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Mathematical Analysis of Information Dissemination Model for Social Networking Services

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Abstract In this paper Homotopy perturbation method (HPM) is implemented to give an approximate analytical solution to the system of non-linear differential equation corresponding to S-SEIR model. The S-SEIR model is constructed for information dissemination characteristics on social network. Our analytical results are compared with the numerical simulation and a satisfactory agreement is noted. The graphical results are shown the effect of information value and user behavior on information dissemination. Using the Homotopy perturbation method we can easily solve other strongly non-linear initial and boundary value problems in engineering and sciences.

Keywords: *information dissemination, social networking services (SNS), information value, user behavior, homotopy perturbation method, numerical simulation*

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1. Introduction

Social networking service is a service which is based on web 2.0 technology of social network. These days the scope of social networking has spread to all areas of the society and has become one of the tools of mass media communication with greater social. Social network has provided great facility by its quick, direct and wide range of information dissemination. On the other hand Information dissemination network has various demerits due to being dispersive, massive and uncontrollable which may result into public opinion problem by spreading false or fake information. Therefore this research paper is an eye opener for more research to be conducted on propagation mechanism of information on social networking which can be a helping hand towards better use of internal facility. The research on propagation mechanism of information on social networking which can be a helping hand towards better use of internet facility and analyze the mode of information dissemination.

The research on propagation mechanism of information dissemination began years ago by Kermark *et al*. [1] who proposed the epidemic mode of SIR with the application of dynamics method in 1927 which was provided an idea to use mathematical tools to study the propagation mechanism of infectious diseases. In 1991 Anderson *et al*. [2] added the exposed state to SIR model and then built a better SEIR model by analyzing propagation patterns of a variety of infectious diseases. In recent times a member of researches on propagation mechanism has been conducted using methods of propagation mechanism of infectious diseases for references. For example Yuan Hua *et al.* [3] modified SEIR model and came up with a new model known as E-SEIR for e-mail viruses spread.

E-SEIR model dealt with the influence of user behavior and anti-virus technology one-mail virus diffusion. Other researchers HuJieying *et al. 1* [4] proposed IM- SEIR model for instant messaging network to study the influence of probabilities of status evolvement on information dissemination.

As per the previous models they made an assumption that the node passed messages to its neighbors in the same probability without taking the influence of information values and the number of infected nodes around every node. The social network of recent times gives the users the opportunity to make a message the moment it's received. Hence the existing mode does not applied to social network. Due to the increase on the numbers of social network the user generally is much involved with its multiple social networks simultaneously. Information transmission from one social network to another is frequent. The S-SEIR model is proposed in this paper keeping in mind information value and user behavior through the analysis of information dissemination mode around different social networks. This S-SEIR model is adaptive to information dissemination characteristics on social network and the influence of correlation parameters on information dissemination is discussed as well.

2. Mathematical Formulation of the Initial Value Problem

As we refer Yancho [5] paper we define users in social network as nodes and the relation between the users as the edge between nodes. Information on social network spread along the edge between the nodes. The nodes are classified into four categories according to the propagation mechanism of information, such as publication node, communication node, immune node and uninfected node. Publication node publishes the original information. When the probability of information audit equals to one, it means that this social network does not have audit mechanism. Communication node receives message from its neighbor nodes and has the capacity to spread the information in or outside website. Immune node receives message from its neighbors but without spread the message. Infected node does not receive or browse the information from its neighbor temporarily but has the opportunity to receive the message. Communication node and immune node together referred as infected node. Considering the propagation mechanism of information on social network and having the basic knowledge of SEIR model, the S-SEIR model is constructed which is shown in Figure (1). In this paper, Hethcote [6] parameter setting is followed information value v_0 should lies between 0 to 1 and the sum of user behavior values are $\delta_1+\delta_2+\delta_3=1$. As in the S-SEIR model [1], we define the following notations; N – Number of node; S – Susceptible node; E – Exposed node; I – Infected node; R – Rejected node.

Considering S-SEIR model, here $S(t)$ is a continuous differentiable function of the time t. The number of nodes receiving information per minute is $\varphi S(t)$, the $\alpha S(t)$ nodes could not receive information and $\delta_1 I(t)$ nodes browsing and spreading the information to the other website. So the equations at time $t+\Delta t$ is as follows

$$S(t+\Delta t)-S(t)=\delta_1 I(t)\Delta t-\varphi S(t)\Delta t-\alpha S(t)\Delta t \quad (1)$$

The above equation converted into differential form as

$$\frac{dS(t)}{dt}=-\varphi S-\alpha S+\delta_1 I \quad (2)$$

Similarly, other differential equations [1] can be worked out and the corresponding differential equations of S-SEIR model are as follows:

$$\frac{dE(t)}{dt}=\varphi S+\delta_2 I-(\beta+\theta)E \quad (3)$$

$$\frac{dI(t)}{dt}=\beta E-(\delta_1+\delta_2+\delta_3)I \quad (4)$$

$$\frac{dR(t)}{dt}=\theta E+\alpha S+\delta_3 I \quad (5)$$

transfer efficiency function $\varphi(t)$ refers to paper of Xiaoyuan [7].

$$\varphi(t)=v_0 e^{(-\lambda t)} \quad (6)$$

Where t -dimensionless time, $S(t)$ -Number of Susceptible node, $E(t)$ - Number of Exposed node, $I(t)$ - Number of Infected node, $R(t)$ -Number of Rejected node, λ -characteristic scale factor, v_0 -Information value, α - Probability value of S→R node, β -Probability value of E→I node, δ_1 -Probability value of S→E node, δ_2 - Probability value of S→E node, δ_3 - Probability value of S→E node, θ -Probability value of E→R node.

The corresponding initial conditions are given by

$$N=10000,\ S(0)=10000,\ E(0)=0,\ I(0)=0\ \text{and}\ R(0)=0. \quad (7)$$

3. Analytical Solution of the Information Dissemination Model Using Homotopy Perturbation Method

Linear and non-linear phenomena are of fundamental importance in various fields of science and engineering. Most models of real – life problems are still very difficult to solve. Therefore, approximate analytical solutions such as Homotopy perturbation method (HPM) [8-22] were introduced. This method is the most effective and convenient ones for both linear and non-linear equations. Perturbation method is based on assuming a small parameter. The majority of non-linear problems, especially those having strong non-linearity, have no small parameters at all and the approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on which the small parameter should exist. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, HPM have been proposed recently.

