# Progressive Review and Analytical Approach for Optimal Solution of Stochastic Transportation Problems (STP) Involving Multi-Choice Cost

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**Abstract** In this paper some general transportation models have been discussed and particularly a multi-choice cost stochastic transportation problem (STP) has been reviewed in the light of progressive research works of previous noteworthy researchers. In addition, an analytical approach for the optimal solution (OS) of the proposed stochastic transportation problem has been demonstrated. The analytical method proposed by us is not only heuristic but also a generalization in threefold. We remark here that some unnecessary complications involved previously have been removed in our proposed method. Finally, by way of demonstrating a numerical illustration some significant conclusive observations have also been drawn in order to highlight the threefold feature.

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## 1. Introduction

In our practical life, we come across many business and production industries which have to usually face problems of economic optimization such as cost minimization of non‑economic items that are most vital to the existence of their firms. The transportation models are one of these economic optimizations which have their roots in operational management and industrial mathematics as well since long back 1941. However, the transportation models have applications not only in limited areas of production industries but also worldwide applications in communication networks, planning and scheduling, we refer Ford and Fulkerson [11] and Fulkerson [12], Garvin [14], and Henderson and Schlaifer [19]. The transportation problem received this name because many of its applications involve in determining how to optimally transport goods. In various problems of economic optimization, the transportation problem is a logistical problem for organizations especially for manufacturing and transport companies. The different methods used in solving different versions of transportation models pay a key role in decision-making and process of allocating

problem in these organizations. For different conventional and modern optimization techniques applied in different dimensions of O.R. models including queueing and machine repair models, we refer recent research works explored by Mahapatra *et al.* [28], Maurya and Maurya [29], Maurya and Garg [30], Maurya *et al.* [31], Maurya [32-38] and Neralić [40] and relevant references therein.

## 2. Literature Review

Transportation model is one of the earliest and most important applications of linear programming problem. Description of a classical transportation problem can be given as follows. The transportation models or problems are primarily concerned with the best possible way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses or customers (called demand destinations). The objective of the classical transportation problem is to determine the optimum shipping schedule which minimizes the total shipping cost with satisfying both supply and demand limits. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of

distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales. Usually the transportation problems may be classified in two categories. The first one is deterministic transportation problem (DTP) and the second is stochastic transportation problem (STP). The classical transportation problem is the basic transportation model lying in the category of deterministic transportation problem (DTP) which is also referred to as a special case of Linear Programming Problem (LPP) and its model is applied to determine an optimal solution of delivery available amount of satisfied demand in which the total transportation cost is minimized. In this connection, for more details we refer to some noteworthy research contributors Charnes *et al.* [6], Cortez [7], Garvin [13], Gass [15], Sharma [46] and Taha [49].

Here, we confess that there is a vast literature on deterministic transportation models with many dimensions. In 1941 Hitchcock [20] developed the basic transportation problem along with the constructive method of solution and later in 1947-1949 Koopmans [24,25] discussed the problem in a more comprehensive manner. Again in 1956 Dantzig [8,9] formulated the transportation problem as linear programming problem and also provided the solution method. Now a day's applications of different versions of transportation problems have acquired prominent place in industrial organizations having several manufacturing units, warehouses and distribution centers.

As suggested also in rigorous study of Maurya & Garg [30] that the best known and the most widely used method for solving the assignment problem; a special case of transportation problem is the 'Hungarian Method'. Previously suggested by Kuhn [26] in 1955, it has appeared in many variants (e.g., [9,15,27,44]). Moreover, the production transportation problem (PTP); a version of classical transportation problem is one of the very important problems in the continuous production industries such as petroleum industry. It deals with the problem of how to plan production and transportation in such an industry given several plants at different locations and large number of customers of their products. The PTP problem has been addressed previously in the literature e.g. Grobatenko and Suvorov [18], Hunjet *et al.* [21], Ivshin [22], Neralić [40,41] and can be formulated as a linear programming problem.

