}$ - vector criterion which everyone a component represents a vector of criteria (characteristics) of technical system (2) which functionally depend on discrete values of a vector of variables X ; $F_1(X)$, $F_2(X)$ - a set of the max and min functions respectively; $I_1(X)$ и $I_2(X)$ set of matrixes of max and min respectively; K_1^{def}, K_2^{def} (*definiteness*), K_1^{unc}, K_2^{unc} (*uncertainty*) the set of criteria of *max* and *min* created in the conditions of definiteness and uncertainty;

in (9) $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$ – a vector function of the restrictions imposed on functioning of technical system $x_j^{\min} \le x_j \le x_j^{\max}$, $j =\overline{1,N}$ – parametrical restrictions.

2.3. Transformation of a Problem of Decision-making in the Conditions of Uncertainty into a Problem of Vector Optimization in the Conditions of Definiteness

Elimination of uncertainty consists in use of qualitative and quantitative descriptions of technical system which can be received, for example, by the principle "entrance exit". Transformation of basic data "entrance exit" to functional dependence is carried out by use of mathematical methods (the regression analysis).

The technical system in which experimental data are presented in the form of a matrix $I_1(X)$, $I_2(X)$ in (7)-(8) is considered in the following designations:

$$I = \begin{bmatrix} X_1 & y_1(X_1) \ldots y_K(X_1) \\ \ldots \\ X_M & y_1(X_M) \ldots y_K(X_M) \end{bmatrix}, or\ I = |X\ Y|, \quad (11)$$

where is considered: $X=\{X_i =\{x_{ij}, j=\overline{1,N}\}, i=\overline{1,M}\}$ - design data of technical system, $\boldsymbol{N}$ – a set of parameters of system, $\boldsymbol{M}$ - a set of alternatives (experiments); $Y=\{y_{ik}, k=\overline{1,K}, i=\overline{1,M}\}$, $\boldsymbol{K}$ – a set of criteria (characteristics) by which each alternative is estimated, [15].

Construction a vector - function (criteria) is carried out on a method of the smallest squares $\min \sum_{i=1}^{M} (y_i - \overline{y}_i)^2$, where by y_i, $i=\overline{1,M}$ - really observed sizes, and $\overline{y_i}$, $i=\overline{1,M}$ their estimates received for one-factorial model by means of function $\overline{y_i} = f(X_i, A)$, $X_i =\{x_i\}$. As $f(X_i, A)$ we use a polynom. In applied part of work the polynom of the second degree is used:

$$\min_A f(A, X) \equiv \sum_{i=1}^{M} \left(y_j - \left(\begin{array}{l} a_0 + a_1 x_{1i} + a_2 x_{1i}^2 + a_3 x_{2i} \\ + a_4 x_{2i}^2 + a_5 x_{1i} * x_{2i} \end{array} \right) \right)^2,$$

Result: Basic data $\{\{f_k(X_i, i=\overline{1,M}\}^T, k=\overline{1,K_1^{unc}}\}, \{f_k(X_i, i=\overline{1,M}\}^T, k=\overline{1,K_2^{unc}}\}\}$ in problems of decision-making in the conditions of uncertainty (7), (8) the functions - $f_k(X)$, $k=\overline{1,K_1^{unc}}$, $f_k(X)$, $k=\overline{1,K_2^{unc}}$ are transformed.

As a result the vector problem (13)-(16) will be transformed into a vector problem in the conditions of definiteness:

$$Opt\ F(X) = \left\{ \begin{array}{l} \max F_1(X) = \{\max f_k(X), k = \overline{1, K_1}\}, \\ \min F_2(X) = \{\min f_k(X), k = \overline{1, K_2}\} \end{array} \right\}, \quad (12)$$

at restrictions

$$f_k^{\min} \le f_k(X) \le f_k^{\max}, k = \overline{1, K}, \quad x_j^{\min} \le x_j \le x_j^{\max}, j = \overline{1, N}, \quad (13)$$

where $F(X)=\{f_k(X), k=\overline{1,K}\}$ - vector criterion which everyone a component submits the characteristic of technical system which is functionally depending on a vector of variables X; subset of criteria $K_1= K_1^{def} \cup K_1^{unc}$, $K_2= K_2^{def} \cup K_2^{unc}$.

3. Theory of Vector Optimization

The theory of vector optimization includes theoretical foundations (axiomatics) and methods of the solution of vector problems with equivalent criteria and with the given criterion priority. The theory is a basis of mathematical apparatus of modeling of technical systems which allows you to select any point from a set of points, optimum across Pareto and to show why she is optimum.

We will present axiomatics and methods of vector optimization, first, at equivalent criteria (sections 3.1, 3.2 in compliance [3] - it is short), and, secondly, at the set criterion priority in a vector problem (sections 3.3, 3.4).

3.1. Axiomatics of Vector Optimization with Equivalent Criteria

Definition 1. (*Definition of a relative assessment of criterion*).

In a vector problem (12)-(13) we will enter designation:

$\lambda_k(X) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}$, $\forall k \in \boldsymbol{K}$ is the relative estimate of a point $X\in S$ k-th criterion;

f_k(X) - k-th criterion at the point $X\in S$; f_k^* - value of the k-th criterion at the point of optimum X_k^*, obtained in vector problem (3)-(6) of individual k-th criterion; f_k^0 is the worst value of the k-th criterion (antioptimum) at the point X_k^0 (Superscript 0 - zero) on the admissible set $\boldsymbol{S}$ in vector problem (3)-(6); the task at *max* (3), (5), (6) the value of f_k^0 is the lowest value of the k-th criterion $f_k^0 = \min_{X\in S} f_k(X)$ $\forall k\in \boldsymbol{K}_1$ and task min f_k^0 is the greatest: $f_k^0 = \max_{X\in S} f_k(X)$ $\forall k\in \boldsymbol{K}_2$. The relative estimate of the λ_k(X), $\forall k\in K$ is first, measured in relative units; secondly, the relative assessment of the $\lambda_k(X)$ $\forall k\in K$ on the admissible set is changed from zero in a point of X_k^0 : $\forall k\in \boldsymbol{K}$ $\lim_{X\to X_k^o} \lambda_k(X)=0$, to the unit at the point of an optimum of X_k^* :$\forall k\in \boldsymbol{K}$ $\lim_{X\to X_k^*} \lambda_k(X)=1$ i.e.: $\forall k\in$ $0\le\lambda_k(X) \le 1$, $X\in S$ this allows the comparison criteria, measured in relative units, among themselves by joint optimization.

Axiom 1. (*About equality and equivalence of criteria in an admissible point of vector problems of mathematical programming*)

In of vector problems of mathematical programming two criteria with the indexes $k\in \boldsymbol{K}$, $q\in \boldsymbol{K}$ shall be considered as equal in $X\in \boldsymbol{S}$ point if relative estimates on k-th and q-th to criterion are equal among themselves in this point, i.e. $\lambda_k(X) = \lambda_q(X)$, $k, q \in \boldsymbol{K}$.

We will consider criteria equivalent in vector problems of mathematical programming if in $X\in S$ point when comparing in the numerical size of relative estimates of $\lambda_k(X)$, $k=\overline{1,K}$, among themselves, on each criterion of $f_k(X)$, $k=\overline{1,K}$, and, respectively, relative estimates of λ_k(X), isn't imposed conditions about priorities of criteria.

Definition 2. (*Definition of a minimum level among all relative estimates of criteria*).

The relative level λ in a vector problem represents the lower assessment of a point of $X\in S$ among all relative estimates of $\lambda_k(X)$, $k = \overline{1,K}$:

$$\forall X \in S, \lambda \le \lambda_k(X), k = \overline{1,K}, \quad (14)$$

the lower level for performance of a condition (14) in an admissible point of $X\in \boldsymbol{S}$ is defined by a formula

$$\forall X \in S, \lambda = \min_{k\in K} \lambda_k(X). \quad (15)$$

Ratios (14) and (15) are interconnected. They serve as transition from operation (15) of definition of min to restrictions (14) and vice versa.

The level λ allows to unite all criteria in a vector problem one numerical characteristic of λ and to make over her certain operations, thereby, carrying out these operations over all criteria measured in relative units. The level λ functionally depends on the $X\in \boldsymbol{S}$ variable, changing X, we can change the lower level - λ. From here we will formulate the rule of search of the optimum decision.

Definition 3. (*The principle of an optimality with equivalent criteria*).

The vector problem of mathematical programming at equivalent criteria is solved, if the point of $X^o\in S$ and a maximum level of λ^o (the top index o - optimum) among all relative estimates such that is found

$$\lambda^o = \max_{X\in S} \min_{k\in K} \lambda_k(X). \quad (16)$$

Using interrelation of expressions (14) and (15), we will transform a maximine problem (16) to an extreme problem

$$\lambda^o = \max_{X\in S} \lambda, \quad (17)$$

$$\lambda \le \lambda_k(X), k = \overline{1,K}. \quad (18)$$

The resulting problem (17)-(18) let's call the λ-problem.

λ-problem (17)-(18) has (N+1) dimension, as a consequence of the result of the solution of λ-problem (17)-(18) represents an optimum vector of $\boldsymbol{X}^o\in \boldsymbol{R}^{N+1}$, ($N$+1) which component an essence of the value of the λ^o, i.e. $\boldsymbol{X}^o=\{x_1^o, x_2^o, ..., x_N^o, x_{N+1}^o\}$, thus $x_{N+1}^o = \lambda^o$, and (N+1) a component of a vector of $\boldsymbol{X}^o$ selected in view of its specificity.

The received a pair of $\{\lambda^o, X^o\}=\boldsymbol{X}^o$ characterizes the optimum solution of λ-problem (17)-(18) and according to vector problem of mathematical programming (12)-(13) with the equivalent criteria, solved on the basis of normalization of criteria and the principle of the guaranteed result. We will call in the optimum solution of $\boldsymbol{X}^o=\{X^o, \lambda^o\}$, X^o - an optimal point, and λ^o - a maximum level.

An important result of the algorithm for solving vector problems (12)-(13) with equivalent criteria is the following theorem.

Theorem 1. (*The theorem of two most contradictory criteria in a vector problem of mathematical programming with equivalent criteria*).

In convex vector problems of mathematical programming at the equivalent criteria which is solved on the basis of normalization of criteria and the principle of the guaranteed result, in an optimum point of $\boldsymbol{X}^o=\{\lambda^o, X^o\}$ two criteria are always - denote their indexes $q\in \boldsymbol{K}$, $p\in \boldsymbol{K}$

(which in a sense are the most contradiction of the criteria $k=\overline{1,K}$), for which equality is carried out:

$$\lambda^o = \lambda_q\left(X^o\right) = \lambda_p\left(X^o\right), q, p \in K, X \in S, \quad (19)$$

and other criteria are defined by inequalities:

$$\lambda^o = \lambda_k\left(X^o\right) \quad \forall k \in K, q \neq p \neq k. \quad (20)$$

3.2. Mathematical Algorithm of the Solution of a Vector Problem with Equivalent Criteria

For the solution of vector problems of mathematical programming (12)-(13) the methods based on axiomatics of normalization of criteria and the principle of the guaranteed result [3,8] are offered. Methods follow from an axiom 1 and the principle of an optimality 1. We will present in the form of a number of steps:

Algorithm 1 of the solution of a vector task (12)-(13) with equivalent criteria [3].