Many authors have applied the Homotopy perturbation method (HPM) to solve the nonlinear boundary value problem in physics and engineering sciences [8,9,15,16]. This method is also used to solve some of the non-linear problem in physical sciences [11-16]. This method is a combination of Homotopy in topology and classic perturbation techniques. Ji-Huan He used to solve the Lighthill equation [11], the Diffusion equation [12] and the Blasius equation [13]. The HPM is unique in its applicability, accuracy and efficiency. The HPM uses the imbedding parameter p as a small parameter, and only a few iterations are needed to search for an asymptotic solution. The approximate analytical solution of eqns.(2)-(5) using the Homotopy perturbation method [8-22] is given by

$$S(t)=\begin{bmatrix} ae^{-\alpha t}+s_{21}\left(e^{-\alpha t}-e^{-(\delta 1+\delta_2+\delta_3)t}\right) \\ +s_{22}\left(e^{-(\lambda+\alpha)t}-e^{-\alpha t}\right) \\ +s_{31}\left(e^{-\alpha t}-e^{-(\delta 1+\delta_2+\delta_3+\lambda)t}\right) \\ +s_{32}\left(e^{-(\lambda+\alpha)t}-e^{-\alpha t}\right)+s_{33}\left(e^{-\alpha t}-e^{-(\lambda+\alpha)t}\right) \\ +s_{34}\left(e^{-(2\lambda+\alpha)t}-e^{-\alpha t}\right)+s_{35}\left(e^{-\alpha t}-e^{-(\beta+\theta)t}\right) \\ +s_{36}\left(e^{-(\delta_1+\delta_2+\delta_3)t}-e^{-\alpha t}\right) \end{bmatrix} \quad (7)$$

$$E(t)=\begin{bmatrix} be^{-(\beta+\theta)t}+e_{21}\left(e^{-(\beta+\theta)t}-e^{-(\lambda+\alpha)t}\right) \\ +e_{22}\left(e^{-(\beta+\theta)t}-e^{-(\delta_1+\delta_2+\delta_3)t}\right) \\ +e_{31}\left(e^{-(\beta+\theta)t}-e^{-(\lambda+\alpha)t}\right) \\ +e_{32}\left(e^{-(\delta_1+\delta_2+\delta_3+\lambda)t}-e^{-(\beta+\theta)t}\right) \\ +e_{33}\left(e^{-(\beta+\theta)t}-e^{-(2\lambda+\alpha)t}\right) \\ +e_{34}\left(e^{-(\lambda+\alpha)t}-e^{-(\beta+\theta)t}\right) \\ +e_{35}\left(e^{-(\delta_1+\delta_2+\delta_3+\lambda)t}-e^{-(\beta+\theta)t}\right) \\ +e_{36}\left(t\,e^{-(\beta+\theta)t}\right) \end{bmatrix} \quad (8)$$

$$I(t)=\begin{bmatrix} ce^{-(\delta_1+\delta_2+\delta_3)t} \\ +i_{21}\left(e^{-(\beta+\theta)t}-e^{-(\delta_1+\delta_2+\delta_3)t}\right) \\ +i_{31}\left(e^{-(\beta+\theta)t}-e^{-(\delta_1+\delta_2+\delta_3)t}\right) \\ +i_{32}\left(e^{-(\delta_1+\delta_2+\delta_3)t}-e^{-(\lambda+\alpha)t}\right) \\ +i_{33}\left(e^{-(\beta+\theta)t}-e^{-(\delta_1+\delta_2+\delta_3)t}\right) \\ -i_{34}\,t\,e^{-(\delta_1+\delta_2+\delta_3)t} \end{bmatrix} \quad (9)$$

$$R(t)=\begin{bmatrix} d+r_{21}\left(1-e^{-(\beta+\theta)t}\right)+a\left(1-e^{-\alpha t}\right) \\ +r_{22}\left(1-e^{-(\delta_1+\delta_2+\delta_3)t}\right)+r_{31}\left(1-e^{-(\beta+\theta)t}\right) \\ +r_{32}\left(e^{-(\lambda+\alpha)t}-1\right)+r_{33}\left(1-e^{-(\beta+\theta)t}\right) \\ +r_{34}\left(e^{-(\delta_1+\delta_2+\delta_3)t}-1\right)+s_{21}\left(1-e^{-\alpha t}\right) \\ +r_{35}\left(e^{-(\delta_1+\delta_2+\delta_3)t}-1\right)+r_{36}\left(1-e^{-(\lambda+\alpha)t}\right) \\ +s_{22}\left(e^{-\alpha t}-1\right)+r_{37}\left(1-e^{-(\beta+\theta)t}\right) \\ +r_{38}\left(e^{-(\delta_1+\delta_2+\delta_3)t}-1\right) \end{bmatrix} \quad (10)$$

where $s_{21}, s_{22}, s_{31}, s_{32}, s_{33}, s_{34}, s_{35}, s_{36}, e_{21}, e_{22}, e_{31}, e_{32}, e_{33}, e_{34}, e_{35}, e_{36}, i_{31}, i_{32}, i_{33}, i_{34}, r_{31}, r_{32}, r_{33}, r_{34}, r_{35}, r_{36}, r_{37}$ *and* r_{38} are defined as follows,

$$s_{21}=\frac{c\delta_1}{\delta_1+\delta_2+\delta_3-\alpha};\; s_{22}=\frac{av_0}{\lambda}; \quad (11)$$

$$s_{31}=\frac{v_0 s_{21}}{\delta_1+\delta_2+\delta_3+\lambda-\alpha}; s_{32}=\frac{s_{21}v_0}{\lambda};\; s_{33}=\frac{s_{22}v_0}{\lambda}; \quad (12)$$

$$s_{34}=\frac{s_{22}v_0}{2\lambda}; s_{35}=\frac{i_{21}\delta_1}{\beta+\theta-\alpha}; \quad (13)$$

