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Mathematical Modeling for Performance Analysis and Inference of k-out of-n Repairable System Integrating Human Error and System Failure

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Abstract Present paper demonstrates mathematical modeling and evaluation of performance measures of k-out of-n repairable system wherein the most influencing constraints of human error and common-cause failure have been taken into consideration. Firstly the mathematical modeling is developed for performance analysis of k-out of-n repairable system with standby units involving human and common-cause failure. Then, a successful attempt has been made to evaluate various important performance measures such as availability of system, steady state availability and mean time of system failure (MTSF), mean operational time(MOT), expected busy period (EBP) and steady state busy period etc. Using the supplementary variable technique, Laplace transforms of various state probabilities are explored. Moreover, a particular case when repair rate follows exponential distribution has also been discussed. In addition, numerical illustration has also been presented in order to enable a better mode for understanding and testing the outcomes explored herein. Finally, tables and graphs for investigated results are displayed for drawing some significant conclusive observations for testing their validity and consistency.

Keywords: *Mathematical modeling, repairable system, human error, common-cause failure, steady state availability, Laplace transform, supplementary variable technique, performance analysis*

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1. Introduction

Literature shows that performance analysis of networks with queues has received considerable attention by a large number of previous noteworthy researchers and it has occupied a prominent place in Operational Research in recent years. Repairable system incorporating human error and system failure is a particular type of queueing system. In series systems, the failures of one or more units result in the system failure. However, there exist systems that are not considered failed until at least k units or components have failed. Such systems are known as k-out of-n systems. Examples of such systems are: large airplanes usually have three or four engines, but two engines may be the minimum number required to provide a safe journey. Similarly, in many power-generation systems that have two or more generators, one generator may be sufficient to provide the power requirements. Performance analysis of wide range of queueing models in different frame works has been confined by several noteworthy researchers, (e.g. [3,4,5,6,7,8,11,13]) and references therein. Among them, recently Maurya [7,8] paid keen attention concerning with performance analysis of $M^X/(G_1,G_2)/1$ queueing model with second phase optional service and Bernoulli vacation schedule and succeeded to investigate significant performance measures.

It is relevant to mention here that mathematical modeling pay a key role for performance analysis of system repair problems like other mathematical techniques. In this direction, some previous researchers (e.g. [3,9,10,16]) and references therein are worth mentioning. Although, reliability and/or performance analysis of system repair problems have been attempted by some previous researchers, (e.g. [2,14,15,16,17]) but human error and common-cause failure have not been yet taken into consideration in their studies. However, it has been keenly observed that human error and common-cause failure are very much influencing constraints in performance and/or reliability analysis of system repair problems. Few earlier research workers confined the reliability behavior of k-out of-n systems taking into account the common-cause failure, for example, Hughes (1987) presented a new approach to common-cause failure in analyzing system repair problems and Who Kee Chang (1987) attempted for reliability analysis of a repairable parallel system with standby involving human failure and common-cause failures yet no attention has been paid to the performance evaluation of k-out of-n repairable system due to human error and system failure constraints. Jain et al (2002) studied a k-out of-n system with dependent failure and standby support and Moustafa (1997) also studied k-out of-n repairable system with dependent failure and imperfect coverage.

In this paper, our main objective is to investigate performance measures and valuable inferences of k-out of-n repairable system with standby units involving human and common-cause failure by using mathematical modeling, particularly to evaluate availability, steady state availability, mean time of system failure (MTSF) etc. The supplementary variable technique has been used here to evaluate Laplace transforms of various state probabilities of the repair model. Numerical computations and graphs for availability and MTSF have also been drawn. Following three particular cases are also developed:

(i). The case when there is repair due to hardware, human and common-cause failures
(ii). The case when there is repair due to hardware and human failures
(iii). The case with no repair

2. Hypotheses of the System

Here, we consider k-out of-n repairable system with standby units involving human and common-cause failure. Following are some assumptions taken into our present consideration for the repair model.

