American Journal of Modeling and Optimization, 2015, Vol. 3, No. 1, 22-25
Available online at http://pubs.sciepub.com/ajmo/3/1/3

DOI:10.12691/ajmo-3-1-3

Mathematical Model and Analysis of Two Phase Hepatic Blood Flow through Arterioles with the Special Reference of Hepatitis A

J. P. Singh[1], Anil Agrawal[1], V Upadhayay[1], Anil Kumar[2,*]

[1]Department of Mathematics Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya Satana MP (India)
[2]Department of Applied Mathematics International Maritime Institute, Greater Noida UP India
*Corresponding author: dranilkumar73@rediffmail.com

Received February 07, 2015; Revised March 02, 2015; Accepted March 11, 2015

Abstract In this investigation, we are considering the Hepatic blood flow in arterioles, keeping in view the nature of Hepatic circulatory system in human being. One of red blood cells and other are plasma are considered. They have applied the Herschel Bulkley Non-Newtonian model in Bio-fluid physiological are investigated. Using experimental values of the parameters, the flow rate for normal and diseased blood arteriols has been computed and compared with corresponding values obtained from a well known experimentally tested model in the literature. The role of Hematocrit is explicit in the determination of blood pressure in the case of Hepatic disease special reference of Hepatitis A.

Keywords: *Hepatitis A, Hematocrit, Blood Flow, Herschel Bulkley Non-Newtonian model, circulatory system*

Cite This Article: J.P. Singh, Anil Agrawal, V Upadhayay, and Anil Kumar, "Mathematical Model and Analysis of Two Phase Hepatic Blood Flow through Arterioles with the Special Reference of Hepatitis A." *American Journal of Modeling and Optimization*, vol. 3, no. 1 (2015): 22-25. doi: 10.12691/ajmo-3-1-3.

1. Introduction

Physiological changes that occur during exposure to weightlessness may induce alterations in blood flow to the liver. Estimation of hepatic blood flow using ground-based weightlessness models may provide insight into functional changes of the liver in crewmembers during flight. In the future study, Hepatic Blood Flow indirectly evaluated by indo cyanine gree clearance, is compared in this area during the normal ambulatory condition and anti orthostatic bed rest.

The fluid is plasma, which itself is a complex mixture of proteins other intergradient in an aqueous base. If the hemoglobin is known, two phase Hepatic blood flow is a study of measuring the blood pressure. The hematocrit is the fraction of the blood composed of red blood cells, as determined by centrifuging blood in a hematocrit tube until the cells become tightly packed in the bottom of the tube. It is impossible to completely pack the red cells together therefore, about 3 to 4 per cent of the plasma remains entrapped among the cells, and the true hematocrit is only about 96 per cent of the measured hematocrit. Hematocrit is three times of hemoglobin concentration. Hepatitis A virus was first characterized in 1973 when scientists detected the virus in stools from human volunteers who were infected with Hepatitis A virus. The second epidemiologic pattern is seen in industrialized countries, where the prevalence of Hepatitis A virus infection is low among children and young adults. A different antibody persists long-term after the infection has cleared. This antibody keeps you immune from future infection. A blood test can detect this second antibody which shows if you have had hepatitis A in the past, and that you are now immune. If hepatitis A is suspected, your doctor may also suggest other blood tests called liver function tests. This measure the activity of chemicals (enzymes) and other substances made in the liver. The liver is the largest gland in body, contributing about 2 per cent of the total body weight, or about 1.5 kg on the average adult human being. The basic functional unit of the liver is the liver lobule, which is a cylindrical structure several millimeters in length and 0.8 to 2 millimeters in diameter. The human liver contains 50,000 to 100, 000 individual lobules.

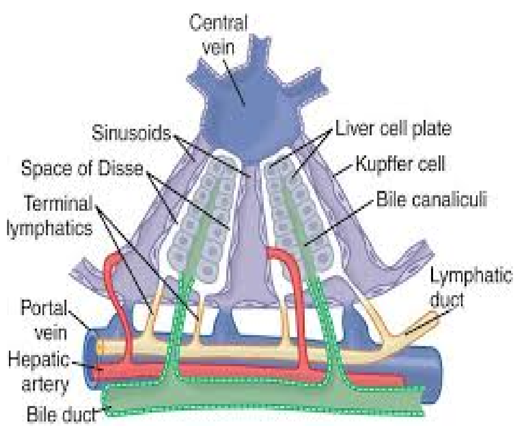

Figure 1. Basic structure of a liver lobule

The liver performs many different functions including (1) filtration and storage of food; (2) metabolism of carbohydrates, proteins, fats, hormones, and foreign chemicals; (3) formation of bile; (4) storage of vitamins and iron; and (5) formation of coagulation factors. It is a discrete organ and its various functions interrelate with another. This becomes especially evident in abnormalities of the liver, because various its functions are disturbed simultaneously. Kumar (2011) studied performance and analysis of blood flow through carotid artery.