For obtaining an optimal solution for transportation problems it was required to solve the problem into two stages. In first stage the initial basic feasible solution (IBFS) was obtained by opting any of the available methods such as 'North West Corner', 'Matrix Minima', 'Least Cost Method', 'Row Minima', 'Column Minima' and 'Vogel's Approximation Method' etc. Then in the next and last stage MODI (Modified Distribution) method was adopted to get an optimal solution. Charnes and Cooper [5] also developed a method for finding an optimal solution from IBFS named as 'Stepping Stone Method'. In subsequent research works, Gleyzal [16] succeeded to explore an algorithm for solving the transportation problem. The well-known 'travelling salesman problem' was analyzed by Flood [10] which was also solved by graph theory subsequently. Beale [4] proposed an algorithm for solving the transportation problem when the shipping cost over each route is convex. Munkres and James [39] developed algorithms for the

assignment and transportation problems and in 1970 Shore [47] also contributed on transportation problem and Vogel's approximation method. Goyal [17] proposed an improved Vogel's approximation method for an unbalanced transportation problem. Later in 1990, Kirca and Stair [23] developed a heuristic method to obtain an efficient initial basic feasible solution.

For recent bibliographies and survey we can refer Ping Ji [43], Maurya and Garg [30], Pandian *et al.* *[42]*, Samuel and Venkatachalapathy [45], Sudhakar *et al.* *[46]*, Mahapatra *et al.* [28]. We further introduce here that Ping Ji [43] performed a dual matrix approach for solving transportation models. Maurya and Garg [30] demonstrated an alternative approach for finding an optimum assignment schedule; a special type of transportation problem. However, Pandian *et al.* *[42]* and Sudhakar *et al.* *[46]* proposed two different methods in 2010 and 2012 respectively for finding an optimal solution directly. It is added with further remarks that Sudhakar et al. [46] planned zero suffix method for finding an optimal solution for transportation problem directly in 2012. Recently, Samuel and Venkatachalapathy [45] applied a modified version of Vogel's approximation method for solving fuzzy transportation problems.

Furthermore, since the simplex method is an iterative algebraic procedure for solving linear programming problems-transportation models. Therefore, keeping in view this feature of the simplex method a few noteworthy previous researchers e.g. Arsham and Khan [1], Arsham [2], Balinski and Gomory [3] among the others confined their attention in this direction and developed optimal solution for deterministic transportation models.

However, as we have revealed earlier that many more research works can be found in the literature both for initial basic feasible solution (IBFS) and optimum solution (OS) dealing with deterministic transportation models yet as far as the method for optimal solution of stochastic transportation models is concerned, we find comparatively a very less previous contributors. In this connection, a very recent research work of Mahapatra *et al.* [28] is mentioning here. We further remark here that Mahapatra *et al.* [28] confined their attention to analyze a multi-choice stochastic transportation problem involving extreme value distribution and succeeded to propose its optimal solution using non-linear mixed integer programming technique.

In the present paper, we confess that the approach applied by Mahapatra et al. [28] is neither heuristic nor an efficient method. Unnecessary time consuming and considerably complicated approach has been presented by them. These unusual features of their proposed sophisticated method motivates us to further explore some better heuristic approach which may be significantly more useful for the management professionals, researchers and scientists of industrial organizations. It is therefore, here our main concern to generalize the previous research work carried out by Mahapatra et al. [28] in following threefold feature:

- The complicated transformation for the objective functions of seven cases has been generalized to finite range.
- An unnecessary multi-choice structure for the transportation cost is avoided.

- An analytical and comparatively more heuristic method as well is suggested to find an optimum solution of the transportation problem taken into consideration by Mahapatra et al. [28].

# 3. Preliminary Ideas and Methodologies Used

In this section, the multi-choice stochastic transportation problem (STP) dealt by Mahapatra *et al.* [28] has been presented to further rectify and solve it by a better heuristic method. Here, we confess that the approach applied by Mahapatra *et al.* [28] is neither heuristic nor an efficient method rather to be complicated too much. By way of demonstrating an analytical method we shall achieve an optimum solution (OS) of the multi choice stochastic transportation problem which will also ensure its threefold generalization as mentioned above. With passing remarks here, it should be specifically asserted that the proposed method making the aforementioned threefold generalization will be definitely advantageous for the researchers, management professionals and scientists of production industries in order to explore transportation models in the future.

We observed in recent studies of Mahapatra *et al.* [28] that both the supplies and demands follow the extreme value distribution such that they convert the probability constraints into deterministic constraints. We also note here that we consider the most complicated model out of three different models studied by Mahapatra *et al.* [28] which includes both supplies and demands follow the extreme value distribution. In addition to this, we assert that our proposed analytical approach can also be straightforwardly applied to find the optimum solution for their other two simple models where either supplies or demands that follow the extreme value distribution.