(i). The system consists of n identical main units and s standby units

(ii). Unit failure, human error and common-cause failure are constant

(iii). Repair rate from failed states due to unit failure, human error and common-cause failure are generally distributed

(iv). Initially n units are operating and s unit are kept as cold standby

(v). The entire system working if at least k out of n units or components are operating

(vi). The system is said to be in one of the failed if k+1 unit have failed due to unit failure, human error and common-cause failure

(vii). When any of the operating units fails, it is replaced immediately by standby unit

(viii). If all the standbys are consumed, the system works as degraded system until k-units works

(ix). No repair will be undertaken until the system has failed due to hardware and human error (i.e. until k+1 unit have failed) or due to common-cause failure

(x). We assume that a repaired system is as good as new

(xi). A perfect switch is used to switch-on the standby units and switch-over time is negligible

3. Notations and Description of the System

In order to analyze our present repair model in the direction as indicated earlier, we use underlying notations;

$P_{i,j}(t)$ Probability that the system is in state (i, j) at time t. State (i, j) is the state of the system when i units failed due to hardware failure and j units due to human error, i, j = 0, 1, 2, …, k+1. State (0, 0) is the initial state at t = 0 and states (i, k+ 1 –i, i = 0, 1,……., k +1) are the failed states of the system. State cc is when the system has failed due to common-cause failure.

λ constant hardware failure rate of a unit.

h constant human error rate of a unit.

λ_c common-cause failure rate

Λ= n λ, H= n h

ρ repair rate when the system is in state (i, j)

ρ_c repair rate when the system is in state cc

$P_{i,j}(y,t)$ Probability density function that the failed system is in state (i, j) and has an elapsed repair time of *y* at time t; j = k +1 –i, i= 0, 1, 2, ……….k +1

$P_{cc}(y,t)$ Probability density function that the failed system is in state cc and has an elapsed repair time of y at time t.

s. Laplace transform variable, $\bar{F}(s)$ Laplace transform of the probability density function f(.)

4. Evaluation of State Probabilities of the System

In view of our assumptions and notations as in sections 2-3, the set of governing differential equations for the model can be expressed as following;

$$\begin{aligned}(\frac{d}{dt}+\Lambda+H+\lambda_c)P_{0,,0}(t) &= \sum_{i=0}^{k+1}\int_0^\infty \rho(y)P_{i,k+1-i}(y,t)dy \\ &+\int_0^\infty \rho_c(y)P_{cc}(y,t)dy\end{aligned} \quad (4.1)$$

$$(\frac{d}{dt}+\Lambda+H+\lambda_c)P_{i,j}(t)=\Lambda P_{i-1,j}(t)+HP_{i,j-1}(t) \quad (4.2)$$

$$i,j=0,1,2,...k, \quad 1\le i,j\le k$$

$$(\frac{d}{dy}+\frac{d}{dt}+\rho(y))P_{i,k+1-i}(y,t)=0 \quad (4.3)$$

$$(\frac{d}{dy}+\frac{d}{dt}+\rho_c(y))P_{cc}(y,t)=0 \quad (4.4)$$

4.1. Boundary Conditions

$$P_{i,k+1-i}(0,t)=\Lambda P_{i-1,j}(t)+HP_{i,j-1}(t);\ i=0,1,2,...k+1 \quad (4.5)$$

$$P_{cc}(0,t)=\lambda_c\sum_{i=0}^{k}\sum_{j=0}^{k-i}P_{i,j}(t) \quad (4.6)$$

$$\sum_{i=0}^{k+1}\sum_{j=0}^{k+1-i}P_{i,j}(t)+P_{cc}(y,t)=1 \quad (4.7)$$

4.2. Initial Conditions

$$P_{0,,0}(t)=1,\ P_{i,j}(t)=0,\ \ i,j=0,1,2,....................k+1 \quad (4.8)$$

5. Solution of the Mathematical Model

By taking the Laplace transform of equations (4.1)-(4.7), one may get a set of following integral and differential equations:

$$\begin{aligned}&(s+\Lambda+H+\lambda_c)\bar{P}_{0,,0}(s) \\ &=\sum_{i=0}^{k+1}\int_0^\infty \rho(y)\bar{P}_{i,k+1-i}(y,s)dy+\int_0^\infty \rho_c(y)\bar{P}_{cc}(y,s)dy\end{aligned} \quad (5.1)$$

$$\bar{P}_{i,L-i}(s) = \binom{L}{i}\Lambda^i H^{k-i}\bar{P}_{0,0}(s)/A^L, \quad i = 0,1,2,.....................L, \;\; L = 1,2,............k \qquad (5.2)$$

$$\left(\frac{d}{dy} + s + \rho(y)\right)\bar{P}_{i,k+1-i}(y,s) = 0 \qquad (5.3)$$