2. Equation of Continuity for Two Phase Blood Flow

The blood flow is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X , this X is replaced by $H/100$, where H is the Hematocrit the volume percentage of blood cells. Then the volume portion covered by the plasma will be $(1-X)$. The mass ratio of blood cells to plasma is given below:

$$r=\frac{X\rho_c}{(1-X)\rho_p} \tag{1}$$

where ρ_c and ρ_p are densities of blood cells and blood plasma respectively. Usually this mass ratio is not constant, even then it may be supposed to constant in present context [Singh and Upadhayay (1986)].

The both phase of blood, i.e. blood cells and plasma move with the common velocity. Campbell and Pitcher has presented a model for two phase of blood separately (1958). The equation of continuity for two phases according to the principle of conservation of mass are as given below [Kapur and Gupta 1963].

$$\frac{\partial\left(X\rho_c\right)}{\partial t}+\left(X\rho_c v^i\right),i=0 \tag{2}$$

and

$$\frac{\partial(1-X)\rho_p}{\partial t}+\left((1-X)\rho_p v^i\right),i=0 \tag{3}$$

where, v is the common velocity of two phase blood cells and plasma. If we define the uniform density of the blood ρ_m as follow

$$\frac{1+r}{\rho_m}=\frac{r}{\rho_c}+\frac{1}{\rho_p} \tag{4}$$

From equation (2)and(3) we can written as,

$$\frac{\partial\rho_m}{\partial t}+\left(\rho_m v^i\right),i=0 \tag{5}$$

3. Equation of Motion for Two Phase Blood Flow

According to [Ruch and Patten (1973)] the hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood (1973). Here viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as given below:

$$X\rho_c\frac{\partial v^i}{\partial t}+\left(X\rho_c v^j\right)v^i_{,j}=-Xp_{,j}g^{ij}+X\eta_c\left(g^{jk}v^i_{,k}\right)_{,j} \tag{6}$$

The equation of motion for plasma will be as follows:

$$\begin{aligned}(1-X)\rho_p\frac{\partial v^i}{\partial t}+\left\{(1-X)\rho_p v^i\right\}v^i_{,j}\\ =-(1-X)p_{,j}g^{ij}+(1-X)\eta_c\left(g^{jk}v^i_{,k}\right)_{,j}\end{aligned} \tag{7}$$

Now adding equation (6) and (7) and using relation (4), the equation of motion for blood flow with the both phases will be as follows:

$$\rho_m\frac{\partial v^i}{\partial t}+\left(\rho_m v^j\right)v^i_{,j}=-p_{,j}+\eta_m\left(g^{jk}v^i_{,k}\right)_{,j} \tag{8}$$

where $\eta_m=X\eta_c+(1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

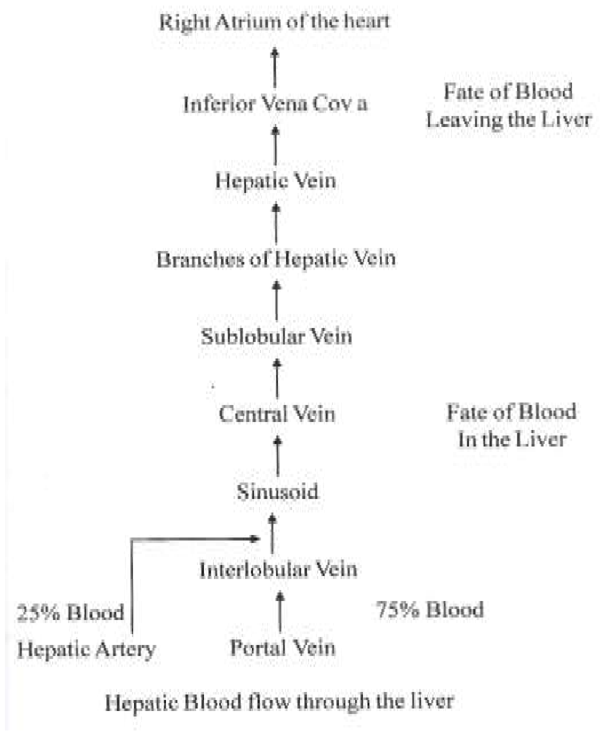

Figure 2. Liver Supply for Hepatic portal Vein and arteries

4. Mathematical Model

In the present communication of the velocity of blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively. The Herschel Bulkley law hold good on the two phase blood flow through veins arterioles, veinules and whose constitutive equation is as follows [18]:

$T^{'} = \eta_m e^n + T_p (T^{'} \geq T_p)$ and $e = 0(T^{'} < T_p)$ where, T_p is the yield stress.