The multi-choice cost stochastic transportation model studied by Mahapatra *et al.* [28] is expressed as following:

$$Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{C_{ij}^{1}, C_{ij}^{2},..., C_{ij}^{k}\right\}x_{ij}, k=1,2,...,K \quad (3.1)$$

subject to constraints:

$$\sum_{i=1}^{m} x_{ij} \geq \alpha_{j}' - \beta_{j}'\left(\ln\left[-\ln\left(1-\delta_{j}\right)\right]\right), i=1,2,...,m \quad (3.2)$$

$$\sum_{j=1}^{n} x_{ij} \leq \alpha_{i} - \beta_{i}\left(\ln\left[-\ln\left(\gamma_{i}\right)\right]\right), j=1,2,...,n \quad (3.3)$$

$$\sum_{i=1}^{m}\alpha_{i} - \beta_{i}\left(\ln\left[-\ln\left(\gamma_{i}\right)\right]\right)$$
$$\geq \sum_{j=1}^{n}\alpha_{j}' - \beta_{j}'\left(\ln\left[-\ln\left(1-\delta_{j}\right)\right]\right) \quad (3.4)$$

with non-negativity constraints:

$$x_{ij} \geq 0, i=1,2,...,m\ and\ j=1,2,...,n \quad (3.5)$$

where $a_{i}$ and $b_{j}$ are two random variables representing respectively the supplies available to $i^{\text{th}}$ source and demands at $j^{\text{th}}$ destination and hence $a_{i}$ and $b_{j}$ satisfy the extreme value distribution, with location factor $\alpha_{i}$, $\alpha_{j}'$ and scale factor $\beta_{i}$, $\beta_{j}'$, respectively, with

$$Pr\left(\sum_{j=1}^{n}x_{ij} \leq a_{i}\right) \geq 1-\gamma_{i} \quad \text{and} \quad Pr\left(\sum_{i=1}^{m}x_{ij} \geq b_{j}\right) \geq 1-\delta_{j} \ ;$$
$$i=1,2,...,m, \text{ and } j=1,2,...,n.$$

# 4. Equivalent Transformation of the Objective Function

We remark here that a sufficient long procedure has been applied by Mahapatra *et al.* [28] in order to convert the objective function as expressed in (2.1). The complexity in their procedure can be reviewed by considering the Case 6, when $K=7$. In order to convert the following objective function to an equivalent form in Case 6, when $K=7$, Mahapatra *et al.* [28] expressed it as following:

$$Min \qquad z = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{C_{ij}^{1}, C_{ij}^{2},..., C_{ij}^{7}\right\}x_{ij} \quad (4.1)$$

to the following two different models,
*Case: Model 6(i)*

$$Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n}\begin{bmatrix} C_{ij}^{1}\left(1-z_{ij}^{1}\right)\left(1-z_{ij}^{2}\right)\left(1-z_{ij}^{3}\right) \\ +C_{ij}^{2}z_{ij}^{1}\left(1-z_{ij}^{2}\right)\left(1-z_{ij}^{3}\right) \\ +C_{ij}^{3}\left(1-z_{ij}^{1}\right)z_{ij}^{2}\left(1-z_{ij}^{3}\right) \\ +C_{ij}^{4}\left(1-z_{ij}^{1}\right)\left(1-z_{ij}^{2}\right)z_{ij}^{3} \\ +C_{ij}^{5}z_{ij}^{1}z_{ij}^{2}\left(1-z_{ij}^{3}\right) \\ +C_{ij}^{6}z_{ij}^{1}\left(1-z_{ij}^{2}\right)z_{ij}^{3} + C_{ij}^{7}\left(1-z_{ij}^{1}\right) \end{bmatrix}z_{ij}^{2}z_{ij}^{3} \quad (4.2)$$