$$\left(\frac{d}{dy} + s + \rho_c(y)\right)\bar{P}_{cc}(y,s) = 0 \qquad (5.4)$$

5.1. Boundary Conditions

$$P_{i,k+1-i}(0,s) = \Lambda\bar{P}_{i-1,k+1-i}(s) + H\bar{P}_{i,k-i}(s); \quad i = 0,1,2,...k+1 \qquad (5.5)$$

$$\bar{P}_{cc}(0,s) = \lambda_c \sum_{i=0}^{k}\sum_{j=0}^{k-i}\bar{P}_{i,j}(s) \qquad (5.6)$$

$$\sum_{i=0}^{k+1}\sum_{j=0}^{k+1-i}P_{i,j}(t) + P_{cc}(y,t) = 1 \qquad (5.7)$$

By integrating equations (5.3), (5.4), we get:

$$\bar{P}_{i,k+1-i}(y,s) = \bar{P}_{i,k+1-i}(0,s)\exp(-sy - \int\rho(y)dy \qquad (5.8)$$

$$\bar{P}_{cc}(y,s) = \bar{P}_{cc}(0,s)\exp(-sy - \int\rho_c(y)dy \qquad (5.9)$$

Again integrating equations (5.8), (5.9), using equations (5.5)-(5.7), we get

$$P_{i,k+1-i}(s) = s^{-1}[1-\bar{F}(s)]\binom{k+1}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^k; \quad i = 0,1,2,......k+1 \qquad (5.10)$$

$$\bar{P}_{cc}(s) = \lambda_c s^{-1}[1-\bar{G}(s)]\sum_{L=1}^{k}\sum_{i=0}^{L}\binom{L}{i}\Lambda^i H^{k-i}\bar{P}_{0,0}(s)/A^L \qquad (5.11)$$

Using equations (5.5)-(5.7), we have following equations also from equations (5.8)-(5.9);

$$\begin{aligned}&\int\rho(y)\bar{P}_{i,k+1-i}(y,s)\\ &= \bar{P}_{i,k+1-i}(0,s)\bar{F}(s)\\ &= \bar{F}(s)\binom{k+1}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^k\end{aligned} \qquad (5.12)$$

$$i = 0,1,2,...k+1$$

$$\begin{aligned}&\int\rho_c(y)\bar{P}_{cc}(y,s)\\ &= \bar{P}_{cc}(0,s)\bar{G}(s)\\ &= \bar{G}(s)\sum_{L=1}^{k}\sum_{i=0}^{L}\binom{L}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^L\end{aligned} \qquad (5.13)$$

Lastly by substituting equations (5.11), (5.12) in equation (5.1), we can easily get

$$\begin{aligned}\bar{P}_{0,,0}(s) = \{&(s+\Lambda+H+\lambda_c)\\ &-[\lambda_c\bar{G}(s)\sum_{L=1}^{k}\sum_{i=0}^{L}\binom{L}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^L]\\ &-[\bar{F}(s)\binom{k+1}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^K]\}\end{aligned}$$

The Laplace transform of the reliability of the system and steady state availability of the system are:

$$\bar{P}_{up}(s) = \sum_{i=1}^{k}\sum_{j=0}^{k-i}\bar{P}_{i,j}(s), \quad \bar{P}_{up}(s) = \lim_{s\to 0} s\sum_{i=1}^{k}\sum_{j=0}^{k-i}\bar{P}_{i,j}(s)$$

6. Particular Case

When repair rate follows exponential distribution. Setting

$$\bar{F}(s) = \frac{\rho}{s+\rho}, \quad \bar{G}(s) = \frac{\rho_c}{s+\rho_c} \qquad (6.1)$$

The Laplace transform of the probabilities of the system are:

$$\bar{P}_{i,L-i}(s) = \sum_{L=1}^{k}\sum_{i=0}^{L}\binom{L}{i}\Lambda^i H^{k-i}\bar{P}_{0,0}(s)/A^L \qquad (6.2)$$

$$P_{i,k+1-i}(s) = \frac{1}{s+\rho}\sum_{i=0}^{k+1}\binom{k+1}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^k \qquad (6.3)$$

$$\bar{P}_{cc}(s) = \frac{\lambda_c}{s+\rho_c}\sum_{L=1}^{k}\sum_{i=0}^{L}\binom{L}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^L \qquad (6.4)$$

$$\begin{aligned}\bar{P}_{0,,0}(s) = \{&(s+\Lambda+H+\lambda_c)\\ &-[\frac{\lambda_c\rho_c}{s+\rho_c}]\sum_{L=1}^{k}\sum_{i=0}^{L}\frac{\binom{L}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)}{A^L}\\ &-[\frac{\rho}{s+\rho}\sum_{i=0}^{k+1}\binom{k+1}{i}\Lambda^i H^{k+1-i}\bar{P}_{0,0}(s)/A^K]\}\end{aligned} \qquad (6.5)$$