Step 1. The problem (12)-(13) by each criterion separately is solved, i.e. for $\forall k \in K_1$ is solved at the maximum, and for $\forall k \in K_2$ is solved at a minimum. As a result of the decision we will receive: X_k^* - an optimum point by the corresponding criterion, $k=\overline{1,K}$; $f_k^* = f_k(X_k^*)$ – the criterion size k-th in this point, $k=\overline{1,K}$.

Step 2. We define the worst value of each criterion on $\boldsymbol{S}$: f_k^0, $k=\overline{1,K}$. For what the problem (12)-(13) for each criterion of $k=\overline{1,K}_1$ on a minimum is solved: $f_k^0 = \min f_k(X)$, $G(X) \le B$, $X \ge 0$, $k=\overline{1,K}_1$.

The problem (12)-(13) for each criterion on a maximum is solved: $f_k^0 = \max f_k(X)$, $G(X) \le B$, $X \ge 0$, $k=\overline{1,K}_2$.

As a result of the decision we will receive: $X_k^0=\{x_j, j=\overline{1,N}\}$ - an optimum point by the corresponding criterion, $k=\overline{1,K}$; $f_k^0=f_k(X_k^0)$ – the criterion size k-th a point, X_k^0, $k=\overline{1,K}$.

Step 3. The analysis of a set of points, optimum across Pareto, for this purpose in optimum points of $X^*=\{X_k^*, k=\overline{1,K}\}$ are defined sizes of criterion functions of $F(X^*)$ and relative estimates $\lambda(X^*)$, $\lambda_k(X) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}$, $\forall k \in \boldsymbol{K}$:

$$F\left(X^*\right) = \left\{f_q(X), q=\overline{1,K}, k=\overline{1,K}\right\} = \begin{vmatrix} f_1(X_1^*),\dots, f_k(X_1^*), \\ \dots \\ f_1(X_k^*),\dots, f_k(X_k^*) \end{vmatrix},$$

$$\lambda\left(X^*\right) = \left\{\lambda_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\right\} = \begin{vmatrix} \lambda_1(X_1^*),\dots, \lambda_k(X_1^*), \\ \dots \\ \lambda_1(X_k^*),\dots, \lambda_k(X_k^*) \end{vmatrix}. \quad (21)$$

As a whole on a problem of accordance with (9) $\forall k\in\boldsymbol{K}$ the relative assessment of $\lambda_k(X)$, $k=\overline{1,K}$ lies within $0 \le \lambda_k(X) \le 1$, $\forall k \in \boldsymbol{K}$.

Step 4. Creation of the λ-problem.

Creation of λ-problem is carried out in two stages: initially built the maximine problem of optimization with the normalized criteria which at the second stage will be transformed to the standard problem of mathematical programming called λ-problem.

For construction maximine a problem of optimization we use definition - relative level $\forall X\in\boldsymbol{S}$ $\lambda = \min_{k\in K} \lambda_k(X)$.

The bottom λ level is maximized on $X\in S$, as a result we will receive a maximine problem of optimization with the normalized criteria.

$$\lambda^o = \max_x \min_k \lambda_k\left(X\right), G\left(X\right) \le B, X \ge 0. \quad (22)$$

At the second stage we will transform a problem (22) to a standard problem of mathematical programming:

$$\lambda^o = \max \lambda, \qquad \lambda^o = \max \lambda, \quad (23)$$

$$\lambda - \lambda_k\left(X\right) \le 0, k=\overline{1,K} \to \lambda - \frac{f_k(X) - f_k^o}{f_k^* - f_k^o} \le 0, k=\overline{1,K}, \quad (24)$$

$$G\left(X\right) \le B, X \ge 0, \qquad G\left(X\right) \le B, X \ge 0, \quad (25)$$

where the vector of unknown of X has dimension of N+1: $X=\{\lambda, x_1, \dots, x_N\}$.

Step 5. Solution of λ-problem.

λ-problem (23)-(25) is a standard problem of convex programming and for its decision standard methods are used.

As a result of the solution of λ-problem it is received:

$\boldsymbol{X^o}=\{\lambda^o, X^o\}$ - an optimum point;

$f_k(X^o)$, $k=\overline{1,K}$ - values of the criteria in this point;

$\lambda_k(X^o) = \frac{f_k(X^o) - f_k^o}{f_k^* - f_k^o}$, $k=\overline{1,K}$ - sizes of relative estimates;

λ^o - the maximum relative estimates which is the maximum bottom level for all relative estimates of $\lambda_k(X^o)$, or the guaranteed result in relative units, λ^o guarantees that all relative estimates of $\lambda_k(X^o)$ more or are equal λ^o :

$$\lambda_k\left(X^o\right) \ge \lambda^o, k=\overline{1,K} \quad \text{or } \lambda^o \le \lambda_k\left(X^o\right), k=\overline{1,K}, X^o \in S, \quad (26)$$

and according to the theorem the 2 [3,8] point of $\boldsymbol{X^o}=\{\lambda^o, x_1, \dots, x_N\}$ is optimum across Pareto.

3.3. Axiomatics of Vector Optimization with a Criterion Priority

For development of methods of the solution of problems of vector optimization with a priority of criterion we will enter definitions:

- priority of one criterion of vector problems with a criterion priority over others criteria;
- numerical expression of a priority;
- the set priority of criterion;

• the lower (minimum) level from all criteria with a priority of one of them;

• about a subset of points, priority by criterion (Axiom 2);

• the principle of an optimality of the solution of problems of vector optimization with the set priority of one of criteria;

and related theorems. For more details see [4,6,11].

Definition 4. (*About the priority of one criterion over the other*).

The criterion of $q \in \boldsymbol{K}$ in vector problem (12)-(13) in a point of $X \in \boldsymbol{S}$ has a priority over other criteria of $k = \overline{1,K}$ relative estimate of $\lambda_q(X)$ by this criterion more or is equal relative estimates of $\lambda_k(X)$ of other criteria, i.e.:

$$\lambda_q(X) \geq \lambda_k(X), k = \overline{1,K},$$

and a strict priority, if at least for one criterion of $t \in \boldsymbol{K}$,

$$\lambda_q(X) \geq \lambda_t(X), t \neq q,$$

and for other criteria of $\lambda_q(X) \geq \lambda_k(X)$, $k=\overline{1,K}$, $k \neq t \neq q$.

Introduction of definition of a priority of criterion in vector problem (12)-(13) executed redefinition of early concept of a priority. If earlier in it the intuitive concept about importance of this criterion was put, now this "importance" is defined by mathematical concept: the more the relative estimate of q-th of criterion over others, the it is more important (more priority), and the highest priority in a point of an optimum of X_k^*, $\forall q \in \boldsymbol{K}$.

From definition of a priority of criterion of $q \in \boldsymbol{K}$ in vector problem (12)-(13) follows that it is possible to reveal a set of points of $\boldsymbol{S_q} \subset \boldsymbol{S}$ which is characterized by that $\lambda_q(X) \geq \lambda_k(X)$ $\forall k \neq q$ $\forall X \in S_q$. But the answer to a question of, as far as criterion of $q \in \boldsymbol{K}$ in this or other point of a set of S_q is more priority than the others, remains open. For clarification of this question we will enter communication coefficient between couple of relative estimates of q and k which in total represent a vector: $P^q(X) = \{p_k^q(X) \mid k=\overline{1,K}\}$ $q \in \boldsymbol{K}$ $\forall X \in S_q$.

Definition 5. (*About numerical expression of a priority of one criterion over another*).

In vector problem (12)-(13) with a priority of criterion of q-th over other criteria of $k = \overline{1,K}$, for $\forall X \in S_q$, a vector of $P^q(X)$ which everyone component shows in how many time a relative estimate of $\lambda_q(X)$, $q \in \boldsymbol{K}$, is more than other relative estimates of $\lambda_k(X)$, $k=\overline{1,K}$, we will call *numerical expression of a priority of q-th of criterion over other criteria* of $k = \overline{1,K}$, i.e.

$$P^q(X) = \left\{ p_k^q(X) = \lambda_q(X)/\lambda_k(X), k = \overline{1,K} \right\},$$
$$p_k^q(X) \geq 1, \forall X \in S_q \subset S, k = \overline{1,K}, \forall q \in K. \quad (27)$$

Definition 6. (*About the set numerical expression of a priority of one criterion over another*).

In vector problem (12)-(13) with a priority of criterion of $q \in \boldsymbol{K}$ for $\forall X \in \boldsymbol{S}$ vector $P^q = \{p_k^q$, $k=\overline{1,K}\}$, is considered the set person making decisions, (decision-maker) if everyone is set a component of this vector. Set by the decision-maker of a component p_k^q, from the point of view of the decision-maker, shows in how many time a relative estimate of $\lambda_k(X)$, $k = \overline{1,K}$ is more than other relative estimates of $\lambda_k(X)$, $k = \overline{1,K}$. The vector of p_k^q, $k = \overline{1,K}$ is the set numerical expression of a priority of q-th of criterion over other criteria of $k=\overline{1,K}$

$$P^q(X) = \left\{ p_k^q, K = \overline{1,K} \right\},$$
$$p_k^q \geq 1, \forall X \in S_q \subset S, k = \overline{1,K}, \forall q \in K. \quad (28)$$

Vector problem (12)-(13) in which the priority any of criteria is set, call Vector problem with the set priority of criterion.

The problem of a task of a vector of priorities arises when it is necessary to determine $X^o \in S$ point by the set vector of priorities.

At operation of comparison of relative estimates with a priority of criterion of $q \in \boldsymbol{K}$, similarly, as well as in a task with equivalent criteria, we will enter the additional numerical characteristic of λ which we will call *level*.

Definition 7. (*About the lower level among all relative estimates with a criterion priority*).

The λ level is the lowest among all relative estimates with a priority of criterion of $q \in$ such that

$$\lambda \leq p_k^q \lambda_k(X), k = \overline{1,K}, q \in K, \forall X \in S_q \subset S; \quad (29)$$

the lower level for performance of a condition (29) is defined

$$\lambda = \min_{k \in K} p_k^q \lambda_k(X), q \in K, \forall X \in S_q \subset S. \quad (30)$$

Ratios (29) and (30) are interconnected and serve further as transition from operation of definition of min to restrictions and vice versa.