$$s_{36}=\frac{i_{21}\delta_1}{\delta_1+\delta_2+\delta_3-\alpha}; i_{21}=\frac{\beta b}{\delta_1+\delta_2+\delta_3-\beta-\theta} \quad (14)$$

$$e_{21}=\frac{av_0}{\lambda+\alpha-\beta-\theta}; e_{22}=\frac{c\delta_2}{\delta_1+\delta_2+\delta_3-\beta-\theta}; \quad (15)$$

$$e_{31}=\frac{s_{21}v_0}{\lambda+\alpha-\beta-\theta}; e_{32}=\frac{s_{21}v_0}{\delta_1+\delta_2+\delta_3+\lambda-\beta-\theta}; \quad (16)$$

$$e_{33}=\frac{s_{22}v_0}{2\lambda+\alpha-\beta-\theta}; e_{34}=\frac{s_{22}v_0}{\lambda+\alpha-\beta-\theta}; \quad (17)$$

$$e_{35}=\frac{i_{21}\delta_2}{\delta_1+\delta_2+\delta_3-\beta-\theta}; e_{36}=\delta_2 i_{21}; \quad (18)$$

$$i_{31}=\frac{e_{21}\beta}{\delta_1+\delta_2+\delta_3-\beta-\theta}; \quad i_{32}=\frac{e_{21}\beta}{\left(\delta_1+\delta_2+\delta_3-\lambda-\alpha\right)}; \quad (19)$$

$$i_{33}=\frac{e_{22}\beta}{\left(\delta_1+\delta_2+\delta_3-\beta-\theta\right)}; i_{34}=\beta e_{22}; r_{21}=\frac{\theta b}{\beta+\theta}; \quad (20)$$

$$r_{22}=\frac{c\delta_3}{\delta_1+\delta_2+\delta_3}; r_{31}=\frac{e_{21}\theta}{\left(\beta+\theta\right)}; \quad (21)$$

$$r_{32}=\frac{e_{21}\theta}{\lambda+\alpha};\; r_{33}=\frac{e_{22}\theta}{\beta+\theta}; r_{34}=\frac{e_{22}\theta}{\delta_1+\delta_2+\delta_3}; \quad (22)$$

$$r_{35}=\frac{\alpha s_{21}}{\delta_1+\delta_2+\delta_3}; r_{36}=\frac{s_{22}\alpha}{\lambda+\alpha}; \quad (23)$$

$$r_{37}=\frac{i_{21}\delta_3}{\beta+\theta}; r_{38}=\frac{i_{21}\delta_3}{\delta_1+\delta_2+\delta_3} \quad (24)$$

4. Numerical simulation

The non-linear differential eqns. (2)-(5) are also solved numerically. We have used the function in Matlab/Scilab software to solve the initial-boundary value problems for the nonlinear differential equations numerically. This numerical solution is compared with our analytical results in figures (2)– (3). Upon comparison, it gives a satisfactory agreement for the values of the dimensionless time t from the range $t=0-4$. The Matlab/Scilab program is also given in Appendix ***C*.**

5. Results and discussions

In this section we discuss the effect of information value v_0, and the user behavior $\delta_1, \delta_2, \delta_3$ in the information dissemination on social network. Figure 2 – Figure 5 represent the function nodes $S(t), E(t), I(t)$ *and* $R(t)$ versus the dimensionless time t for different values of dimensionless parameters. From Figure 2 and Figure 4 it is clear that when v_0 increases the number of susceptible node $S(t)$ is decreases from 10000. At the same time, the number of exposed node $E(t)$, the number of infected node $I(t)$ and the number of rejected node $R(t)$ are increases from 0 in some fixed values of $\alpha, \beta, \delta_1, \delta_2, \delta_3$ *and* λ. The number of infected node and the number of exposed node are remarkably increases when the information $v_0 = 0.75$. When the information value $v_0 = 0.12$, the system is inactive status. From Figure 3 and Figure 5 it is clear that when changing the user behavior values $\delta_1, \delta_2, \delta_3$ show some impact in the information dissemination in some fixed value of $v_0 = 0.25$ *and* $\lambda = 4$. The changes in the value of the parameter δ_3 affects the number of rejected nodes.