6.1. Case I

Consider the case when n=2, s=1, k=1, ρ, ρ_c are the repair rates when system is in state (i, j), and in cc states respectively. In this case we have: $\Lambda = 2\lambda$, $H = 2h$, $A = s + 2(\lambda + h) + \lambda_c$

Moreover, from equations (6.2)-(6.5), one may get

$$P_{0,1}(s) = \frac{2h}{[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \qquad (6.1.1)$$

$$P_{1,0}(s) = \frac{2\lambda}{[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \qquad (6.1.2)$$

$$P_{0,2}(s) = \frac{4h^2}{(s+\rho)[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \qquad (6.1.3)$$

$$P_{1,1}(s) = \frac{8\lambda h}{(s+\rho)[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \qquad (6.1.4)$$

$$P_{2,0}(s) = \frac{4\lambda^2}{(s+\rho)[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \qquad (6.1.5)$$

$$P_{cc}(s) = \frac{\lambda_c(s+4(h+\lambda)+\lambda_c)}{(s+\rho_c)[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \qquad (6.1.6)$$

$$P_{0,0}(s) = \frac{(s+\rho)(s+\rho_c)(s+2(\lambda+h)+\lambda_c)}{C(s)} \qquad (6.1.7)$$

where :

$$C(s) = (s+\rho)(s+\rho_c)(s+2(h+\lambda)+\lambda_c)^2$$

$$-\lambda_c\rho_c(s+\rho)(s+4(h+\lambda)+\lambda_c) - 4\rho(\lambda+h)^2(s+\rho_c)$$

Therefore the Laplace transform of probability of the system is in up-state is:

$$P_{up}(s) = P_{0,0}(s) + P_{0,1}(s) + P_{1,0}(s)$$

$$P_{up}(s) = \frac{(s+\rho)(s+\rho_c)(s+4(\lambda+h)+\lambda_c)}{s(s^3+k_1s^2+k_2s+k_3)} \quad (6.1.8)$$

where

$$k_1 = \rho + \rho\_\{c\} + 2\lambda_c + 4(h+\lambda)$$

$$k_2 = \rho_c(\rho-\lambda_c) + (2(h+\lambda)+\lambda_c)^2 + 2(\rho+\rho_c)(2(h+\lambda)+\lambda_c)$$

$$k_3 = \rho_c(\rho-\lambda_c)(4h+4\lambda+\lambda_c) + (\rho+\rho_c(2(h+\lambda)+\lambda_c)^2 - 4\rho(h+\lambda)^2$$

$$P_{down}(s) = P_{0,2}(s) + P_{1,1}(s) + P_{2,0}(s) + P_{cc}(s)$$

$$P_{down}(s) = \bar{P}_{cc}(s) + \bar{P}_{i,k+1-i}(s) = \frac{\lambda_c(s+4(h+\lambda)+\lambda_c)(s+\rho)+4(h+\lambda)^2(s+\rho_c)}{C(s)}$$

Taking inverse Laplace transform of equation (6.1.8), one may get:

$$P_{up}(t) = k_4e^{r_1t} + k_5e^{r_2t} + k_6e^{r_3t} + k_7e^{r_4t} \quad (6.1.9)$$

where

$$k_4 = \rho\rho_c(4(\lambda+h)+\lambda_c))/(-r_1r_2r_3)$$

$$k_5 = \frac{((r_1+\rho)(r_1+\rho_c)(r_1+4(\lambda+h))}{(r_1(r_1-r_2)(r_1-r_3))}$$

$$k_6 = \frac{((r_2+\rho)(r_2+\rho_c)(r_2+4(\lambda+h)+\lambda_c))}{(r_2(r_2-r_1)(r_2-r_3))}$$

$$k_7 = \frac{((r_3+\rho)(r_3+\rho_c)(r_3+4(\lambda+h)+\lambda_c))}{(r_3(r_3-r_1)(r_3-r_2))}$$

and $r_{1,}r_2, r_3$ are the roots of the equation $s^3 + k^1s^2 + k^2s + k^3s = 0$