$$z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} \leq 2, \quad (4.3)$$

$$z_{ij}^{p} = 0/1; p=1,2,3; \forall i\ and\ j. \quad (4.4)$$

*Case: Model 6(ii)*

$$Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n}\begin{bmatrix} C_{ij}^{1}z_{ij}^{1}\left(1-z_{ij}^{2}\right)\left(1-z_{ij}^{3}\right) \\ +C_{ij}^{2}z_{ij}^{2}\left(1-z_{ij}^{1}\right)\left(1-z_{ij}^{3}\right) \\ +C_{ij}^{3}z_{ij}^{3}\left(1-z_{ij}^{1}\right)\left(1-z_{ij}^{2}\right) \\ +C_{ij}^{4}z_{ij}^{1}z_{ij}^{2}\left(1-z_{ij}^{3}\right) + C_{ij}^{5}z_{ij}^{1}z_{ij}^{3}\left(1-z_{ij}^{2}\right) \\ +C_{ij}^{6}z_{ij}^{2}z_{ij}^{3}\left(1-z_{ij}^{1}\right) + C_{ij}^{7}z_{ij}^{1}z_{ij}^{2}z_{ij}^{3} \end{bmatrix} \quad (4.5)$$

$$z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} \geq 1, \quad (4.6)$$

$$z_{ij}^{p} = 0/1; p=1,2,3; \forall i\ and\ j. \quad (4.7)$$

# 5. Reasonable Explanation and Generalization of the Model

In this section, our major concern is to propose a generalized heuristic approach for transformation of the objective function. Before we present our generalization for their objective functions of Cases 1-7, our endeavor is to first make available a reasonable explanation for the equation explored in (4.2) and inequality constraint expressed in (4.3) for the model 6 (i). Here, to serve our present purpose for the aforementioned reasonable explanation, we add a dummy transportation cost $C_{ij}^8$ to the objective function expressed in equation (4.2) with the coefficient as $C_{ij}^8 z_{ij}^1 z_{ij}^2 z_{ij}^3$. Since the coefficient of $z_{ij}^1 z_{ij}^2 z_{ij}^3$ required always be zero in the light of derivation explored by Mahapatra *et al.* [28]. This might be the reason why they did not consider that $z_{ij}^1 = 1 = z_{ij}^2 = z_{ij}^3$ resulting $z_{ij}^1 + z_{ij}^2 + z_{ij}^3 = 3$ which violates the inequality constraint (4.3). Consequently, Mahapatra *et al.* [28] improved the original condition $0 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 3$ to the desired restriction $0 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 2$ as expressed in (4.3).

Now we search for providing a reasonable explanation for the equation explored in (4.5) and an inequality constraint expressed in (4.6) for the model 6 (ii). Proceeding in a similar way as in model 6 (i), we consider the dummy transportation cost $C_{ij}^8$ with the coefficient as $C_{ij}^8 \left(1 - z_{ij}^1\right)\left(1 - z_{ij}^2\right)\left(1 - z_{ij}^3\right)$. As per their derivation explored by Mahapatra *et al.* [28], they needed the coefficient of $\left(1 - z_{ij}^1\right)\left(1 - z_{ij}^2\right)\left(1 - z_{ij}^3\right)$ always be equal to

zero. Therefore, this might be the reason why they did not assume $z_{ij}^1 = 0 = z_{ij}^2 = z_{ij}^3$ in order to avoid ruled out the equation $z_{ij}^1 + z_{ij}^2 + z_{ij}^3 = 0$. As a consequence, Mahapatra *et al.* [28] revised the original condition $0 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 3$ to meet out the desired restriction $1 \le z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \le 3$ so that they could be succeed to derive their result of $z_{ij}^1 + z_{ij}^2 + z_{ij}^3 \ge 1$.

Moreover, in order to present our generalization for their objective functions of Cases 1-7 we assume that

$$Min\ z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(b_{ij}^1 C_{ij}^1 + b_{ij}^2 C_{ij}^2 + ... + b_{ij}^K C_{ij}^K\right) x_{ij} \quad (5.1)$$

Subject to constraints:

$$b_{ij}^p = 0/1;\ p = 1, 2, 3..., K \quad (5.2)$$

With usual condition of total probability that

$$\sum_{p=1}^{K} b_{ij}^p = 1 \quad (5.3)$$

Here, we further remark that our present approach is to generalize the special case with $K \le 8$ to any finite range of $p$.