In section 3.1 we have given definition of a point of $X^o \in S$, optimum across Pareto, with equivalent criteria.

Considering this definition as initial, we will construct a number of the axioms dividing an admissible set of $\boldsymbol{S}$, first, into a subset of points of S^o, optimum across Pareto, and, secondly, on subsets of points of $S_q \subset S$, $q \in \boldsymbol{K}$, priority on q-th to criterion.

Axiom 2. (*About a subset of points, priority by criterion*).

In vector problem (12)-(13) the subset of points of $\boldsymbol{S}_q \subset \boldsymbol{S}$ is called as area of a priority of criterion of $q \in \boldsymbol{K}$ over other criteria, if $\forall X \in \boldsymbol{S_q}$ $\forall k \in \boldsymbol{K}$ $\lambda_q(X) \geq \lambda_k(X)$, $q \neq k$.

This definition extends and on a set of points of $\boldsymbol{S^o}$, optimum across Pareto that is given by the following definition.

Axiom 2a. (*About a subset of points, priority by criterion, on Pareto's great number in Vector problem*).

In a vector problem of mathematical programming the subset of points of $S_q^o \subset \boldsymbol{S^o} \subset \boldsymbol{S}$ is called as area of a priority of criterion of $q \in \boldsymbol{K}$ over other criteria, if

$$\forall X \in S_q^o \ \ \forall k \in K \ \ \lambda_q(X) \geq \lambda_k(X), q \neq k.$$

We will give some explanations.

Axiom 2 and 2a allowed to break in vector problem (12)-(13) an admissible set of points of $\boldsymbol{S}$, including a subset of points, optimum across Pareto, $\boldsymbol{S^o} \subset \boldsymbol{S}$, into subsets:

one subset of points of $S' \in S$ where criteria are equivalent, and a subset of points of S', being crossed with a subset of points of S^o, allocates a subset of points, optimum across Pareto, at equivalent criteria of $S^{oo} = S' \cap S^o$ which as it will be shown further, consists of one point of $X^o \in S$, i.e. $X^o = S^{oo} = S' \cap S^o$, $S' \in S$, $S^o \in S$;

"K" of subsets of points where each criterion of $q=\overline{1,K}$ has a priority over other criteria of $k=\overline{1,K}$, $q \neq k$, thus breaks, first, sets of all admissible points of S, into subsets of $S_q \subset S$, $q=\overline{1,K}$ and, secondly, a set of points, optimum across Pareto, S^o, into subsets $S_q^o \subset S^o \subset S$, $q=\overline{1,K}$,

From here the following ratios are right:

$$S' \mathrm{U}\left(\underset{q\in K}{U} S_q^o\right) = S^o, S_q^o \subset S^o \subset S, q=\overline{1,K}.$$

We will notice that the subset of points of S_q^o on the one hand is included in area (a subset of points) priority of criterion of $q \in K$ over other criteria:

$$S_q^o \subset S_q \subset S, \tag{31}$$

and with another, in a subset of points, optimum across Pareto:

$$S_q^o \subset S^o \subset S. \tag{32}$$

The axiom 2 and numerical expression of a priority of criterion (*Definition 5*) allow to identify each admissible point of $X \in S$ (by means of vector $P^q(X)=\{ p_k^q(X) = \lambda_q(X)/\lambda_k(X), k=\overline{1,K} \}$), to form and choose:

- subset of points by priority criterion of S_q which is included in a set of points of S, $\forall q \in K$ $X \in S_q \subset S$, (such subset of points can be used in *problems of a clustering*, but it is beyond article);
- subset of points by priority criterion of S_q^o which is included in a set of points of S^o, optimum across Pareto, $\forall q \in K$, $X \in S_q^o \subset S^o$.

Thus, full identification of all points in a vector problem (12)-(13) in sequence is executed:

Set of admissible points of $X \in S \rightarrow$	Subset of points, optimum across Pareto, $X \in S^o \subset S \rightarrow$	Subset of points, optimum across Pareto $X \in S_q^o \subset S^o \subset S \rightarrow$	Separate point of a $\forall X \in S$ $X \in S_q^o \subset S^o \subset S$

It is the most important result which will allow to output the principle of an optimality and to construct methods of a choice of any point of Pareto's great number.

Definition 8. (*Principle of an optimality 2. The solution of a vector problem with the set criterion priority*).

Vector problem (12)-(13) with the set priority of q-th of criterion of p_k^q, $k=\overline{1,K}$ is considered solved if the point of X^o and a maximum level of λ^o among all relative estimates such that is found

$$\lambda^o = \max_{X\in S} \min_{k\in K} p_k^q \lambda_k(X), q \in K. \tag{33}$$

Using interrelation (29) and (30), we will transform a maximine problem (33) to an extreme problem of the form

$$\lambda^o = \max_{X\in S} \lambda, \tag{34}$$

$$\lambda \le p_k^q \lambda_k(X), k=\overline{1,K}. \tag{35}$$

Problem (34)-(35) we will call λ-problem with a priority of q-th of criterion.

The result of solution the λ-problem will be point $X^o=\{X^o, \lambda^o\}$ – it is result also of the solution of vector problem (12)-(13) with the set priority of the criterion, solved on the basis of normalization of criteria and the principle of the guaranteed result. In the optimum solution of $X^o=\{X^o, \lambda^o\}$, X^o - an optimum point, and λ^o - the maximum bottom level. The point of X^o and the λ^o level correspond to restrictions (15), which can be written as:

$$\lambda^o \le p_k^q \lambda_k\left(X^o\right), k=\overline{1,K}. \tag{36}$$

These restrictions are a basis of an assessment of correctness of results of the decision in practical vector problems of optimization.

Definition 1 and 2 "Principles of optimality" follows the opportunity to formulate the concept of the operation «opt».

Definition 9. (*Mathematical operation "opt"*).

In vector problem (12)-(13) which part criteria of "max" and "min" are, the mathematical operation "opt" consists in definition of a point of X^o and the maximum λ^o bottom level to which all criteria measured in relative units are lifted:

$$\lambda^o \le \lambda_k\left(X^o\right) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}, k=\overline{1,K}. \tag{37}$$

i.e. all criteria of $\lambda_k(X^o)$, $k=\overline{1,K}$ are equal or more maximum level of λ^o, (therefore λ^o also is called as the guaranteed result).

Theorem 2. (*The theorem of the most inconsistent criteria in vector problem with the set priority).*

If in a convex vector problem of mathematical programming of maximizing (12)-(13) the priority of q-th of criterion of p_k^q, $k=\overline{1,K}$, $\forall q \in K$ over other criteria is set, in a point of an optimum of $X^o \in S$ received on the basis of normalization of criteria and the principle of guaranteed result, always there will be two criteria with the indexes $r \in K$, $t \in K$, for which strict equality is carried out:

$$\lambda^o = p_k^r \lambda_r\left(X^o\right) = p_k^t \lambda_t\left(X^o\right), r, t \in K, \tag{38}$$

and other criteria are defined by inequalities:

$$\lambda^o \le p_k^q\left(X^o\right), k=k=\overline{1,K}, \forall q \in K, q \ne r \ne t. \tag{39}$$

Criteria with the indexes $r \in K$, $t \in K$ for which equality (38) is carried out are called the most inconsistent.

Proof. Similar to the theorem 2 [7].

We will notice that in (38) and (39) indexes of criteria of r, $t \in K$ can coincide with the $q \in K$ index.

Consequence of the theorem 1. *About equality of an optimum level and relative estimates in vector problem with two criteria with a priority of one of them.*

In a convex vector problem of mathematical programming with two equivalent criteria, solved on the basis of normalization of criteria and the principle of the guaranteed result, in an optimum point of X^o equality is always carried out: at a priority of the first criterion over the second:

$$\lambda^o = \lambda_1\left(X^o\right), p_2^1\left(X^o\right)\lambda_2\left(X^o\right), X^o \in S, \quad \text{where } p_2^1\left(X^o\right) = \lambda_1\left(X^o\right)/\lambda_2\left(X^o\right), \tag{40}$$

at a priority of the second criterion over the first:

$$\lambda^o = p_1^2\left(X^o\right)\lambda_1\left(X^o\right) = \lambda_2\left(X^o\right), X^o \in S, \quad \text{where } p_1^2\left(X^o\right) = \lambda_2\left(X^o\right)/\lambda_1\left(X^o\right).$$

Algorithm 2 of the decision in problems of vector optimization with a criterion priority [4].

Step 1. *We solve a vector problem with equivalent criteria*. The algorithm of the decision is presented in section 3.2. As a result of the decision we will receive:

optimum points by each criterion separately X_k^*, $k=\overline{1,K}$ and sizes of criterion functions in these points of $f_k^* = f_k(X_k^*)$, $k=\overline{1,K}$ which represent boundary of a set of points, optimum across Pareto;

anti-optimum points by each criterion of $X_k^0 = \{x_j, j=\overline{1,N}\}$ and the worst unchangeable part of criterion of $f_k^0 = f_k(X_k^0)$, $k=\overline{1,K}$;

$\boldsymbol{X^o}=\{\lambda^o, X^o\}$ - an optimum point, as result of the solution of VPMP at equivalent criteria, i.e. result of the solution of a maximine problem and λ-problem constructed on its basis;

λ^o - the maximum relative assessment which is the maximum lower level for all relative estimates of $\lambda_k(X^o)$, or the guaranteed result in relative units, λ^o guarantees that all relative estimates of $\lambda_k(X^o)$ more or are equal to λ^o:

$$\lambda^o \le \lambda_k\left(X^o\right), k = \overline{1,K}, X^o \in S. \tag{41}$$

The person making the decision, carries out the analysis of results of the solution of vector problem at equivalent criteria. If the received results satisfy the decision-maker, the end, differently the subsequent calculations.

We will in addition calculate:

• in each point of X_k^*, $k=\overline{1,K}$ we will determine sizes of all criteria of $q=\overline{1,K}$:

$\{f_q(X_k^*), q=\overline{1,K}\}$, $k=\overline{1,K}$, and relative estimates $\lambda(X^*) = \{\lambda_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\}$, $\lambda_k(X) = \dfrac{f_k(X) - f_k^o}{f_k^* - f_k^o}$, $\forall k \in \boldsymbol{K}$:

$$F\left(X^*\right) = \begin{vmatrix} f_1(X_1^*),\dots, f_k(X_1^*), \\ \dots \\ f_1(X_k^*),\dots, f_k(X_k^*) \end{vmatrix}, \quad \lambda\left(X^*\right) = \begin{vmatrix} \lambda_1(X_1^*),\dots, \lambda_k(X_1^*), \\ \dots \\ \lambda_1(X_k^*),\dots, \lambda_k(X_k^*) \end{vmatrix}. \tag{42}$$

Matrixes of criteria of $F(X^*)$ and relative estimates of $\lambda(X^*)$ show sizes of each criterion of $k=\overline{1,K}$ upon transition from one optimum point of X_k^*, $k\in\boldsymbol{K}$ to another to X_q^*, $q\in\boldsymbol{K}$, i.e. on border of a great number of Pareto.