6.1.1. Mean Operational Time (MOT)

From equation (6.1.9) one may get:

$$\mu_{up}(t) = \int_o^\infty P_{up}(t)dt = \frac{k_4}{r_1}(e^{r_1t}-1) + \frac{k_5}{r_2}(e^{r_2t}-1) + \frac{k_6}{r_3}(e^{r_3t}-1) + \frac{k_7}{r_4}(e^{r_4t}-1) \quad (6.1.10)$$

6.1.2. Busy Period

$$B(s) = \bar{P}_{cc}(s) + \bar{P}_{i,k+1-i}(s) = \frac{\lambda_c(s+4(h+\lambda)+\lambda_c)(s+\rho)+4(h+\lambda)^2(s+\rho_c)}{s(s^3+k_1s^2+k_2s+k_3)} \quad (6.1.11)$$

Taking inverse Laplace transform of equation (6.1.11), it is fairly easy to get busy period:

$$B(t) = k_4^1e^{r_1t} + k_5^1e^{r_2t} + k_6^1e^{r_3t} + k_7^1e^{r_4t} \quad (6.1.12)$$

where

$$k_4^1 = (\lambda_c\rho(4(h+\lambda)+\lambda_c)+4\rho_c(h+\lambda)^2)/(-r_1r_2r_3)$$

$$k_5^1 = \frac{(\lambda_c(r_1+4(h+\lambda)+\lambda_c)(r_1+\rho)+4(h+\lambda)^2(r_1+\rho_c))}{(r_1(r_1-r_2)(r_1-r_3))}$$

$$k_6^1 = \frac{(\lambda_c(r_2+4(h+\lambda)+\lambda_c)(r_2+\rho)+4(h+\lambda)^2(r_2+\rho_c))}{(r_2(r_2-r_1)(r_2-r_3))}$$

$$k_7^1 = (\lambda_c(r_3+4(h+\lambda)+\lambda_c)(r_3+\rho)+4(h+\lambda)^2(r_3+\rho_c)$$

6.1.3. Expected Busy Period (EBP)

From equation (6.1.12) one may get the expected busy period as following;

$$\mu_B(t) = \int_o^\infty B(t)dt = \frac{k_4^1}{r_1}(e^{r_1t}-1) + \frac{k_5^1}{r_2}(e^{r_2t}-1) + \frac{k_6^1}{r_3}(e^{r_3t}-1) + \frac{k_7^1}{r_4}(e^{r_4t}-1) \quad (6.1.13)$$

6.1.4. Steady State Availability of the System

$$P_{up} = \lim_{s\to 0} sP_{up}(s) = \rho\rho_c(4(\lambda+h)+\lambda_c)/C'(0) \quad (6.1.14)$$

where:

$$C'(0) = (2(h+\lambda)+\lambda_c)^2(\rho+\rho_c) + 2\rho\rho_c(2(h+\lambda)+\lambda_c) - \lambda_c\rho_c(4(h+\lambda)+\lambda_c) - 4\rho(h+\lambda)^2 - \lambda_c\rho\rho_c)$$

6.1.5. Steady State Busy Period

$$B(\infty) = (\lambda_c(s+4(h+\lambda)+\lambda_c)(s+\rho) + 4(h+\lambda)^2(s+\rho_c))/C'(0) \quad (6.1.15)$$

6.1.6. Mean Time of System Failure (MTSF)

$$MTSF = \lim_{s\to 0} P_{up}(s) = \rho\rho_c(4(\lambda+h)+\lambda_c)/C(0) \quad (6.1.16)$$

where :

$$C(0) = \rho\rho_c(2(\lambda+h)+\lambda_c)^2 - \lambda\lambda_c\rho\rho_c(4(\lambda+h)+\lambda_c) - 4\rho\rho_c(\lambda+h)^2$$