## 6. Numerical Illustration and Redundancy of Multi-Choice Cost

**Table 1. Multi-choice transportation cost for route $x_{ij}$ and optimal $C_{ij}^k$**

| Sl. No. | Route: $x_{ij}$ | Transportation cost $C_{ij}^k$ | Optimal solution by [28] | |
|---|---|---|---|---|
| | | | $C_{ij}^k$ | $x_{ij}$ |
| 1 | (1,1): $x_{11}$ | 10 or 11 or 12 | 10 | 615.8808 |
| 2 | (1,2): $x_{12}$ | 15 or 16 | 15 | 0 |
| 3 | (1,3): $x_{13}$ | 21 or 22 or 23 or 24 | 21 | 0 |
| 4 | (1,4): $x_{14}$ | 21 or 23 or 25 | 23 | 0 |
| 5 | (2,1): $x_{11}$ | 15 or 17 or 19 or 21 or 23 or 25 | 15 | 0 |
| 6 | (2,2): $x_{22}$ | 10 or 12 or 14 or 16 or 18 or 20 | 10 | 382.1037 |
| 7 | (2,3): $x_{23}$ | 9 or 10 or 11 | 9 | 408.3479 |
| 8 | (2,4): $x_{24}$ | 18 or 19 | 18 | 0 |
| 9 | (3,1): $x_{31}$ | 20 or 21 or 22 or 23 or 24 or 25 or 26 | 22 | 0 |
| 10 | (3,2): $x_{32}$ | 10 or 11 or 12 or 13 or 14 or 15 or 16 or 17 | 10 | 129.7771 |
| 11 | (3,3): $x_{33}$ | 20 or 22 or 25 | 22 | 0 |
| 12 | (3,4): $x_{34}$ | 15 or 20 | 15 | 305.2464 |

This section deals with demonstration of a numerical example to explore that the multi-choice of transportation cost considered by Mahapatra *et al.* [28] is redundant. Here, in order to manifest our assertion for existence of redundancy of multi-choice cost, we consider the same example already discussed by Mahapatra *et al.* [28] and we reproduce their Table 1 for transportation cost from three supplies to four demands. In addition to this, we also list their findings of $C_{ij}^k$ and $x_{ij}$ in our Table 1 for further comparative analysis. From findings of Mahapatra *et al.* [28] in the Table 1, we observe that three values of transportation cost specified by $C_{14} = 23$, $C_{31} = 22$ and

$C_{33} = 22$ are not the lowest cost in $C_{ij}^k$, since $\min C_{14}^k = 21$, $\min C_{31}^k = 20$ and $\min C_{33}^k = 20$. It may demonstrate that their approach is reasonable to find the optimal solution under multi-choice transportation cost environment. However, our next section will manifestly demonstrate their unnecessary their approach for multi-choice transportation cost.

## 7. Discussions for Multi-Choice Transportation Cost

In view of equations (4.2) or (4.5) for $K = 7$, and also as per our generalization of equation (5.1), where $z_{ij}^p$, $p = 1, 2, 3$ or $b_{ij}^k$, $k = 1, 2, ..., K$ are decision variables for the minimization problem of stochastic transportation model, we can directly find the least value among $C_{ij}^k$. In the Table 1 of our numerical illustration, we should directly take the least value for $x_{32}$ as following:

$$\min C_{32}^k = \min \{10, 11, 12, 13, 14, 15, 16, 17\} = 10 \quad (7.1)$$

Here, we further remark that our above assertion of redundancy of multi-choice transportation cost is violated by the optimal solution explored by Mahapatra *et al.* [28] with $C_{14} = 23$, $C_{31} = 22$ and $C_{33} = 22$. However, it may be critically observed that the values of $x_{14} = x_{31} = x_{33} = 0$ in their optimal solution resulting the outcomes of $C_{14}$, $C_{31}$ and $C_{33}$ will not influence the minimum transportation cost. Recalling the stochastic transportation model taken into consideration of Mahapatra *et al.* [28] is a problem of its minimization type and under assumption that if the decision maker is allowed to select the transportation cost from a supply (source) to a demand (destination) then the smallest cost will be the most favorable choice. As a result, keeping in view of these facts we proposed to directly take the least value for the transportation cost to dramatically simplify the complicated multi-cost structure model of Mahapatra *et al.* [28] in the next section.