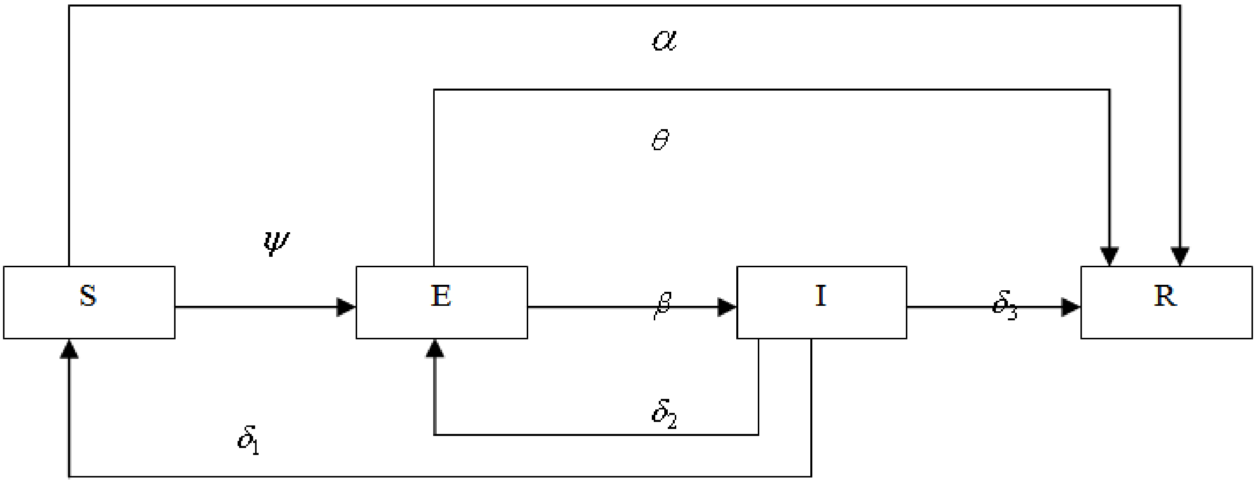

Figure 1. S-SEIR model for information dissemination

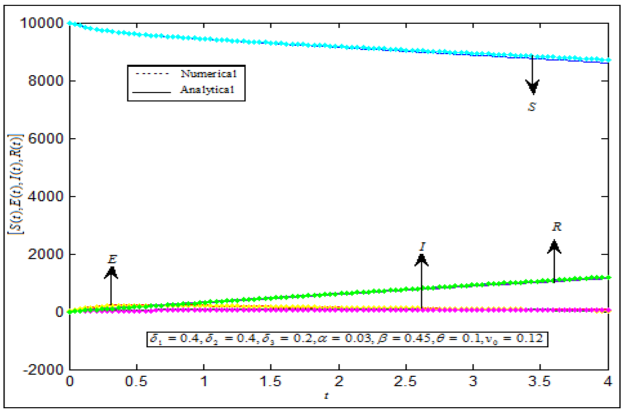

Figure 2 (a) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn. (2)-(5) for the fixed value $\delta_1, \delta_2, \delta_3, \alpha, \beta, \theta,$ *and* v_0 where $t = 0$ *to* 4.

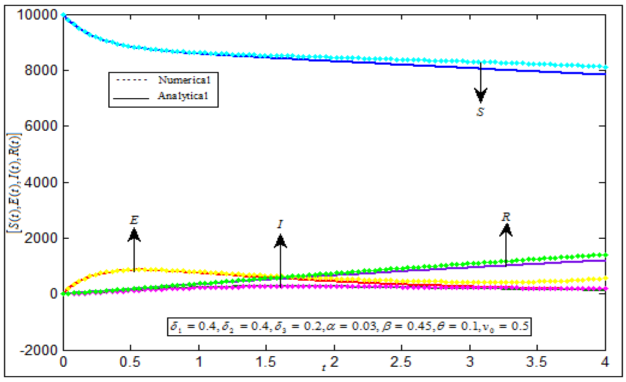

Figure 2(c). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn. (2)-(5) for the fixed values of $\delta_1, \delta_2, \delta_3, \alpha, \beta, \theta,$ *and* v_0 where $t = 0$ *to* 4.

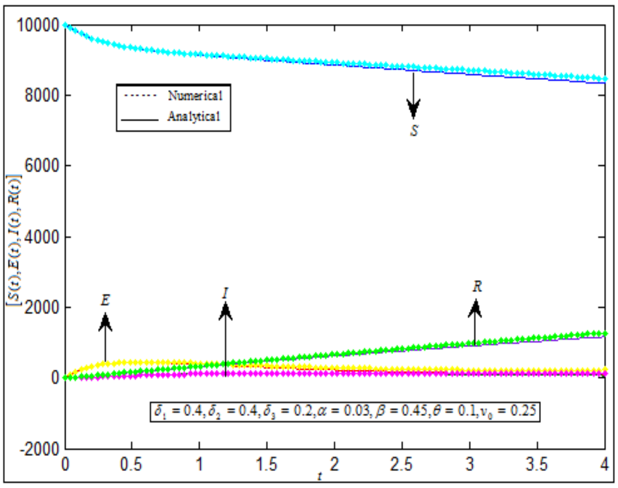

Figure 2(b). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed values of $\delta_1, \delta_2, \delta_3, \alpha, \beta, \theta,$ *and* v_0 where $t = 0$ *to* 4.

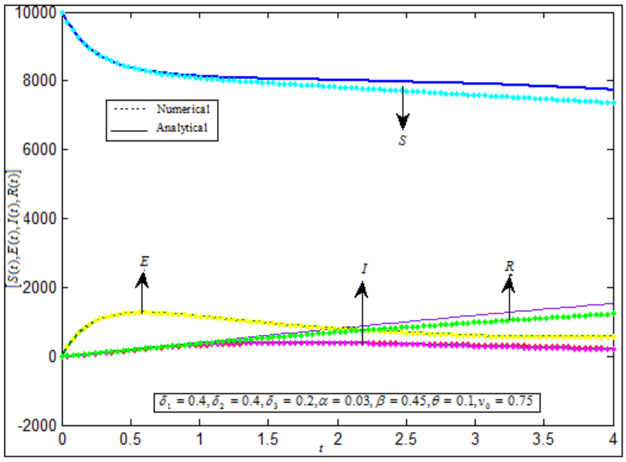

Figure 2(d). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn. (2)-(5) for the fixed values of $\delta_1, \delta_2, \delta_3, \alpha, \beta, \theta,$ *and* v_0 where $t = 0$ *to* 4.

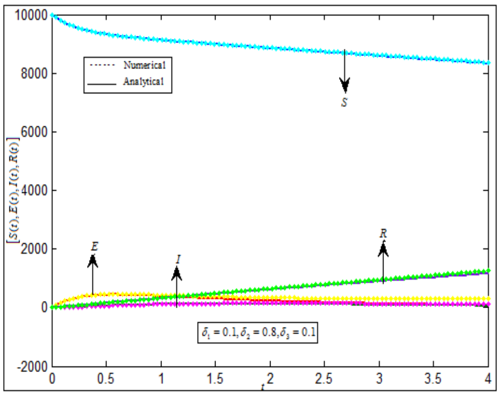

Figure 3(a). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ where $t = 0$ *to* 4.

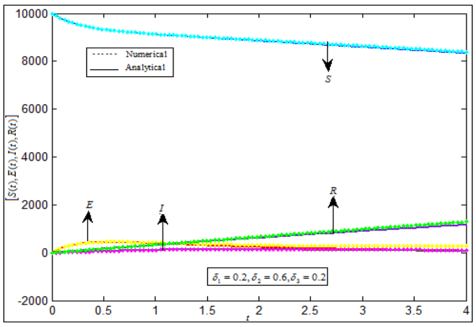

Figure 3(b) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ where $t = 0$ *to* 4.

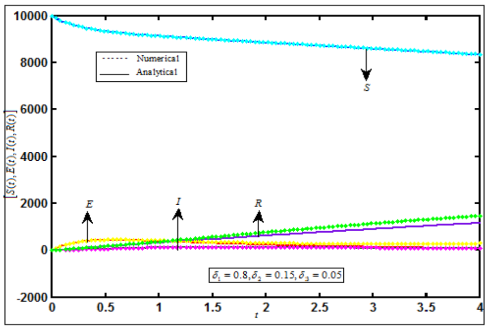

Figure 3(c) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ where $t = 0$ *to* 4.