6.2. Case II

We consider the case when n=2, s=1, k=1, ρ are the repair occurs when system is in state (i, j). In this case, we have: $\Lambda = 2\lambda, H = 2h, A = s+2(\lambda+h)+\lambda_c$ and from equations (6.2)-(6.5), one may get

$$P_{0,1}(s) = \frac{2h}{[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.2.1)$$

$$P_{1,0}(s) = \frac{2\lambda}{[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.2.2)$$

$$P_{0,2}(s) = \frac{4h^2}{(s+\rho)[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.2.3)$$

$$P_{1,1}(s) = \frac{8\lambda h}{(s+\rho)[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.2.4)$$

$$P_{2,0}(s)=\frac{4\lambda^2}{(s+\rho)[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.2.5)$$

$$P_{cc}(s)=\frac{\lambda_c(s+4(h+\lambda)+\lambda_c)}{s[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.2.6)$$

$$P_{0,0}(s)=\frac{(s+\rho)(s+4(\lambda+h)+\lambda_c)}{(s+\rho)(s+2(\lambda+h)+\lambda_c)^2-4\rho(\lambda+h)^2} \quad (6.2.7)$$

Therefore the Laplace transform of probability of the system is in up-state is:

$$P_{up}(s)=P_{0,0}(s)+P_{0,1}(s)+P_{1,0}(s)$$

$$P_{up}(s)=\frac{(s+\rho)(s+4(\lambda+h)+\lambda_c)}{s^3+k_1s^2+k_2s+k_3} \quad (6.2.8)$$

where

$$k_1=\rho+4(h+\lambda)+2\lambda_c$$

$$k_2=(2(h+\lambda)+\lambda_c)^2+\rho(4(h+\lambda)+2\lambda_c)$$

$$k_3=\lambda_c\rho(4(h+\lambda)+\lambda_c)$$

$$P_{down}(s)=P_{0,2}(s)+P_{1,1}(s)+P_{2,0}(s)+P_{cc}(s)$$

$$P_{down}(s)=\frac{4s(h+\lambda)^2+2\lambda_c(s+\rho)(\lambda+h)}{(s+\rho)(s+2(h+\lambda)+\lambda_c))}$$

Taking inverse Laplace transform of equation (6.1.8), one may get after a little simplification:

$$P_{up}(t)=k_4+k_5e^{r_1t}+k_6e^{r_2t}+k_7e^{r_3t} \quad (6.2.9)$$

where

$$k_4=((r_1+\rho)(r_1+4(\lambda+h)+\lambda_c))/((r_1-r_2)(r_1-r_3)))$$
$$k_5=((r_2+\rho)(r_2+4(\lambda+h)+\lambda_c))/((r_2-r_1)(r_2-r_3)))$$
$$k_6=((r_3+\rho)(r_3+4(\lambda+h)+\lambda_c))/((r_3-r_1)(r_3-r_2)))$$

and r_1, r_2, r_3 are the roots of the equation $s^3+k_1s^2+k_2s+k_3=0$

$$\frac{P_{down}\lambda\_\{c\}(s+\rho)(s+4(h+\lambda)+\lambda\_\{c\})+4(\lambda+h)^2}{((s+\rho)(s+2(\lambda+h)+\lambda_c)^2-4\rho(\lambda+h)^2))}$$

6.2.1. Expected Operational Time

From equation (6.2.9), one may get the expected operational time as below;

$$\mu_{up}(t)=\int_o^\infty P_{up}(t)dt$$
$$=k_4+\frac{k_5}{r_1}(e^{r_1t}-1)+\frac{k_6}{r_2}(e^{r_2t}-1)+\frac{k_7}{r_3}(e^{r_3t}-1) \quad (6.2.10)$$

6.2.2. Busy Period

$$B(s)=\overline{P}_{i,k+1-i}(s)$$
$$=\frac{4(h+\lambda)^2}{(s+\rho)(s+2(\lambda+h)+\lambda_{\{c\}})^2-4\rho(\lambda+h)^2} \quad (6.2.11)$$

where

$$k_1=\rho+2\lambda_c+4(h+\lambda)$$
$$k_2=(2(h+\lambda)+\lambda_c)(2(h+\lambda)+2\rho+\lambda_c)$$
$$k_3=\lambda_c\rho(4h+4\lambda+\lambda_c)$$

Taking inverse Laplace transform of equation (6.2.11) and after a little simplification, one may get:

$$B(t)=k_4^1e^{r_1t}+k_5^1e^{r_2t}+k_6^1e^{r_3t} \quad (6.2.12)$$

where

$$k_4^1=4(h+\lambda)^2/(r_1-r_2)(r_1-r_3)$$
$$k_5^1=4(h+\lambda)^2/(r_2-r_1)(r_2-r_3)$$
$$k_6^1=4(h+\lambda)^2/(r_3-r_1)(r_3-r_2)$$