# 8. Present Heuristic Approach

In this section, a little attempt has been made to demonstrate our simplified approach by way of solving the same minimization problem of the transportation model dealt in Mahapatra *et al.* [28]. Its successive procedure includes as following:

$$Min \; z = 10x_{11} + 15x_{12} + 21x_{13} + 21x_{14} + 15x_{21} + 10x_{22}$$
$$+9x_{23} + 18x_{24} + 20x_{31} + 10x_{32} + 20x_{33} + 15x_{34} \quad (8.1)$$

Subject to constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 987.782536 \quad (8.2)$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 790.4516176 \quad (8.3)$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 692.4721905 \quad (8.4)$$

$$x_{11} + x_{21} + x_{31} \geq 651.880781 \quad (8.5)$$

$$x_{12} + x_{22} + x_{32} \geq 511.880781 \quad (8.6)$$

$$x_{13} + x_{23} + x_{33} \geq 408.3478976 \quad (8.7)$$

$$x_{14} + x_{24} + x_{34} \geq 305.2463882 \quad (8.8)$$

where upper bounds for supplies and lower bounds for demands have been directly expressed in studies of Mahapatra *et al.* [28].

In the following, a comparative transportation cost with the same demand (destination) to rule out those expansive suppliers (sources) has been demonstrated. We compare the cost to find that the maximum of them is 21. To achieve the optimal solution for the transportation model with minimization type problem, we assumed here that $x_{13} = 0$ and $x_{14} = 0$.

Under the condition expressed by equation (8.7) and assumption $x_{13} = 0$, we compare the unit cost for $x_{23}$ and $x_{33}$ to find that $9 < 20$ such that we derive that $x_{33} = 0$ and $x_{23} = 408.3478976$.

Further, based on equation (8.8) and unit costs of $x_{24}$ and $x_{34}$ with $15 < 18$, we find that $x_{24} = 0$ and $x_{34} = 305.2463882$.

Similarly, based on equation (8.5), and unit costs of $x_{11}$, $x_{21}$ and $x_{31}$ with $10 < 15 < 20$, we may easily get an implication that $x_{21} = 0$, $x_{31} = 0$ and $x_{11} = 651.880781$.

Now we publicize our findings to equations (8.3)-(8.4) to convert them as expressed respectively below:

$$x_{22} \leq 382.1037200 \quad (8.9)$$

$$x_{32} \leq 387.2258023 \quad (8.10)$$

Moreover, based on equation (8.6), and unit costs of $x_{12}$, $x_{22}$ and $x_{32}$ with $10 = 10 < 15$, we imply that $x_{12} = 0$ and

$$x_{22} + x_{32} = 511.880781 \quad (8.11)$$

Finally, we remark that by making use of analytical approach we succeeded to explore an optimum solution rather to refer any sophisticated programming package. In addition to above remark, we also propose that the analytical approach demonstrated herein is a generalization. In support of our proposed analytical approach for its generalized technique, a little fact is sufficient to present that the finding of Mahapatra *et al.* [28] with $x_{22} = 382.1037$ and $x_{32} = 129.7771$ is a special result of our findings of equations (8.9)-(8.11).

# 9. Observational Conclusions

In this paper, a multi-choice stochastic transportation model has been discussed for providing an analytical approach to explore the optimal solution rather to select any sophisticated programming package. More specifically, after critical review of the multi-choice stochastic transportation problem (STP) studied by Mahapatra *et al.* [28], an analytical approach for its optimal solution is proposed which includes its following fourfold feature:

i. The approach applied by Mahapatra *et al.* [28] for finding an optimal solution is time consuming and as well as an inefficient.

ii. The proposed analytical approach is simpler than the Mahapatra *et al.* [28].

iii. The proposed analytical approach is an efficient and time saving.

iv. The proposed analytical approach does not refer any sophisticated programming package.

v. The findings of Mahapatra *et al.* [28] are special case of our results explored herein.

However, the derivation process proposed by Mahapatra *et al.* [28] may be fascinating to use the

minimum number for the possible parameter of the coefficient yet the character for the multi-choice is that only one cost will be adopted for the objective function such that the derivation of a compact expression with the minimum number of parameters is not important. Instead of this, their longer and complicated transformation will detour the attention for usual practitioners to focus on that there is only one cost will be adopted. Finally, with passing above remarks we further conclude that the proposed analytical technique for solving stochastic transportation problem (STP) will be categorically useful for the researchers, management professionals and scientists of manufacturing industries dealing with transportation models in the future.

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