• in an optimum point at equivalent criteria of X^o we will calculate sizes of criteria and relative estimates:

$$f_k\left(X^o\right), k = \overline{1,K}; \lambda_k\left(X^o\right), k = \overline{1,K}, \tag{43}$$

which satisfy to an inequality (41). In other points of $X\in\boldsymbol{S^o}$ smaller of criteria in relative units of $\lambda = \min_{k\in K} \lambda_k(X)$ is always less than λ^o. Are remembered given λ-problem (23)-(25).

This information also is a basis for further studying of structure of a great number of Pareto.

Step 2. *Choice of priority criterion of* $q\in\boldsymbol{K}$.

From the theory (see the theorem 1) it is known that in an optimum point of X^o always there are two most inconsistent criteria, $q\in\boldsymbol{K}$ and $v\in\boldsymbol{K}$ for which in relative units exact equality is carried out:

$$\lambda^o = \lambda_q\left(X^o\right) = \lambda_v\left(X^o\right), q, v \in K, X \in S, \tag{44}$$

and for the others it is carried out inequalities: $\lambda^o \le \lambda_k(X^o)$ $\forall k \in \boldsymbol{K}$, $q \ne v \ne k$.

As a rule, the criterion which the decision-maker would like to improve gets out of this couple, such criterion is called as "priority criterion", we will designate it $q\in\boldsymbol{K}$.

Step 3. *Numerical limits of change of size of a priority of criterion of* $q\in\boldsymbol{K}$ are defined.

For priority criterion of $q\in\boldsymbol{K}$ from a matrix (42) we will define numerical limits of change of size of criterion:

• in physical units of

$$f_q\left(X^o\right) \le f_q\left(X\right) \le f_q\left(X_q^*\right), k \in K, \tag{45}$$

where $f_q(X_q^*)$ undertakes from a matrix (42) $F(X^*)$, all criteria showing sizes measured in physical units;

• in relative units of

$$\lambda_q\left(X^o\right) \le \lambda_q\left(X\right) \le \lambda_q\left(X_q^*\right), k \in K, \tag{46}$$

where $\lambda_q(X_q^*)$ undertakes from a matrix (42) $\lambda(X^*)$, all criteria showing sizes measured in relative units (we will notice that $\lambda_q(X_q^*)=1$); $\lambda_q(X^o)$ from (43).

As a rule, results (45)-(46) are given for the display for the analysis.

Step 4. *Choice of size of priority criterion*. (Decision-making).

The person making the decision, carries out the analysis of results of calculations (42) and from an inequality (45) chooses the numerical size f_q of criterion of $q\in\boldsymbol{K}$:

$$f_q\left(X^o\right) \le f_q \le f_q\left(X_q^*\right), q \in K. \tag{47}$$

For the chosen size of criterion of f_q it is necessary to define a vector of unknown X^{oo}, for this purpose we carry out the subsequent calculations.

Step 5. *Calculation of a relative assessment*.

For the chosen size of priority criterion of f_q the relative assessment is calculated:

$$\lambda_q = \frac{f_q - f_q^o}{f_q^* - f_q^o}, \quad (48)$$

which upon transition from X^o point to X_q^* according to (46) lies in limits: $\lambda_q(X^o) \leq \lambda_q \leq \lambda_q(X_q^*)=1$.

Step 6. *Calculation of coefficient of linear approximation*.

Assuming linear nature of change of criterion of $f_q(X)$ in (45) and according to a relative assessment of $\lambda_q(X)$ in (29), using standard methods of linear approximation, we will calculate proportionality coefficient between $\lambda_q(X^o)$, λ_q, which we will call ρ:

$$\rho = \frac{\lambda_q - \lambda_q(X^o)}{\lambda_q(X_q^*) - \lambda_q(X^o)}, q \in K. \quad (49)$$

Step 7. *Calculation of coordinates of priority criterion with the size f_q.*

Assuming linear nature of change of a vector of X = {x_1 x_2} we will determine coordinates of a point of priority criterion with the size f_q with a relative assessment (48):

$$x^q = \begin{cases} x_1 = X^o(1) + \rho\left(X_q^*(1) - X^o(1)\right), \\ x_2 = X^o(2) + \rho\left(X_q^*(2) - X^o(2)\right) \end{cases}. \quad (50)$$

Step 8. *Calculation of the main indicators of a point of x_q.*

For the received x_q point, we will calculate:

all criteria in physical units $f_k(x^q)=\{f_k(x^q), k=\overline{1,K}\}$;

all relative estimates of criteria $\lambda^q=\{\lambda_k^q, k=\overline{1,K}\}$, $\lambda_k(x^q)= \frac{f_k(x^q) - f_k^o}{f_k^* - f_k^o}$, $k=\overline{1,K}$ $\overline{1,K}$;

vector of priorities $P^q=\{p_k^q = \frac{\lambda_q(x^q)}{\lambda_k(x^q)}, k=\overline{1,K}\}$;

maximum relative assessment $\lambda^{oq}=\min(p_k^q \lambda_k(x^q), k=\overline{1,K})$.

Any point from Pareto's set $\boldsymbol{X}_t^o=\{\lambda_t^o, X_t^o\}\in \boldsymbol{S}^o$ can be similarly calculated

Analysis of results. The calculated size of criterion $f_q(X_t^o)$, $q\in\boldsymbol{K}$ is usually not equal to the set f_q. The error of the choice of $\Delta f_q=|f_q(X_t^o) - f_q|$ is defined by an error of linear approximation

4. Results. Numerical Problem of Modeling of Technical System (Research and the Solution of a Problem with Equivalent Criteria and with the Set Criterion Priority)

We will consider a task "Numerical modeling of technical system" in which data on some set of functional characteristics (definiteness conditions), discrete values of characteristics (an uncertainty condition) and the restrictions imposed on functioning of technical system are known [3]. We will add the fifth criterion to a vector task [3]. We will conduct research, first, as the set of points will change, optimum across Pareto, secondly, we will calculate a point on a great number of Pareto in which one of criteria has a priority over other criteria.

The numerical problem of modeling of technical system is considered with equivalent criteria and with the set criterion priority.

It is given. The technical system, which functioning is defined by two parameters 1 $X=\{x_1, x_2\}$ – a vector (operated) variables. Basic data for the solution of a task are five characteristics (criterion) of

$$F(X)=\{f_1(X), f_2(X), f_3(X), f_4(X), f_5(X)\},$$

which size of an assessment depends on a vector of X. For characteristics of $f_1(X)$, $f_2(X)$, $f_5(X)$ functional dependence on parameters X (a definiteness condition) is known:

$$\begin{aligned} &f_1(X) = 67.425 + 0.02225 * x_1 + 0.00239 * x_1^2 \\ &-0.05625 * x_2 + 0.00029 * x_2^2 + 0.0021232 * x_1 * x_2, \\ &f_2(X) = 4456.3 - 2.315 * x_1 + 0.239 * x_1^2 \\ &+2.805 * x_2 - 0.037 * x_2^2 - 0.22192 * x_1 * x_2, \\ &f_5(X) = 281.7 - 0.7 * x_1 + 0.01 * x_1^2 + 0.36 * x_2 \\ &-0.019 * x_2^2 + 0.022 * x_1 * x_2. \end{aligned} \quad (51)$$

Functional restrictions:

$$\begin{aligned} &3800 \leq f_2(X) \equiv 4456.3 - 2.315 * x_1 + 0.239 * x_1^2 \\ &+2.805 * x_2 - 0.037 * x_2^2 - 0.22192 * x_1 * x_2 \leq 5500 \end{aligned} \quad (52)$$

Parametrical restrictions:

$$25 \leq x_1 \leq 100, 25 \leq x_2 \leq 100. \quad (53)$$

For the third and fourth characteristic results of experimental data are known: sizes of parameters and corresponding characteristics (uncertainty condition). Numerical values of parameters X and characteristics of $y_3(X)$, $y_4(X)$ are presented in Table 1.

Table 1. Numerical values of parameters and characteristics of technical system

x_1	x_2	$y_3(X)$→**max**	$y_4(X)$→**min**
25	25	1148	490.9
25	50	1473	483.1
25	75	1798	557.3
25	100	2122	521.5
50	25	725	498.1
50	50	968	521.5
50	75	1212	549.9
50	100	1456	578.3
75	25	440	507.3
75	50	572	549.9
75	75	734	592.5
75	100	897	635.1
100	25	202	521.5
100	50	284	578.3
100	75	385	635.1
100	100	446	691.9

1 Practical problems of simulation of technical systems on this algorithm can be solved with dimensionality of parameters X more than two N>2. The structure of the software becomes complicated. Geometrical interpretation of N=3,4 … isn't possible. The choice of two parameters selected from three (N '=2) ⊂ (N=3) is possible. In this direction it is carried further researches and development of the appropriate algorithms.

In the made decision, assessment size of the first, second and the third characteristic (criterion) is possible to receive above (max), for the fourth and five characteristic is possible below (min). Parameters $X=\{x_1, x_2\}$ change in the following limits: $x_1, x_2 \in [25.\ 50.\ 75.\ 100]$.

It is required. To construct model of technical system in the form of a vector problem. To solve a vector problem with equivalent criteria. To choose priority criterion. To establish numerical value of priority criterion. To make the best decision (optimum).

Methodology of modeling of technical system in the conditions of definiteness and uncertainty.

1. Creation of mathematical model of technical system.

1.1. *Construction in the conditions of definiteness* is defined by functional dependence of each characteristic and restrictions on parameters of technical system. In our example three characteristics (35) and restrictions (36)-(37) are known:

$$\begin{aligned} f_1(X) &= 67.425 + 0.02225*x_1 + 0.00239*x_1^2 \\ &-0.05625*x_2 + 0.00029*x_2^2 + 0.0021232*x_1*x_2, \\ f_2(X) &= 4456.3 - 2.315*x_1 + 0.239*x_1^2 \\ &+2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2, \\ f_5(X) &= 281.7 - 0.7*x_1 + 0.01*x_1^2 \\ &+0.36*x_2 - 0.019*x_2^2 + 0.022*x_1*x_2. \end{aligned} \quad (54)$$

Functional restrictions:

$$\begin{aligned} 3800 \le f_2(X) &\equiv 4456.3 - 2.315*x_1 + 0.239*x_1^2 \\ &+2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2 \le 5500 \end{aligned} \quad (55)$$

Parametrical restrictions:

$$25 \le x_1 \le 100,\ 25 \le x_2 \le 100. \quad (56)$$

These data are used further at creation of mathematical model of technical system.