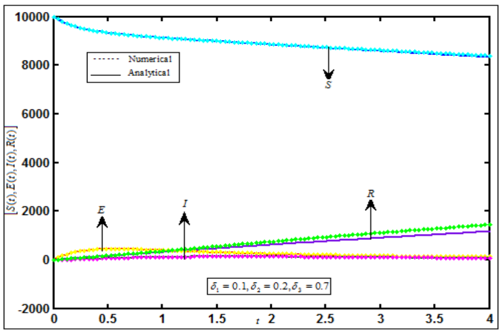

Figure 3(d) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ where $t = 0$ *to* 4.

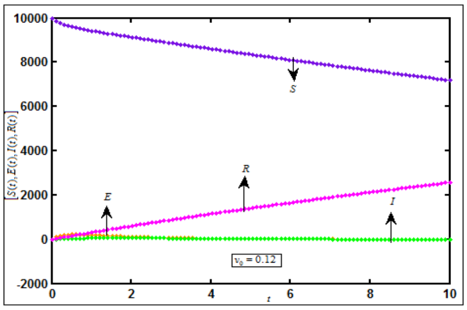

Figure 4(a) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $\alpha = 0.03, \beta = 0.45, \delta_1 = 0.4, \delta_2 = 0.4, \delta_3 = 0.2$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

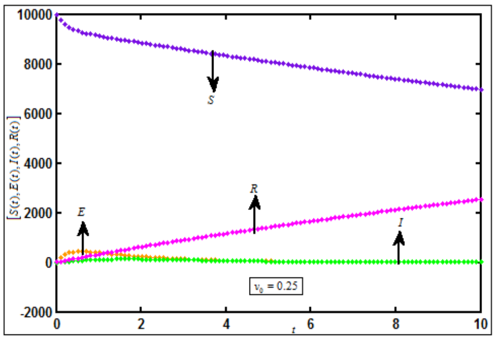

Figure 4(b). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $\alpha = 0.03, \beta = 0.45, \delta_1 = 0.4, \delta_2 = 0.4, \delta_3 = 0.2$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

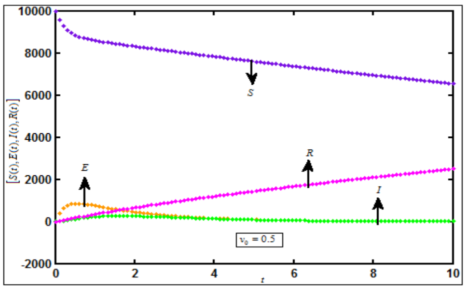

Figure 4(c). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $\alpha = 0.03, \beta = 0.45, \delta_1 = 0.4, \delta_2 = 0.4, \delta_3 = 0.2$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

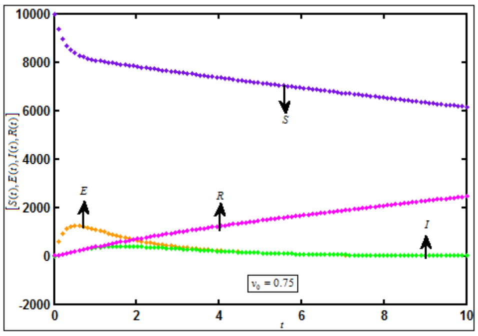

Figure 4(d). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $\alpha = 0.03, \beta = 0.45, \delta_1 = 0.4, \delta_2 = 0.4, \delta_3 = 0.2$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

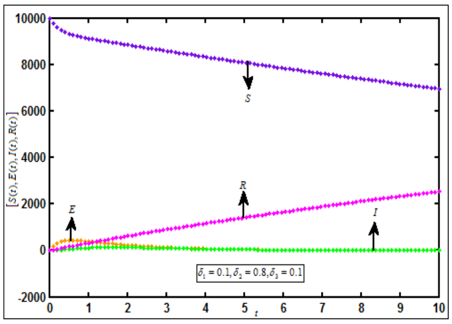

Figure 5(a). The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

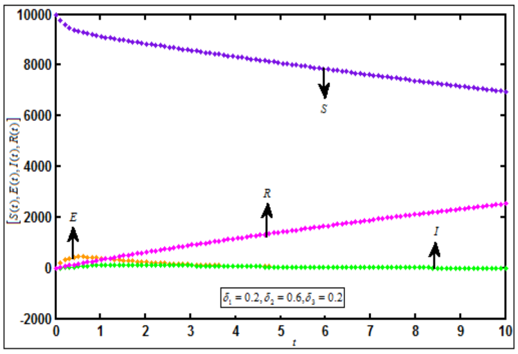

Figure 5(b) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

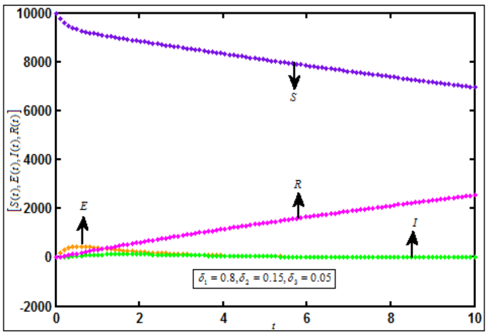

Figure 5(c) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

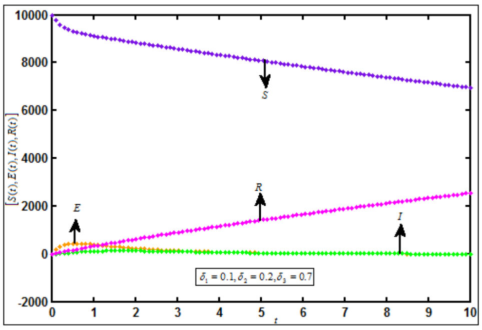

Figure 5(d) The dimensionless function nodes $S(t), E(t), I(t)$ *and* $R(t)$ are computed with respect to the dimensionless time t using the eqn.(2)-(5) for the fixed value of $v_0 = 0.25$ *and* $\lambda = 4$ where $t = 0$ *to* 10.