When strain $e = 0(T^{'} < T_p)$ a core region is formed which is flows just like a plug. Let the radius of the plug be r_p .The stress acting on the surface of plug will be T_p. Equating the forces acting on the plug, we get,

$$P\pi r_p^2 = T_p 2\pi r_p \Rightarrow r_p = 2\frac{T_p}{P} \quad (9)$$

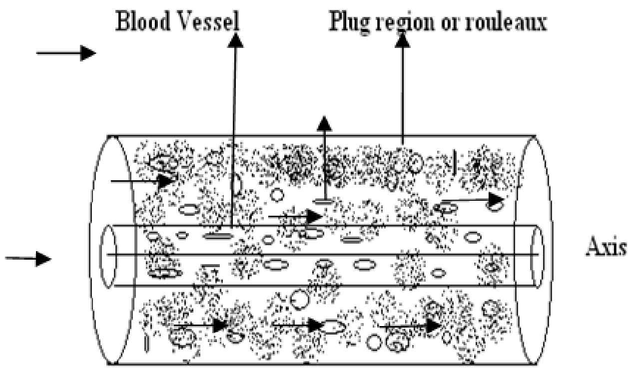

Figure 3. Geometical structure of Herschel Bulkley blood flow model

The Constitutive equation for test part of the blood vessel is given by

$$T^{'} = \eta_m e^n + T_p \; or \; T^{'} - T_p = \eta_m e^n = T_e$$

where, T_e =effective stress, whose generalized form are given below:

$T^{ij} = -Pg^{ij} + T_e^{ij}$ where, $T_e^{ij} = \eta_m \left(e^{ij}\right)^n$ while $e^{ij} = g^{jk} v_k^i$

Now we describe the basic equations for Herschel Bulkley blood flow as follows:

The equation of Continuity is given below:

$$\frac{1}{\sqrt{g\sqrt{\left(gV^i\right)}}_{,i}} = 0 \quad (10)$$

The equation of motion **is:**

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m V^j V_{,j}^i = -T_{e,j}^{ij} \quad (11)$$

Where all the symbols have their usual meanings, since, the blood vessels are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates.

As we know, $X^1 = r, X^2 = \theta, X^3 = z,$ Matrix of metric tensor in cylindrical co-ordinates is $\left[g_{ij}\right]$ and matrix of conjugate metric tensor is $\left[g^{ij}\right]$ whereas the Chritoffel's symbols of 2nd kind are as follows:

$\begin{Bmatrix} 1 \\ 2 \; 2 \end{Bmatrix} = -r, \begin{Bmatrix} 1 \\ 2 \; 2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \; 2 \end{Bmatrix} = \frac{1}{r}$ Remaining others are zero.

The governing tensorial equations can be transformed into cylindrical forms which are as follows:

The equation of continuity are given below

$$\frac{\partial v}{\partial z} = 0$$

The equation of motion- r-component: $-\frac{\partial p}{\partial z} = 0, \Theta$ - component =0 and z-component: $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\left[r\left(\frac{\partial v_z}{\partial r}\right)^n\right]$.

Here, this fact has been taken in view that the blood flow in axially symmetric in arteries concerned, i.e. $v_\theta = 0$ and v_r, v_z and p do not depend upon θ.

We get $v_z = v(r)$ and $p = p(z)$ and

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r}\left[r\left(\frac{dv}{dz}\right)^n\right] \quad (12)$$

And the pressure gradient $-\frac{dp}{dz} = p$

$r\left(\frac{dv}{dz}\right)^n = -\frac{pr^2}{2\eta_m} + A,$ we apply boundary condition: at $r = 0. V = V_0$ then $A = 0$.

Or $-\frac{dv}{dr} = -\left(\frac{p_r}{2\eta_m}\right)^{\frac{1}{n}}$, Replace r from $r - r_p$

$$-\frac{dv}{dr} = \left(\frac{\frac{1}{2}pr - \frac{1}{2}pr_p}{\eta_m}\right)^{\frac{1}{n}} \Rightarrow \frac{dv}{dr} = -\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left(r - r_p\right)^{\frac{1}{n}} \quad (13)$$

Integrating above equation (13) under the no slip boundary condition: $v = 0$ at $r = R$ we get:

$$V = \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[\left(R - r_p\right)^{\frac{1}{n}+1} - \left(r - r_p\right)^{\frac{1}{n}+1}\right] \quad (14)$$

This is the formula for velocity of blood flow in arterioles, veinules and veins. Putting $r = r_p$ to get the velocity v_p of plug flow are given by,

$$V_p = \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\left(R - r_p\right)^{\frac{1}{n}+1} \quad (15)$$

Where the value of r_p is taken from equation (7), we have.