1.2. Construction in the conditions of uncertainty consists in use of the qualitative and quantitative descriptions of technical system received by the principle "entrance exit" in Table 1. Transformation of information (basic data of $y_3(X)$, $y_4(X)$) to a functional type of $f_3(X)$, $f_4(X)$ is carried out by use of mathematical methods (the regression analysis).

Basic data of Table 1 are created in Matlab system in the form of a matrix

$$I = |X, Y| = \{x_{i1}\ x_{i2}\ y_{i3}\ y_{i4}, i = \overline{1, M}\}. \quad (57)$$

For each set experimental these y_k, $k=\overline{3,4}$ function of regression on a method of the smallest squares in *Matlab* system is formed. A_k,- polynom defining interrelation of factors of $X_i=\{x_{1i}, x_{2i}\}$ (57) and functions $\bar{y}_{ki} = f(X_i, A_k)$, $k=\overline{3,4}$ is constructed.

As a result of calculations we received system of coefficients of $A_k=\{A_{0k}, A_{1k}, A_{2k}, A_{k3}, A_{4k}, A_{5k}\}$ which define coefficients of a polynom (function):

$$\begin{aligned} f_k(X, A) &= A_{0k} + A_{1k}x_1 + A_{2k}x_1^2 \\ &+ A_{3k}x_2 + A_{4k}x_2^2 + A_{5k}x_1*x_2, k = \overline{3,4} \end{aligned} \quad (58)$$

As a result of calculations of coefficients of A_k, k=3, we received the $f_3(X)$ function:

$$\begin{aligned} f_3(X) &= 1273.5 - 19.919*x_1 + 0.0854*x_1^2 \\ &+16.071*x_2 + 0.001*x_2^2 - 0.13034*x_1*x_2, \end{aligned} \quad (59)$$

The graphical representation of the $f_3(X)$ function is shown in Figure 1[3].

We showed in Figure 1 [13].X_3^*, X_3^0 the best (maximum) and worst (minimum) decision, according to $f_3(X_3^*)$, $f_3(X_3^0)$ – sizes of functions.

As a result of calculations of coefficients of A_k, k =4, we received the $f_4(X)$ function:

$$\begin{aligned} f_4(X) &= 481.7 - 0.6915*x_1 + 0.0047*x_1^2 \\ &+0.3535*x_2 - 0.0023*x_2^2 + 0.021808*x_1*x_2, \end{aligned} \quad (60)$$

The graphical representation of the $f_4(X)$ function is shown in Figure 2 [3].

We showed in Figure 2 [3] X_4^*, X_4^0 the best (minimum) and worst (maximum) decision, according to $f_4(X_4^*)$, $f_4(X_4^0)$ – sizes of functions.

Parametrical restrictions are similar (56): $25 \le x_1 \le 100$, $25 \le x_2 \le 100$.

1.3. Creation of mathematical model of technical system (The general part for conditions of definiteness and uncertainty).

For creation of mathematical model of technical system we used:

the functions received conditions of definiteness (54) and uncertainty (59), (60);

functional restrictions (55);

parametrical restrictions (56).

We considered functions (54) and (59), (60) as the criteria defining focus of functioning of technical system. A set of criteria ***K***=5 included three criteria of $f_1(X)$, $f_2(X)$, $f_3(X)$ →max and two $f_4(X)$, $f_5(X)$ →min. As a result model of functioning of technical system was presented a vector problem of mathematical programming:

$$\begin{aligned} opt\ F(X) &= \{max\ F_1(X) \\ &= \{max\ f_1(X) \equiv 67.425 + 0.02225*x_1 + 0.00239*x_1^2 \quad (61) \\ &- 0.05625*x_2 + 0.00029*x_2^2 + 0.0021232*x_1*x_2, \end{aligned}$$

$$\begin{aligned} max\ f_2(X) &\equiv 4456.3 - 2.315*x_1 + 0.239*x_1^2 \\ &+2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2, \end{aligned} \quad (62)$$

$$\begin{aligned} max\ f_3(X) &\equiv 1273.5 - 19.919*x_1 + 0.0854*x_1^2 \\ &+16.071*x_2 + 0.001*x_2^2 - 0.13034*x_1*x_2\}, \end{aligned} \quad (63)$$

$$\begin{aligned} &min\ F_2(X) \\ &= \{min\ f_4(X) \equiv 481.7 - 0.6915*x_1 + 0.0047*x_1^2 \\ &+0.3535*x_2 - 0.0023*x_2^2 + 0.021808*x_1*x_2, \\ &min\ f_5(X) = 281.7 - 0.7*x_1 + 0.01*x_1^2 \\ &+0.36*x_2 - 0.019*x_2^2 + 0.022*x_1*x_2\}\} \end{aligned} \quad (64)$$

at restrictions

$$\begin{aligned} 3800 \le f_2(X) &\equiv 4456.3 - 2.315*x_1 + 0.239*x_1^2 \\ &+2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2 \le 5500 \end{aligned} \quad (65)$$

$$25 \le x_1 \le 100,\; 25 \le x_2 \le 100. \qquad (66)$$

The vector problem of mathematical programming represents model of adoption of the optimum decision in the conditions of definiteness and uncertainty in total.

2. The solution of a vector problem of mathematical programming - model of technical system with equivalent criteria and with the set criterion priority.

2.1. Algorithm 1 of the decision in problems of vector optimization with equivalent criteria

The solution of a vector problem (61)-(62) with was submitted as sequence of steps.

Step 1. Problems (61)-(66) were solved by each criterion separately, thus used the function *fmincon* (…) of *Matlab* system [14], the appeal to the function *fmincon* (…) is considered in [8].

As a result of calculation for each criterion we received optimum points: X_k^* and $f_k^* = f_k(X_k^*)$, $k=\overline{1,K}$ – sizes of criteria in this point, i.e. the best decision on each criterion:

$$X_1^* = \{x_1 = 100, x_2 = 100\},\; f_1^* = f\left(X_1^*\right) = -112.06;$$

$$X_2^* = \{x_1 = 97.16, x_2 = 48.09\},\; f_2^* = f_2\left(X_2^*\right) = -5500.0;$$

$$X_3^* = \{x_1 = 25.0, x_2 = 100.0\},\; f_3^* = f_3\left(X_3^*\right) = -2120.15;$$

$$X_4^* = \{x_1 = 25.0, x_2 = 25.0\},\; f_4^* = f_4\left(X_4^*\right) = 488.38;$$

$$X_5^* = \{x_1 = 25.0, x_2 = 68.41\},\; f_5^* = f_5\left(X_5^*\right) = 243.78.$$

Restrictions (66) and points of an optimum in coordinates $\{x_1, x_2\}$ are presented on Figure 1. (Compare to Figure 3 [3]).

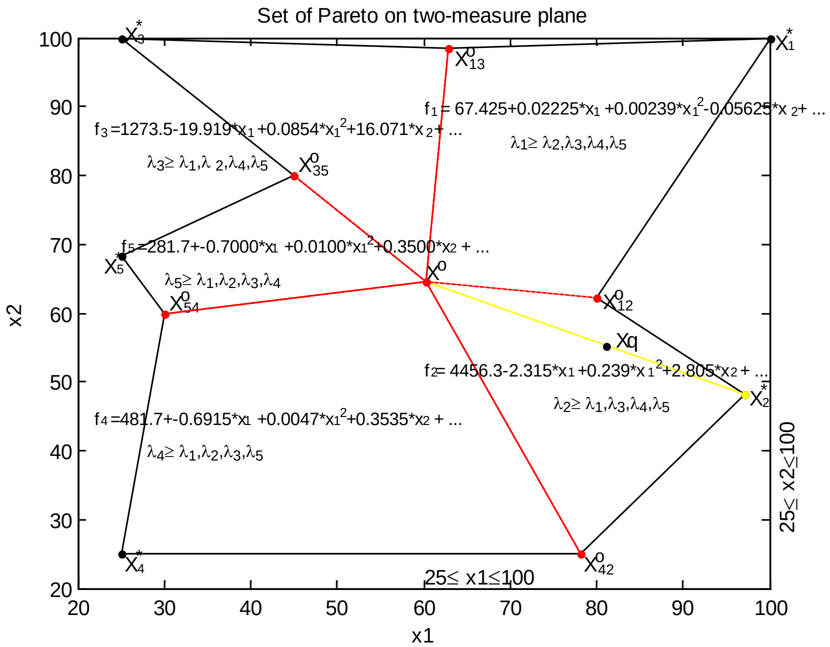

Figure 1. Pareto's great number, $S^o \subset S$ in two-dimensional system of coordinates

Step 2. We defined the worst unchangeable part of each criterion (anti-optimum):

$$X_1^0 = \{x_1 = 25.0, x_2 = 25.0\},\; f_1^0 = f_1\left(X_1^0\right) = 69.57;$$

$$X_2^0 = \{x_1 = 37.32, x_2 = 98.88\},\; f_2^0 = f_2\left(X_2^0\right) = 3800;$$

$$X_3^0 = \{x_1 = 83.1, x_2 = 25.0\},\; f_3^0 = f_3\left(X_3^0\right) = 339.7;$$

$$X_4^0 = \{x_1 = 83.1, x_2 = 25.0\},\; f_4^0 = f_4\left(X_4^0\right) = -689.9.$$

(Top index zero).

Step 3. We made the analysis of a set of points, optimum across Pareto. In points of an optimum of $X^* = \{X_1^*, X_2^*, X_3^*, X_4, X_5^{**}\}$ sizes of criterion functions of $F(X^*) = \left\| f_q(X_k^*) \right\|_{q=\overline{1,K}}^{k=\overline{1,K}}$ determined. Calculated a vector of $D=(d_1\ d_2\ d_3\ d_4\ d_5)^T$ - deviations by each criterion on an admissible set of $\boldsymbol{S}$: $d_k = f_k^* - f_k^0$, $k=\overline{1,5}$, and matrix of relative estimates of

$$\lambda\left(X^*\right) = \left\| \lambda_q(X_k^*) \right\|_{q=\overline{1,K}}^{k=\overline{1,K}},$$

where $\lambda_k(X) = (f_k^* - f_k^0)/d_k$.

$$F\left(X^*\right) = \begin{vmatrix} -112.1 & -4306.1 & -449.3 & 690.0 & 377.7 \\ -100.0 & -5500.0 & -310.5 & 572.5 & 384.3 \\ -72.1 & -3903.5 & -2120.2 & 534.2 & 171{,}4 \\ -69.6 & -4456.1 & -1149.8 & 488.4 & 281.3 \\ -70.6 & -4187.0 & 1710.1 & 518.1 & 243.8 \end{vmatrix},$$

$$D = \begin{vmatrix} 42.48 \\ 1700.0 \\ 1780.5 \\ -201.6 \\ -154.2 \end{vmatrix},$$

$$\lambda\left(X^*\right) = \begin{vmatrix} 1.0000 & 0.2977 & 0.0616 & 0 & 0.1312 \\ 0.7170 & 1.0000 & -0.0164 & 0.5829 & 0.0887 \\ 0.0584 & 0.0609 & 1.0000 & 0.7726 & 1.4692 \\ 0 & 0.3859 & 0.4550 & 1.0000 & 0.7564 \\ 0.0244 & 0.2276 & 0.7697 & 0.8527 & 1.0000 \end{vmatrix}.$$

Discussion. The analysis of sizes of criteria in relative estimates showed that in points of an optimum of $X^* = \{X_1^*, X_2^*, X_3^*, X_4^*, X_5^*\}$ the relative assessment is equal to unit.