6. Conclusion

The primary result of the work is to give an analytical and numerical solution of the non-linear S-SEIR model.

A simple closed form of analytical expressions of the number susceptible node eqn. (7) and the number of exposed node eqn. (8) and the number of infected node eqn. (9) and the number of rejected node eqn. (10) are given in terms of $\alpha, \beta, \delta_1, \delta_2, \delta_3, v_0$ *and* λ. The analytical expressions for the S-SEIR model are derived using the Homotopy perturbation method. We have also presented graphical notations for impacting factors in S-SEIR model. A good agreement between the analytical results and a numerical simulation is observed. This HPM method is an extremely simple method and it is also a promising method to solve other non-linear differential equations.

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Appendix A

Basic Concept of Homotopy Perturbation Method

To explain this method, let us consider the following function:

$$D_o(u) - f(r) = 0, \qquad r \in \Omega \qquad \text{(A. 1)}$$

with the boundary conditions of

$$B_o(u, \frac{\partial u}{\partial n}) = 0, \qquad r \in \Gamma \qquad \text{(A. 2)}$$

where D_o is a general differential operator, B_o is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. In general, the operator D_o can be divided into a linear part L and a non-linear part N. Eqn. (A.1) can therefore be written as

$$L(u) + N(u) - f(r) = 0 \qquad \text{(A. 3)}$$

By the Homotopy technique, we construct a Homotopy $v(r, p): \Omega \times [0,1] \to \Re$ that satisfies

$$H(v, p) = \begin{bmatrix} (1-p)[L(v) - L(u_0)] \\ + p[D_o(v) - f(r)] \end{bmatrix} = 0. \qquad \text{(A. 4)}$$

$$H(v, p) = \begin{bmatrix} L(v) - L(u_0) \\ + pL(u_0) \\ + p[N(v) - f(r)] \end{bmatrix} = 0. \qquad \text{(A. 5)}$$

where $p \in [0, 1]$ is an embedding parameter, and u_0 is an initial approximation of eqn. (A. 1) that satisfies the boundary conditions. From eqn. (A. 4) and eqn. (A. 5), we have

$$H(v, 0) = L(v) - L(u_0) = 0 \qquad \text{(A. 6)}$$

$$H(v, 1) = D_o(v) - f(r) = 0 \qquad \text{(A. 7)}$$

When p=0, eqn. (A.4) and eqn. (A.5) become linear equations. When p =1, they become non-linear equations. The process of changing p from zero to unity is that of $L(v) - L(u_0) = 0$ to $D_o(v) - f(r) = 0$. We first use the embedding parameter p as a "small parameter" and assume that the solutions of eqn. (A. 4) and eqn. (A. 5) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + ... \qquad \text{(A. 8)}$$

Setting $p = 1$ results in the approximate solution of Eqn. (A. 1):

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + ... \qquad \text{(A. 9)}$$

This is the basic idea of the HPM.

Appendix B

Solution of the Initial Value Problem eqns.(2)-(5) Using Homotopy Perturbation Method

In this Appendix, we indicate how the eqns. (7)-(10) are derived in this paper. To find the solutions of eqns. (2)-(5), we construct the homotopy as follows:

$$\begin{bmatrix} (1-p)\left[\dfrac{dS}{dt} + \alpha S\right] \\ + p\begin{bmatrix} \dfrac{dS}{dt} + \alpha S \\ + v_0 e^{(-\lambda t)} S - \delta_1 I \end{bmatrix} \end{bmatrix} = 0 \qquad \text{(B. 1)}$$

$$\begin{bmatrix} (1-p)\left[\dfrac{dE}{dt} + (\beta + \theta) E\right] \\ + p\begin{bmatrix} \dfrac{dE}{dt} + (\beta + \theta) E \\ - v_0 e^{(-\lambda t)} S - \delta_2 I \end{bmatrix} \end{bmatrix} = 0 \qquad \text{(B. 2)}$$

$$(1-p)\left[\frac{dI}{dt} + \delta_1 I\right] + p\left[\frac{dI}{dt} + \delta_1 I - \beta E\right] = 0 \qquad \text{(B. 3)}$$

$$(1-p)\frac{dR}{dt} + p\left[\frac{dR}{dt} - \theta E - \alpha S - \delta_3 I\right] = 0 \qquad \text{(B. 4)}$$

The analytical solutions of (B. 1), (B. 2), (B. 3) and (B. 4) are

$$S = S_0 + p^1 S_1 + p^2 S_2 + p^3 S_3. \qquad \text{(B. 5)}$$

$$E = E_0 + p^1 E_1 + p^2 E_2 + p^3 E_3. \qquad \text{(B. 6)}$$

$$I = I_0 + p^1 I_1 + p^2 I_2 + p^3 I_3. \qquad \text{(B. 7)}$$

$$R = R_0 + p^1 R_1 + p^2 R_2 + p^3 R_3. \qquad \text{(B. 8)}$$

Substituting the eqns. (B. 5) to (B. 6) in (B. 1) to (B. 4) respectively we get

$$\left[\begin{array}{l}(1-p)\left[\begin{array}{l}\dfrac{d\left(S_0+p^1S_1+p^2S_2+p^3S_3...\right)}{dt}\\+\alpha\left(S_0+p^1S_1+p^2S_2+p^3S_3...\right)\end{array}\right]+\\p\left[\begin{array}{l}\dfrac{d\left(S_0+p^1S_1+p^2S_2+p^3S_3...\right)}{dt}\\+\alpha\left(S_0+p^1S_1+p^2S_2+p^3S_3...\right)\\+v_0e^{(-\lambda t)}\left(S_0+p^1S_1+p^2S_2+p^3S_3...\right)\\-\delta_1\left(I_0+p^1I_1+p^2I_2+p^3I_3...\right)\end{array}\right]\end{array}\right]=0 \quad \text{(B. 9)}$$