6.2.3. Expected Busy Period

From equation (6.2.12) , one may get

$$\mu_B(t)=\int_o^\infty B(t)dt$$
$$=\frac{k_4^1}{r_1}(e^{r_1t}-1)+\frac{k_5^1}{r_2}(e^{r_2t}-1)+\frac{k_6^1}{r_3}(e^{r_3t}-1) \quad (6.2.13)$$

6.2.4. Steady State Availability of the System

$$P_{up}=\lim_{s\to 0}sP_{up}(s)$$
$$=\frac{\rho(4(\lambda+h)+\lambda_c)}{(2(h+\lambda)+\lambda_c)^2+2\rho(2(h+\lambda)+\lambda_c)} \quad (6.2.14)$$

6.2.5. Steady State Busy Period

$$B(\infty)=\frac{(4(h+\lambda)^2)}{(2(h+\lambda)+\lambda_c)^2+2\rho(2(h+\lambda)+\lambda_c)} \quad (6.2.15)$$

6.2.6. Mean Time of System Failure (MTSF)

$$MTSF=\lim_{s\to 0}P_{up}(s)$$
$$=\frac{(\rho(4(\lambda+h)+\lambda_c)}{(\rho(2(\lambda+h)+\lambda_c)^2-4\rho(\lambda+h)^2))} \quad (6.2.16)$$

6.3. Case III

Consider the case when n=2, s=1, k=1, and no repair, we have: $\Lambda=2\lambda, H=2h, A=s+2(\lambda+h)+\lambda_c$

And from equations (6.2)-(6.5), one may get

$$P_{0,1}(s)=\frac{2h}{[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.3.1)$$

$$P_{1,0}(s)=\frac{2\lambda}{[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.3.2)$$

$$P_{0,2}(s)=\frac{4h^2}{s[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.3.3)$$

$$P_{1,1}(s)=\frac{8\lambda h}{s[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.3.4)$$

$$P_{2,0}(s)=\frac{4\lambda^2}{s[s+2(h+\lambda)+\lambda_c]}P_{0,0}(s) \quad (6.3.5)$$

$$P_{cc}(s) = \frac{\lambda_c(4(h+\lambda)+\lambda_c)}{s[s+2(h+\lambda)+\lambda_c]} P_{0,0}(s) \qquad (6.3.6)$$

$$P_{0,0}(s) = \frac{1}{(s+2(\lambda+h)+\lambda_c)} \qquad (6.3.7)$$

Therefore the Laplace transform of probability of the system is in upstate is:

$$P_{up}(s) = P_{0,0}(s) + P_{0,1}(s) + P_{1,0}(s)$$

$$P_{up}(s) = \frac{(s+4(\lambda+h)+\lambda_c)}{s^3 + k_1 s^2 + k_2} \qquad (6.3.8)$$

where

$$k_1 = 4(h+\lambda) + 2\lambda_c$$

$$k_2 = (2(h+\lambda) + \lambda_c)$$

$$P_{down}(s) = P_{0,2}(s) + P_{1,1}(s) + P_{2,0}(s) + P_{cc}(s)$$

$$P_{down}(s) = \frac{\lambda_c(4(h+\lambda)+\lambda_c) + 4(\lambda+h)^2}{s(s+2(h+\lambda)+\lambda_c)^2}$$

Taking inverse Laplace transform of equation (6.1.8) one may get:

$$P_{up}(t) = k_3 e^{r_1 t} + k_4 e^{r_2 t} \qquad (6.3.9)$$

where

$$k_3 = (4(\lambda+h)+\lambda_c))/(r_1 - r_2)$$

$$k_4 = (4(\lambda+h)+\lambda_c))/(r_2 - r_1)$$

and $r_{1,} r_{2,}$ are the roots of the equation $s^2 + k_1 s + k_2 = 0$

6.3.1.Mean Operational Time (MOT)

From equation (6.3.9), one may get:

$$\mu_{up}(t) = \int_o^\infty P_{up}(t)dt = \frac{k_3}{r_1}(e^{r_1 t} - 1) + \frac{k_4}{r_2}(e^{r_2 t} - 1) \quad (6.3.10)$$