Other criteria there is much less than unit. It is required to find such point (parameters) at which relative estimates are closest to unit. The step 4 is directed on the solution of this problem.

Step 4. Creation of λ-problem is carried out in two stages: originally the maximine problem of optimization with the normalized criteria is under construction:

$$\lambda^o = \max_x \min_k \lambda_k(X), G(X) \le 0, X \ge 0,$$

which at the second stage was transformed to a standard problem of mathematical programming (λ-problem):

$$\lambda^o = \max \lambda, \quad (67)$$

at restrictions

$$\lambda - \frac{\left(\begin{array}{l} 67.4 + 0.02225 * x_1 + 0.0024 * x_1^2 \\ -0.05625 * x_2 + 0.00029 * x_2^2 \\ +0.002123 * x_1 * x_2 - f_1^o \end{array}\right)}{f_1^* - f_1^o} \le 0, \quad (68)$$

$$\lambda - \frac{\left(\begin{array}{l} 4456.3 - 2.315 * x_1 + 0.239 * x_1^2 + 2.605 * x_2 \\ -0.037 * x_2^2 - 0.222 * x_1 * x_2 - f_2^o \end{array}\right)}{f_2^* - f_2^o} \le 0, \quad (69)$$

$$\lambda - \frac{\left(\begin{array}{l} 1273.5 - 19.92 * x_1 + 0.0854 * x_1^2 + 16.07 * x_2 \\ +0.01 * x_2^2 - 0.1303 * x_1 * x_2 - f_3^o \end{array}\right)}{f_3^* - f_3^o} \le 0, \quad (70)$$

$$\lambda - \frac{\left(\begin{array}{l} 481.7 - 0.6915 * x_1 + 0.0047 * x_1^2 \\ +0.3535 * x_2 - 0.0023 * x_2^2 \\ +0.0218 * x_1 * x_2 - f_4^o \end{array}\right)}{f_4^* - f_4^o} \le 0, \quad (71)$$

$$\lambda - \frac{\left(\begin{array}{l} 281.7 - 0.7 * x_1 + 0.01 * x_1^2 + 0.36 * x_2 \\ -0.019 * x_2^2 + 0.022 * x_1 * x_2 - f_5^o \end{array}\right)}{f_5^* - f_5^o} \le 0, \quad (72)$$

$$\begin{array}{l} 3800 \le f_2(x) \le 5500; 0 \le \lambda \le 1, \\ 25 \le x_1 \le 100, 25 \le x_2 \le 100, \end{array} \quad (73)$$

where the vector of unknown had dimension of N+1: $\boldsymbol{X}=\{x_1, \dots, x_N, \lambda\}$. Appeal to function fmincon(…), [14]:

[Xo,Lo]=fmincon('Z_TehnSist_4Krit_L',X0,Ao,bo,Aeq,beq,lbo,ubo,'Z_TehnSist_LConst',options).

As a result of the solution of a vector problem of mathematical programming (61)-(66) at equivalent criteria and λ-problem corresponding to it (67)-(73) received:

$\boldsymbol{X^o}=\{X^o, \lambda^o\}=\{X^o=\{x_1=60.36, x_2=64.52, \lambda^o=0.3236\}$ - an optimum point – design data of technical system, point $\boldsymbol{X^o}$ is presented in Figure 1;

$f_k(X^o)$, $k=\overline{1,K}$ - sizes of criteria (characteristics of technical system):

$$\left\{\begin{array}{l} f_1(X^o) = 83.3, f_2(X^o) = 4350.1, \\ f_3(X^o) = 915.8, f_4(X^o) = 555.2, f_5(X^o) = 305.7 \end{array}\right\}; \quad (74)$$

$\lambda_k(X^o)$, $k=\overline{1,K}$ - sizes of relative estimates:

$$\left\{\begin{array}{l} \lambda_1(X^o) = 0.3236, \lambda_2(X^o) = 0.3236, \\ \lambda_3(X^o) = 0.3236, \\ \lambda_4(X^o) = 0.6683, \lambda_5(X^o) = 0.5984 \end{array}\right\}; \quad (75)$$

λ^o=0.3236 is the maximum lower level among all relative estimates measured in relative units: : $\lambda^o=min(\lambda_1(X^o), \lambda_2(X^o), \lambda_3(X^o), \lambda_4(X^o), \lambda_5(X^o))$=0.3236. A relative assessment - λ^o call the guaranteed result in relative units, i.e. $\lambda_k(X^o)$ and according to the characteristic of technical $f_k(X^o)$ system it is impossible to improve, without worsening thus other characteristics.

Discussion. We will notice that according to the theorem 1, in $\boldsymbol{X^o}$ point criteria 1, 2, 3 are contradictory. This contradiction is defined by equality of $\lambda_1(X^o)=\lambda_2(X^o)=\lambda_3(X^o)=\lambda^o$=0.3236, and other criteria an inequality of $\{\lambda_4(X^o)=0.6683, \lambda_5(X^o)=0.5984\}>\lambda^o$.

Thus, the theorem 1 forms a basis for determination of correctness of the solution of a vector task. In a vector problem of mathematical programming, as a rule, for two criteria equality is carried out:

$$\lambda^o = \lambda_q(X^o) = \lambda_p(X^o), q, p \in K, X \in S,$$

(in our example of such criteria three)

and for other criteria is defined as an inequality: $\lambda^o \le \lambda_k(X^o)$ $\forall k \in \boldsymbol{K}$, $q \ne p \ne k$.

In an admissible set of points of $\boldsymbol{S}$ formed by restrictions (73), optimum points X_1^*, X_2^*, X_3^*, X_4^*, X_5^* united in a contour, presented a set of points, optimum across Pareto, to $\boldsymbol{S^o} \subset \boldsymbol{S}$. For specification of border of a great number of Pareto calculated additional points: X_{12}^o, X_{13}^o, X_{35}^o, X_{54}^o, X_{42}^o which lie between the corresponding criteria.

For definition of a point of X_{12}^o the vector problem was solved with two criteria (68), (69), (73).

Results of the decision: X_{12}^o={79.99 62.31}, $\lambda^o(X_{12}^o)$ =0.5445; F_{12} ={92.7 4725.6 582.1 578.3 348.0}; L_{12} = {0.5445 0.5445 0.1362 0.5541 0.3238}.

Other points were similarly defined: X_{13}^o ={62.94 98.44}, $\lambda^o(X_{13}^o)$=0. 4507; F_{13} = {88.7 3800.0 1142.2 604.4 264.9}; L_{13} = (0.4507 0.0 0.4507 0.4243 0.8630}; X_{35}^o ={45.0 80.0}, $\lambda^o(X_{35}^o)$=0. 83; F_{35} = {-78.3 -4024.8 -1372.9 552.2 256.9}; L_{35} = {0.2045 0.1322 0.5803 0.6836 0.9152}; X_{54}^o={30.0 60.0}, $\lambda^o(X_{54}^o)$=0. 83; F_{35} = {-71.7 -4236.7 -1486.0 517.4 262.5}; L_{54} = {0.0508 0.2574 0.6429 0.8562 0.8786}; X_{42}^o ={78.14 25.0}, $\lambda^o(X_{42}^o)$=0.9108; F_{42} = {86.7 5348.3 386.2 506.4 382.2}; L_{42} = {0.4026 0.9108 0.0261 0.9108 0.4526}.

Points: X_{12}^o, X_{13}^o, X_{35}^o, X_{54}^o, X_{42}^o are presented in Figure 1. Coordinates of these points, and also characteristics of technical system in relative units of $\lambda_1(X)$, $\lambda_2(X)$, $\lambda_3(X)$,

$\lambda_4(X)$, $\lambda_5(X)$ are shown in Figure 5 in three measured space, where the third axis of λ - a relative assessment.

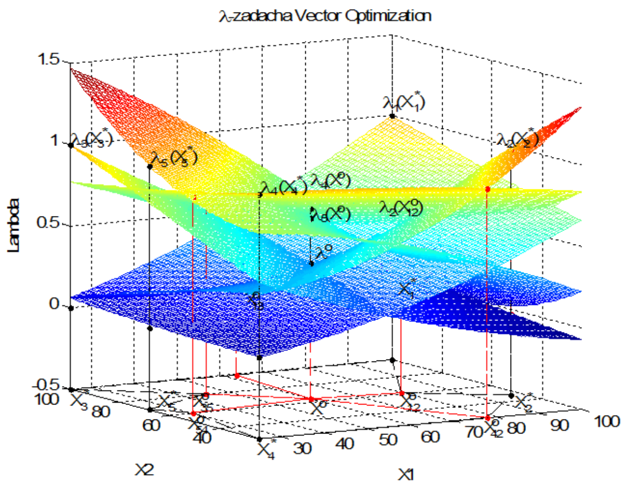

Figure 2. The solution of λ-problem in three-dimensional system of coordinates of x_1, x_2 and λ

2.2. Algorithm 2 of the decision in problems of vector optimization with a criterion priority

***Step** 1. We solve a vector problem with equivalent criteria*. The algorithm of the decision is presented in section 3.2. Numerical results of the solution of a vector task are given above. Pareto's great number of $S^o \subset S$ lies between optimum points $X_1^* X_{13}^o\ X_3^*\ X_{35}^o\ \ X_5^*\ X_{54}^o\ X_4^*\ X_{42}^o\ \ X_2^*\ X_{12}^o\ X_1^*$.