$$\left[\begin{array}{l}(1-p)\left[\begin{array}{l}\dfrac{d\left(E_0+p^1E_1+p^2E_2+p^3E_3...\right)}{dt}\\+(\beta+\theta)\left(E_0+p^1E_1+p^2E_2+p^3E_3...\right)\end{array}\right]\\+p\left[\begin{array}{l}\dfrac{d\left(E_0+p^1E_1+p^2E_2+p^3E_3...\right)}{dt}\\+(\beta+\theta)\left(E_0+p^1E_1+p^2E_2+p^3E_3...\right)\\-v_0e^{(-\lambda t)}\left(S_0+p^1S_1+p^2S_2+p^3S_3....\right)\\-\delta_2\left(I_0+p^1I_1+p^2I_2+p^3I_3...\right)\end{array}\right]\end{array}\right]=0 \quad \text{(B. 10)}$$

$$\left[\begin{array}{l}(1-p)\left[\begin{array}{l}\dfrac{d\left(I_0+p^1I_1+p^2I_2+p^3I_3..\right)}{dt}\\+\delta_1\left(I_0+p^1I_1+p^2I_2+p^3I_3..\right)\end{array}\right]\\+p\left[\begin{array}{l}\dfrac{d\left(I_0+p^1I_1+p^2I_2+p^3I_3..\right)}{dt}\\+\delta_1\left(I_0+p^1I_1+p^2I_2+p^3I_3..\right)\\-\beta\left(E_0+p^1E_1+p^2E_2+p^3E_3..\right)\end{array}\right]\end{array}\right]=0 \quad \text{(B. 11)}$$

$$\left[\begin{array}{l}(1-p)\left[\dfrac{d\left(R_0+p^1R_1+p^2R_2+p^3R_3...\right)}{dt}\right]\\+p\left[\begin{array}{l}\dfrac{dR_0+p^1R_1+p^2R_2+p^3R_3........}{dt}\\-\theta\left(E_0+p^1E_1+p^2E_2+p^3E_3....\right)\\-\alpha\left(S_0+p^1S_1+p^2S_2+p^3S_3...\right)\\-\delta_3\left(I_0+p^1I_1+p^2I_2+p^3I_3.....\right)\end{array}\right]\end{array}\right]=0 \quad \text{(B. 12)}$$

The comparing the coefficient of like powers of p in (B. 9), (B. 10), (B. 11) and (B. 12) we get

$$p^0:\frac{dS_0}{dt}+\alpha S_0=0 \quad \text{(B. 13)}$$

$$p^0:\frac{dE_0}{dt}+(\beta+\theta)E_0=0 \quad \text{(B. 14)}$$

$$p^0:\frac{dI_0}{dt}+(\delta_1+\delta_2+\delta_3)I_0=0 \quad \text{(B. 15)}$$

$$p^0:\frac{dR_0}{dt}=0 \quad \text{(B. 16)}$$

$$p^1:\frac{dS_1}{dt}+\alpha S_1+v_0e^{(-\lambda t)}S_0-\delta_1I_0=0 \quad \text{(B. 17)}$$

$$p^1:\frac{dE_1}{dt}+(\beta+\theta)E_1-v_0e^{(-\lambda t)}S_0-\delta_2I_0=0 \quad \text{(B. 18)}$$

$$p^1:\frac{dI_1}{dt}+(\delta_1+\delta_2+\delta_3)I_1-\beta E_0=0 \quad \text{(B. 19)}$$

$$p^1:\frac{dR_1}{dt}-\theta E_0-\alpha S_0-\delta_3I_0=0 \quad \text{(B. 20)}$$

$$p^2:\frac{dS_2}{dt}+\alpha S_2+v_0e^{(-\lambda t)}S_1-\delta_1I_1=0 \quad \text{(B. 21)}$$

$$p^2:\frac{dE_2}{dt}+(\beta+\theta)E_2-v_0e^{(-\lambda t)}S_1-\delta_2I_1=0 \quad \text{(B. 22)}$$

$$p^2:\frac{dI_2}{dt}+(\delta_1+\delta_2+\delta_3)I_2-\beta E_1=0 \quad \text{(B. 23)}$$

$$p^2:\frac{dR_2}{dt}-\theta E_1-\alpha S_1-\delta_3I_1=0 \quad \text{(B. 24)}$$

The initial approximations are given by

$$S_0(0)=10000, E_0(0)=0, I_0(0)=0 \text{ and } R_0(0)=0, \quad \text{(B. 25)}$$

$$S_i(0)=0, E_i(0)=0, I_i(0)=0, R_i(0)=0, i=1,2,3... \quad \text{(B. 26)}$$

Solving the eqns. (B. 13)-(B. 24) and using the initial approximations the eqns. (B. 25) & (B. 26). We can obtain the following results

$$S_0=ae^{-\alpha t} \quad \text{(B. 27)}$$

$$S_1=\left[\begin{array}{l}\dfrac{c\delta_1}{\delta_1+\delta_2+\delta_3-\alpha}\left(e^{-\alpha t}-e^{-(\delta_1+\delta_2+\delta_3)t}\right)\\+\dfrac{av_0}{\lambda}\left(e^{-(\lambda+\alpha)t}-e^{-\alpha t}\right)\end{array}\right] \quad \text{(B. 28)}$$

$$S_2=\left[\begin{array}{l}s_{31}\left(e^{-\alpha t}-e^{-(\delta_1+\delta_2+\delta_3+\lambda)t}\right)\\+s_{32}\left(e^{-(\lambda+\alpha)t}-e^{-\alpha t}\right)\\+s_{33}\left(e^{-\alpha t}-e^{-(\lambda+\alpha)t}\right)\\+s_{34}\left(e^{-2(\lambda+\alpha)t}-e^{-\alpha t}\right)\\+s_{35}\left(e^{-\alpha t}-e^{-(\beta+\theta)t}\right)\\+s_{36}\left(e^{-(\delta_1+\delta_2+\delta_3)t}-e^{-\alpha t}\right)\end{array}\right] \quad \text{(B. 29)}$$

$$E_0=be^{-(\beta+\theta)t} \quad \text{(B. 30)}$$

$$E_1 = \begin{bmatrix} \frac{av_0}{\lambda+\alpha-\beta-\theta}\left(e^{-(\lambda+\alpha)\,t} - e^{-\alpha\,t}\right) \\ +\frac{c\delta_2}{\delta_1+\delta_2+\delta_3-\beta-\theta}\left(e^{-\alpha\,t} - e^{-(\delta_1+\delta_2+\delta_3)\,t}\right) \end{bmatrix} \quad \text{(B. 31)}$$