5. Results

Observations : SHARDA HOSPITAL GR. NOIDA

Patient Name:Mr Paritosh Kumar (ID20142290473).

Diagnosis: Dr Mamta Kumari

S No.	Dated	HB	BP	Hematocrit
1	29/12/2014	15.10	110/72	45.30
2	30/12/2014	15.45	113/73	46.35
3	31/12/2014	15.60	117/78	46.8

The flow flux of two phased blood flow in arterioles, veinules and veins is given below:

$$Q = \int_0^{r_p} 2\pi r V_p dr + \int_{r_p}^{R} 2\pi r V dr$$

$$= \int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} \left(R - r_p\right)^{\frac{1}{n}+1} dr +$$

$$\int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} \left[\left(R - r_p\right)^{\frac{1}{n}+1} - \left(r - r_p\right)^{\frac{1}{n}+1}\right] dr$$

Using (13) and (15), we get

$$= \int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} \left(R - r_p\right)^{\frac{1}{n}+1} dr +$$

$$\int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} \left[\left(R - r_p\right)^{\frac{1}{n}+1} - \left(r - r_p\right)^{\frac{1}{n}+1}\right] dr \quad (16)$$

6. Conclusion

A simple survey of the table between blood pressure and Hematocrit in Hepatitis A patient depicts that when Hematocrit increased, Blood pressure is also increased, therefore the Hematocrit is proportional to the blood pressure. If this would have been possible to get blood Pressure on the particular tissue (Liver) during operation of Hepatitis A patient then the relation between blood pressure and hemoglobin has been measured more accurately. Infection with hepatitis A virus can cause an unpleasant illness, but most people fully recover. Some people have no symptoms at all. Symptoms include flu-like symptoms and yellowing of your skin and the whites of your eyes (jaundice) which then gradually clear without treatment. Serious problems are very rare.

Acknowledgement

We are thankful and sincere to Dr Mamta Kumari Sharada Hospital Greater Noida UP India for the data are provided to me. Also, the corresponding authors are thankful to the learned referees for their fruitful suggestions for improving the presentation of this work.

References

[1] Berkow, Robert, ed. Merck Manual of Medical Information. Whiehouse Station, NJ: Merck Research Laboratories, 1997.

[2] **A**.K. Berry, A.S.kapoor, R. Nagabhushanam, Animal phisiology, 1981.

[3] Whynne HA, Cope L.H. Mutch E at al. The effect of age upon liver, 9, 297-301, 1989.

[4] Cirrhosis overview National Digestive Information Clearing house, Retrieved on 01-22, 2010.

[5] Dienstag JL, Hepatitis vairus infection, N Engel j med. 359; 1486-1500, 2008.

[6] Perrillo R. Hepatitis A and D. In Feldman M, Friedman L.S, Brandt L.J, eds Sleisenger and Fortran Gastrointestional and liver diesses 9th ed. Philadelphia, Pa: Saunders Elsevier, Chap 18, 2010.

[7] Sorrel MF, Belongia EA, Costa J, Gareen IF, Inadomi JM,et al, National Institutes OF Health consensus Development conference statement, Management of Hepatitis A, ann Intern Med. 150: 104-10; 2009.

[8] V. Upadhyay; some phenomena in two phase blood flow; 2000.

[9] Guyton, A.C.: Medical Physiology; W.B.S. 207, 1981.

[10] Mishra R.S. Tensors and Riemannian Geometry, Pothishala Pvt. Ltd. Allahabad, 1990

[11] Glenn Elert. : The Physics Hypertext book. By Glenn Elert". Hypertextbook.com. Retrieved, 09-14, 2010.

[12] Singh P. and Upadhyay K.S.; a new approach for the shock propogation in the two phase system; NAT. acad. Sc.; Letters, vol 8, No 2, 1986.

[13] Kapur, J.N. and Gupta, R.C; Power law fluid flow in the inlet length of a circular pipe; the math, seminar; 3, 55-67; 1963.

[14] Ruch, T.C and H.D.; patton (ends); physiology and bio-physics-vols (II) and (III) W.B.S., 1973.

[15] Pinson AG, Philbricr JT, Lindbeck GH; Schorling JB; ED manarbment of actute pyelonephrits in women; 1994.

[16] Dientag, J.L. Hepatitis A virus infection, N. Engle J Med, 359 (14) 2008.

[17] Anil Kumar Gupta: Performance and analysis of blood flow through carotid artery, International Journal of Engineering and Business Management, ISSN: 1847-9790, 2011, Vol. 3, No. 4, 1-6, ISSN: 1847-9790, Croatia

[18] Anil Kumar C L Varshney and VP Singh: Performance study on effect of cognitive states on Heart Rate, blood pressure and respiration rate Indian Journal of Biomechanics: Special Issue (NCBM 7-8 March 2009), pp 8-12, 2009.