We will carry out the analysis of a great number of Pareto $S^o \subset S$. For this purpose we will connect auxiliary points: X_{12}^o, X_{13}^o, X_{35}^o, X_{54}^o, X_{42}^o with a point X^o which conditionally represents the center of a great number of Pareto. As a result have received five subsets of points $X \in\ S_q^o \subset \boldsymbol{S}^o \subset \boldsymbol{S}$, $q=\overline{1,5}$. The subset of $S_1^o \subset \boldsymbol{S}^o \subset \boldsymbol{S}$ is characterized by the fact that the relative assessment of $\lambda_1 \geq \lambda_2$, λ_3 , λ_4, λ_5, i.e. in the field of S first criterion has a priority over the others. Similar to $\boldsymbol{S}_2^o$, $\boldsymbol{S}_3^o$, $\boldsymbol{S}_4^o$, $\boldsymbol{S}_5^o$ - subsets of points where the second - the fifth criterion has a priority over the others respectively. Set of points, optimum across Pareto we will designate $\boldsymbol{S}^o = \boldsymbol{S}_1^o \cup \boldsymbol{S}_2^o \cup \boldsymbol{S}_3^o \cup \boldsymbol{S}_4^o \cup \boldsymbol{S}_5^o$. Coordinates of all received points and relative estimates are presented in two-dimensional space in Figure 1. These coordinates are shown in three measured space $\{x_1, x_2, \lambda\}$ from a point of X_4^* in Figure 2 where the third axis of λ- a relative assessment. Restrictions of a set of points, optimum across Pareto, in Figure 2 it is lowered to-0.5 (that restrictions were visible). This information is also a basis for further research of structure of a great number of Pareto. The person making decisions, as a rule, is the designer of technical system. If results of the solution of a vector task with equivalent criteria don't satisfy the person making the decision, then the choice of the optimal solution is carried out from any subset of points of $\boldsymbol{S}_1^o, \boldsymbol{S}_2^o, \boldsymbol{S}_3^o, \boldsymbol{S}_4^o, \boldsymbol{S}_5^o$.

***Step** 2. Choice of priority criterion of $q \in \boldsymbol{K}$.* From the theory (see the theorem 1) it is known that in an optimum point of X^o always there are two most inconsistent criteria, $q \in \boldsymbol{K}$ and $v \in \boldsymbol{K}$ for which in relative units exact equality is carried out: $\lambda^o = \lambda_q(X^o) = \lambda_p(X^o)$, q, $v \in \boldsymbol{K}$, $X \in \boldsymbol{S}$, and for the others it is carried out inequalities: $\lambda^o \leq \lambda_k(X^o)\ \forall k \in \boldsymbol{K}$, $q \neq v \neq k$.

In model of technical system (61)-(66) and the corresponding λ-problem (67)-(73) such criteria are the first, second and third:

$$\lambda^o = \lambda_1\left(X^o\right) = \lambda_2\left(X^o\right) = \lambda_3\left(X^o\right) = 0.3236. \quad (76)$$

We will show them in Figure 3.

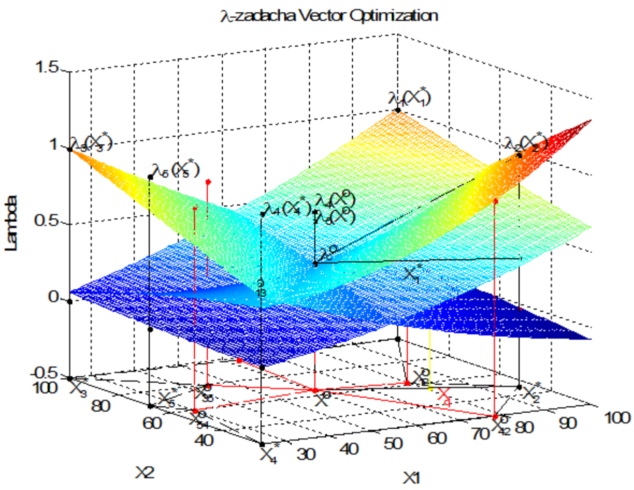

Figure 3. The solution of λ-problem (1, 2, 3 criterion) in three-dimensional system of coordinates of x_1, x_2 and λ

As a rule, the criterion which the decision-maker would like to improve gets out of couple of contradictory criteria. Such criterion is called "priority criterion", we will designate it $q=2 \in \boldsymbol{K}$. This criterion is investigated in interaction with the third criterion of $k=3 \in \boldsymbol{K}$. We will allocate these two criteria from all set of the criteria $\boldsymbol{K}$=5 shown in Figure 3. We will present criteria of $q=2$ k=3 in separate drawing of Figure 4 in order that was the picture of construction and choice of size of priority criterion is visible.

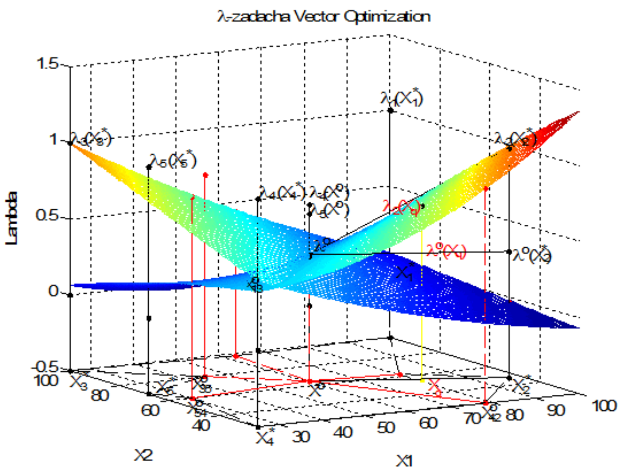

Figure 4. The solution of λ-problem (2, 3 criterion) in three-dimensional system of coordinates of x_1, x_2 and λ

On the display the message is given:

q=input ('Enter priority criterion (number) of q =') - Have entered: q=2.

Step 3. *Numerical limits of change of size of a priority of criterion of q=2∈**K*** are defined.

For priority criterion of $q=2$ numerical limits in physical units upon transition from a point of an optimum of X^o (74) to the point of X_q^* received on the first step are defined.

Information about the criteria for q=2 are given on the screen:

$$f_q\left(X^o\right)=4350.06 \le f_q\left(X\right) \le 5500 = f_q\left(X_q^*\right), q \in K. \quad (77)$$

In relative units the criterion of q=2 changes in the following limits:

$$\lambda_q\left(X^o\right) = 0.3236 \le \lambda_q\left(X\right) \le \lambda = \lambda_q(X_q^*), q = 2 \in K. \quad (78)$$

These data it is analyzed.

Step 4. *Choice of size of priority criterion. q∈**K**.* (Decision-making).

The message is displayed: "Enter the size of priority criterion f_q=" - we enter, for example, f_q =5000.

Step 5. *Calculation of a relative assessment.*

For the chosen size of priority criterion of f_q =5000 the relative assessment is calculated:

$$\lambda_q = \frac{f_q - f_q^o}{f_q^* - f_q^o} = \frac{5000 - 3800}{5500 - 3800} = 0.7059, \quad (78)$$

which upon transition from X^o point to X_q^* according to (75) lies in limits: 0. 3236 =$\lambda_2(X^o) \le \lambda_2 = 0.7059 \le \lambda_2(X_2^*)$=1, $q \in \boldsymbol{K}$.

Step 6. *Calculation of coefficient of linear approximation*.

Assuming linear nature of change of criterion of $f_q(X)$ in (77) and according to a relative assessment of $\lambda_q(X)$, using standard methods of linear approximation, we will calculate proportionality coefficient between $\lambda_q(X^o)$, λ_q, which we will call ρ:

$$\rho = \frac{\lambda_q - \lambda_q(X^o)}{\lambda_q(X_q^*) - \lambda_q(X^o)} = \frac{0.7059 - 0.3236}{1 - 0.3236} = 0.5652, \quad (79)$$
$$q = 2 \in K.$$

Step 7. *Calculation of coordinates of priority criterion with the size f_q.*

Assuming linear nature of change of a vector of $X^q=\{x_1\ x_2\}$, q=2 we will determine coordinates of a point of priority criterion with the size f_q with a relative assessment (79):

$$X^q = \left\{ \begin{array}{l} x_1 = X^o(1) + \rho\left(X_q^*(1) - X^o(1)\right), \\ x_2 = X^o(2) + \rho\left(X_q^*(2) - X^o(2)\right) \end{array} \right\}. \quad (80)$$

$$X^q = \{x_1 = 76.5478, x_2 = 42.9348\}.$$

where $X^o=\{x_1=60.36, x_2=64.52\}$, $X_2^*=\{x_1=97.16, x_2=48.09\}$.

As a result of calculations we have received point coordinates: $X^q=\{x_1=76.5478, x_2=42.9348\}$.

Step 8. *Calculation of the main indicators of a point of X_q.*

For the received X_q point, we will calculate:

all criteria in physical units $f_k(X^q)=\{f_k(X^q), k=\overline{1,K}\}$:

$$f\left(X^q\right) = \left\{ \begin{array}{l} f_1\left(X^q\right) = 92.3, f_2\left(X^q\right) = 4889.9, \\ f_3\left(X^q\right) = 525.8, f_4\left(X^q\right) = 566.8, \\ f_5\left(X^q\right) = 351.3 \end{array} \right\};$$

all relative estimates of criteria $\lambda^q=\{\lambda_k^q, k=\overline{1,K}\}$, $\lambda_k(X^q)= \frac{f_k(X^q) - f_k^o}{f_k^* - f_k^o}$, $k=\overline{1,K}$:

$$\lambda_k\left(X^q\right) = \left\{ \begin{array}{l} \lambda_1\left(X^q\right) = 0.5342, \lambda_2\left(X^q\right) = 0.6411, \\ \lambda_3\left(X^q\right) = 0.1045, \lambda_4\left(X^q\right) = 0.6110, \\ \lambda_5\left(X^q\right) = 0.3025 \end{array} \right\};$$

vector of priorities $P^q=\{p_k^q = \frac{\lambda_q(X^q)}{\lambda_k(X^q)}, k=\overline{1,K}\}$:

$$P^q = \left[\begin{array}{l} p = 1.2002, p = 1.0, p = 6.1325, \\ p = 1.0493, p = 2.1193 \end{array} \right];$$

minimum relative assessment: minLXq=min(LXq): minLXq=min($\lambda_k(X^q)$) = 0.1045;

relative assessment taking into account a criterion priority:

$$*P^q = \left\{ \begin{array}{l} p_1^2\lambda_1\left(X^q\right), p_2^2\lambda_2\left(X^q\right), p_3^2\lambda_3\left(X^q\right), \\ p_4^2\lambda_4\left(X^q\right), p_5^2\lambda_5\left(X^q\right) \end{array} \right\}$$
$$= \{0.6411, 0.6411, 0.6411, 0.6411, 0.6411\};$$

the minimum relative assessment taking into account a criterion priority:

$$\lambda^{oo} = \min\left(\begin{array}{l} p_1^2\lambda_1\left(X^q\right), p_2^2\lambda_2\left(X^q\right), \\ p_3^2\lambda_3\left(X^q\right), p_4^2\lambda_4\left(X^q\right), p_5^2\lambda_5\left(X^q\right) \end{array} \right) = 0.6411.$$

Any point from Pareto's set $\boldsymbol{X}_t^o=\{\lambda_t^o, X_t^o\} \in \boldsymbol{S}^o$ can be similarly calculated: $X_t^o=\{x_1=76.5478, x_2=42.9348\}$.