$$E_2 = \begin{bmatrix} e_{31}\left(e^{-(\beta+\theta)t} - e^{-(\lambda+\alpha)t}\right) \\ +e_{32}\left(e^{-(\delta_1+\delta_2+\delta_3+\lambda)t} - e^{-(\beta+\theta)t}\right) \\ +e_{33}\left(e^{-(\beta+\theta)t} - e^{-(2\lambda+\alpha)t}\right) \\ +e_{34}\left(e^{-(\lambda+\alpha)t} - e^{-(\beta+\theta)t}\right) \\ +e_{35}\left(e^{-(\delta_1+\delta_2+\delta_3+\lambda)t} - e^{-(\beta+\theta)t}\right) \\ +e_{36}\left(t\,e^{-(\beta+\theta)t}\right) \end{bmatrix} \quad \text{(B. 32)}$$

$$I_0 = ce^{-(\delta_1+\delta_2+\delta_3)t} \quad \text{(B. 33)}$$

$$I_1 = \frac{\beta b}{\delta_1+\delta_2+\delta_3-\beta-\theta}\left(e^{-(\beta+\theta)t} - e^{-(\delta_1+\delta_2+\delta_3)t}\right) \quad \text{(B. 34)}$$

$$I_2 = \begin{bmatrix} i_{31}\left(e^{-\alpha\,t} - e^{-(\delta_1+\delta_2+\delta_3)\,t}\right) \\ +i_{32}\left(e^{-(\beta+\theta)t} - e^{-(\lambda+\alpha)t}\right) \\ +i_{33}\left(e^{-(\delta_1+\delta_2+\delta_3+\lambda)t} - e^{-(\beta+\theta)t}\right) \\ -i_{34}t\,e^{-(\delta_1+\delta_2+\delta_3+\lambda)t} \end{bmatrix} \quad \text{(B. 35)}$$

$$R_0 = d \quad \text{(B.36)}$$

$$R_1 = \begin{bmatrix} \frac{\theta b}{\beta+\theta}\left(1-e^{-(\beta+\theta)t}\right) + a\left(1-e^{-\alpha\,t}\right) \\ +\frac{c\delta_3}{\delta_1+\delta_2+\delta_3}\left(1-e^{-(\delta_1+\delta_2+\delta_3)t}\right) \end{bmatrix} \quad \text{(B. 37)}$$

$$R_2 = \begin{bmatrix} r_{31}\left(1-e^{-(\beta+\theta)t}\right) + r_{32}\left(e^{-(\lambda+\alpha)t} - 1\right) \\ +r_{33}\left(1-e^{-(\beta+\theta)t}\right) + r_{34}\left(e^{-(\delta_1+\delta_2+\delta_3)t} - 1\right) \\ +\frac{c\delta_1}{\delta_1+\delta_2+\delta_3-\alpha}\left(1-e^{-\alpha\,t}\right) \\ +r_{35}\left(e^{-(\delta_1+\delta_2+\delta_3)t} - 1\right) + r_{36}\left(1-e^{-(\lambda+\alpha)t}\right) \\ +\frac{av_0}{\lambda}\left(e^{-\alpha\,t} - 1\right) + r_{37}\left(1-e^{-(\beta+\theta)t}\right) \\ +r_{38}\left(e^{-(\delta_1+\delta_2+\delta_3)t} - 1\right) \end{bmatrix} \quad \text{(B. 38)}$$

According to the HPM, we can conclude that,

$$S = \lim_{p\to 1} S(t) = S_0 + S_1 + S_2 \quad \text{(B. 39)}$$

$$E = \lim_{p\to 1} E(t) = E_0 + E_1 + E_2 \quad \text{(B. 40)}$$

$$I = \lim_{p\to 1} I(t) = I_0 + I_1 + I_2 \quad \text{(B. 41)}$$

$$R = \lim_{p\to 1} R(t) = R_0 + R_1 + R_2 \quad \text{(B. 42)}$$

After putting (B. 27)-(B. 29) in (B. 39); (B. 30)-(B. 32) in (B. 40); (B. 31)-(B. 35) in (B. 41) and (B. 36)-(B. 38) in (B. 42). We obtain the solution in the text the eqns. (7)-(10).

Appendix C

Matlab/Scilab Program to Find the Numerical Solution of non-linear Equations (2)-(5)

```
function s-seir
options= odeset ('RelTol', 1e-6, 'Stats', 'on');
%initial conditions
x0 = [10000; 0; 0;0];
tspan = [0,10];
tic[t,x]=ode45(@TestFunction, tspan, x0, options);
toc
figure
hold on
plot(t, x(:,1))
%plot(t, x(:,2))
%plot(t, x(:,3))
%plot(t, x(:,4))
legend('S','E','I','R')
ylabel('x')
xlabel('t')
return
function [dx_dt]= TestFunction(t,x)
n=4; a1=0.03; a2=0.45; a3=0.1; d1=0.4; d2=0.4; d3=0.2; v0=0.12;
dx_dt(1)=(-v0)*(exp(-n*t))*x(1)-a1*x(1)+d1*x(3);
dx_dt(2)=((v0)*(exp(-n*t)))*x(1)+d2*x(3)-((a2+a3)*x(2));
dx_dt(3)=(a2*x(2))-((d1+d2+d3)*x(3));
dx_dt(4)=(a3*x(2))+(a1*x(1)+d3*x(3));
dx_dt = dx_dt';
return
```

Appendix D

Nomenclature

Symbol	Meaning
t	time
ψ	Probability value of S →E node
θ	Probability value of E→R node
α	Probability value of S→R node
β	Probability value of E→I node
δ_1	Probability value of S→E node
δ_2	Probability value of S→E node
δ_3	Probability value of S→E node
v_0	Information value
λ	Characteristic scale factor
$S(t)$	Susceptible node
$E(t)$	Exposed node
$I(t)$	Infected node
$R(t)$	Rejected node