Analysis of results. The calculated size of criterion $f_q(X_t^o)$, $q \in \boldsymbol{K}$ is usually not equal to the set f_q. The error of the choice of $\Delta f_q=|f_q(X_t^o) - f_q|$ =|4889.9 - 5000|=110.1 is defined by an error of linear approximation, $\Delta f_{q\%}$= 2.2%.

In the course of modeling parametrical restrictions (66), functional restrictions (65) can be changed, i.e. some set of optimum decisions is received. Choose a final version which in our example included from this set of optimum decisions:

- parameters of technical system $X^o=\{x_1=60.36, x_2=64.52\}$;
- the parameters of the technical system at a given priority criterion q=2: $X^q=\{x_1=76.5478, x_2=42.9348\}$.

We represent these parameters in a two-dimensional x_1, x_2 and three dimensional coordinate system x_1, x_2 and λ in Figure 1, Figure 3, Figure 4, and also in physical units for each function $f_1(X)$, … , $f_5(X)$ on Figure 5, ..., Figure 9, respectively.

The first characteristic $f_1(X)$ in physical units show in Figure 5.

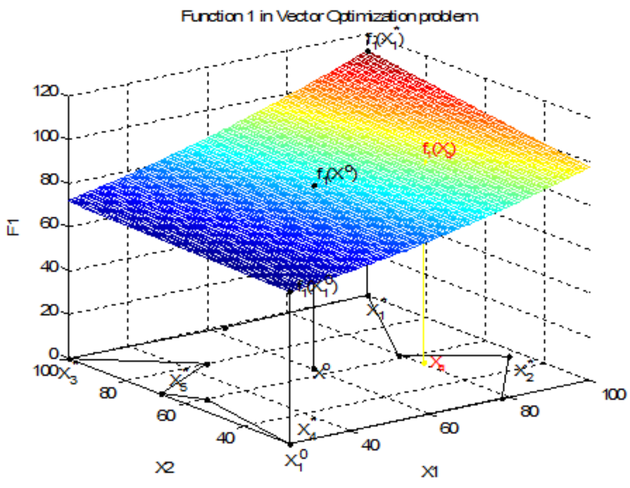

Figure 5. The first characteristics of $f_1(X)$ of technical system in natural indicator

In point X^o, X^q of the second characteristic of $f_2(X)$ will assume to the look presented in Figure 6.

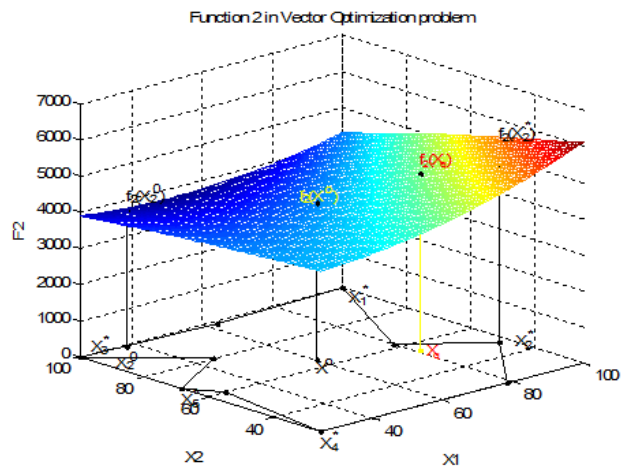

Figure 6. The second characteristics of $f_2(X)$ of technical system in natural indicator

In point X^o, X^q of the third characteristic of $f_3(X)$ will assume to the look presented in Figure 7;

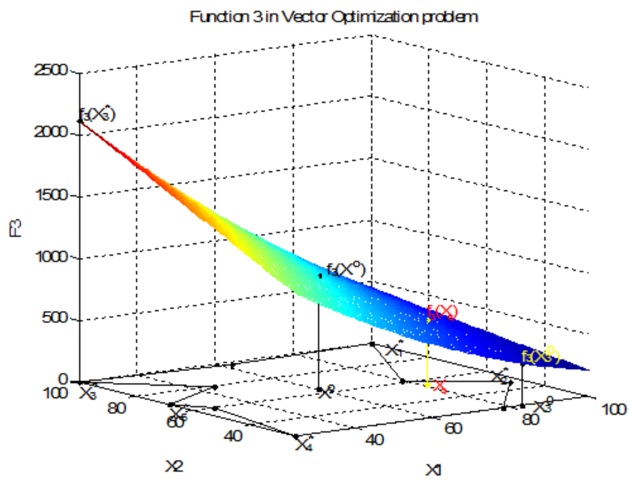

Figure 7. The third characteristics of $f_3(X)$ of technical system in natural indicator

In point X^o, X^q of the fourth characteristic of $f_4(X)$ will assume to the look presented in Figure 8;

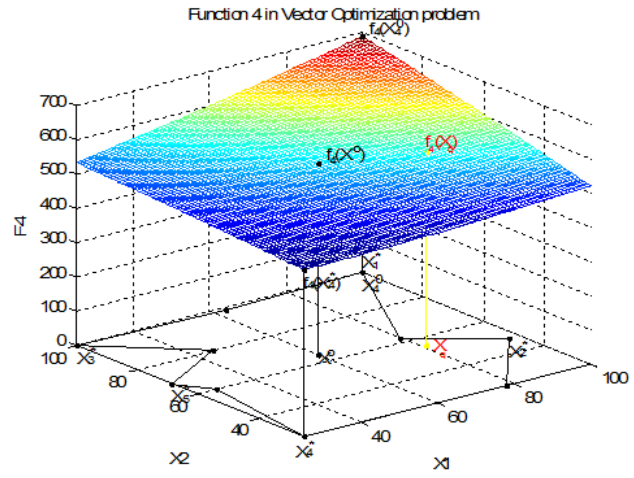

Figure 8. The fourth characteristics of $f_4(X)$ of technical system in natural indicator

In point X^o, X^q of the five characteristic of $f_5(X)$ will assume to the look presented in Figure 9.

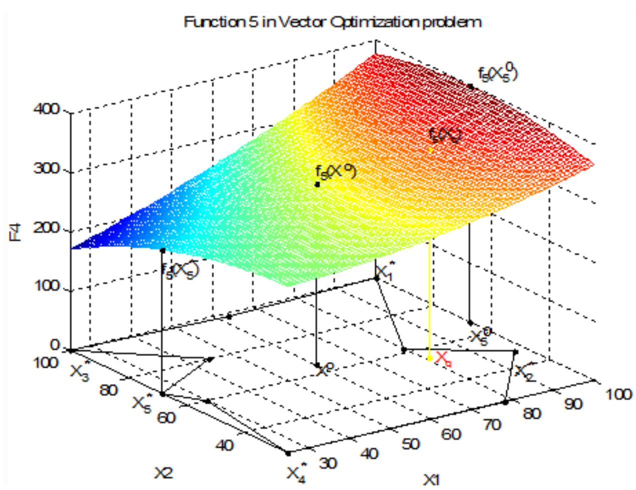

Figure 9. The fourth characteristics of $f_5(X)$ of technical system in natural indicator

Collectively, *the submitted version*:

• point - X^o; characteristics of $f_1(X^o)$, $f_2(X^o)$, $f_3(X^o)$, $f_4(X^o)$, $f_5(X^o)$;

• relative estimates of $\lambda_1(X^o)$, $\lambda_2(X^o)$, $\lambda_3(X^o)$, $\lambda_4(X^o)$, $\lambda_5(X^o)$;

• maximum λ^o relative level such that $\lambda^o \leq \lambda_k(X^o)\ \forall k \in \boldsymbol{K}$

- there is *an optimum decision at equivalent criteria* (characteristics), and procedure of receiving is *adoption of the optimum decision at equivalent criteria* (characteristics).

• point – X^q; characteristics of $f_1(X^q)$, $f_2(X^q)$, $f_3(X^q)$, $f_4(X^q)$, $f_5(X^q)$;

• relative estimates of $\lambda_1(X^q)$, $\lambda_2(X^q)$, $\lambda_3(X^q)$, $\lambda_4(X^q)$, $\lambda_5(X^q)$;

• maximum λ^o relative level such that $\lambda^o \leq \lambda_k(X^q)\ \forall k \in \boldsymbol{K}$

- there is an optimal solution at the set priority of the second criterion (characteristic) in relation to other criteria. Procedure of receiving a point is X^q *adoption of the optimal solution at the set priority of the second criterion.*

Theory of vector optimization, methods of solution of the vector problems with equivalent criteria and given priority of criterion can choose any point from the set of points, optimum across Pareto, and show the optimality of this point.

5. Conclusions

The problem of adoption of the optimum decision in difficult technical system on some set of functional characteristics is one of the most important tasks of the system analysis and design. In work the new technology (methodology) of creation of mathematical model of technical system in the conditions of definiteness and uncertainty in the form of a vector problem of mathematical programming is presented.

For the first time in domestic and foreign literature, we have submitted the theory of vector optimization and methods for the choice of any point, from Pareto's great number. The principles of an optimality of a point are shown in the theory, first, at equivalent criteria, secondly, at the set criterion priority. These methods can be used at design of technical systems of various branches: electro technical [2], aerospace, metallurgical, etc.

At creation of characteristics in the conditions of uncertainty regression methods of transformation of information are used. The methodology of modeling and adoption of the optimum decision is based on normalization of criteria and the principle of the guaranteed result (maxmin). Methods allow solving vector problems at equivalent criteria and with the set criterion priority. Results of the decision are a basis for decision-making on the studied technical system on all set of point's optimum across Pareto.

This methodology has system character and can be used when modeling both technical and economic systems. Authors are ready to participate in the solution of vector problems of linear and nonlinear programming.

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[2] We mention the work of V.L. Levitskii "Simulation and Optimization of Parameters of Magnetoelectric Linear Inductor Electric Direct Current Motor" [5, p. 50–120]. It deals with designing an augmented electric motor (AEM) with its model reduced to vector mathematical programming problem (3)–(6). The vector of design parameters $X = (X_1, \ldots, X_5)$ consisted of X_1 for the air clearance δ, X_2 for the tooth pitch, X_3 for the number of teeth, X_4 for the height of the concentrator, and X_5 for the pole overlap coefficient. The vector of design criteria $F(X) = (f(X), p(X), \eta(X), \ldots)$ included $f(X)$ for the nominal towing force, $p(X)$ for the nominal power, $\eta(X)$ for the nominal efficiency and so on, ten indices in total. The central orthogonal plan of the second order was used to construct the dependencies of f on the listed design parameters X [5, p. 96]. The work "…Multiobjec_tive Optimization of Static Modes of Mass_Exchange Processes by the Example of Absorption in Gas Separation" [13] is an example from another industry. Thus, experimental data both from the AEM problem and from similar ES of other industries can be represented as theoretical (system) problem (3)-(6).