6.3.2. Steady State Availability of the System

$$P_{up} = \lim_{s\to 0} sP_{up}(s) = \frac{(4(\lambda+h)+\lambda_c)}{(2(h+\lambda)+\lambda_c)^2} \qquad (6.3.11)$$

6.3.3. Mean Time of System Failure (MTSF)

$$MTSF = \lim_{s\to 0} P_{up}(s) = \frac{(4(\lambda+h)+\lambda_c)}{(2(\lambda+h)+\lambda_c)^2} \qquad (6.3.12)$$

7. Numerical Illustration

In this section, our endeavor is to test the validity of various performance measures explored herein by way of numerical illustration. Setting $\lambda = .02$, $h = .01$, $\lambda_c = .002$, $\rho = .8$, $\rho_c = .008$, one can compute results case wise as following;

7.1. Case I

when n=2, s=1, k=1, ρ, ρ_c are the repair rates when system is in state (i, j), and in cc states respectively

7.1.1. Availability Analysis

$$\begin{aligned} P_{up}(t) &= 4.2907\times10^{-2}\exp(-0.12737t) \\ &+ 0.1193\exp(-9.1338\times10^{-3}t) \\ &- 6.8221\times10^{-3}\exp(-0.79463t) + 0.84461 \end{aligned} \qquad (7.1.1)$$

7.1.2. Expected Operational Time

$$\begin{aligned} \mu_{up}(t) &= 0.84461t - 0.33687(\exp(-0.12737t)-1) \\ &-13.061(\exp(-9.1338\times10^{-3}t)-1) \\ &+8.5853\times10^{-3}(\exp(-0.79463t)-1) \end{aligned} \qquad (7.1.2)$$

7.1.3. Busy Period

$$\begin{aligned} B(t) &= 6.8167\times10^{-3}(\exp(-0.79463t) \\ &-0.20564(\exp(-9.1338\times10^{-3}t) \\ &-4.3484\times10^{-2}(\exp(-0.12737t) + 0.24231 \end{aligned} \qquad (7.1.3)$$

7.1.4. Expected Busy Period

$$\begin{aligned} \mu(t) &= 0.24231t - 8.5785\times10^{-3}(\exp(-0.79463t)-1) \\ &+22.514\times(\exp(-9.1338\times10^{-3}t)-1) \\ &+0.34140(\exp(-0.12737t)-1) \end{aligned} \qquad (7.1.4)$$

7.2. Case II

when n=2, s=1, k=1, ρ are the repair occurs when system is in state (i, j)

7.2.1. Availability Analysis

$$\begin{aligned} P_{up} &= 4.3691\times10^{-2}(\exp(-0.12744t) \\ &-6.8295\times10^{-3}\exp(-0.79463t) \\ &+0.96314\exp(-1.9276\times10^{-3}t) \end{aligned} \qquad (7.2.1)$$

$$\begin{aligned} \mu_{up}(t) &= 8.5782\times10^{-3}(\exp(-0.79464t)-1) \\ &-0.34226(\exp(-0.12743t)-1) \\ &-499.66(\exp(-1.9277\times10^{-3}t)-1) \end{aligned} \qquad (7.2.2)$$

7.2.2. Busy Period

$$\begin{aligned} B(t) &= (6.8068t\times10^{-3}\exp(-0.79463t) \\ &-4.2990\times10^{-2}(\exp(-0.12744t)) \\ &+3.6183\times10^{-2}(\exp(-1.9276\times10^{-3}t) \end{aligned} \qquad (7.2.3)$$

7.2.3. Expected Busy Period

$$\begin{aligned} \mu(t) &= -8.5660\times10^{-3}\exp(-0.79463t)-1) \\ &+0.33734(\exp(-0.12744t)-1) \\ &-18.771(\exp(-1.9276\times10^{-3}t)-1) \end{aligned} \qquad (7.2.4)$$

7.3. Case III

when n=2, s=1, k=1, and no repair

7.3.1. Availability Analysis

$$P_{up}(t) = 1.0\exp\{-0.062t\} + 0.06t\exp\{-0.062t\} \quad (7.3.1)$$

$$\mu_{up}(t) = 32.258(1-\exp\{-0.062t\}) - t\exp\{-0.062t\} \quad (7.3.2)$$

Setting $t = 0,1,2,............$, in equations (7.1.1), (7.2.1) and (7.3.1), one gets Table 1. Variation of availability w.r.t. time in three cases is shown in Figure 1.

Table 1. Availability of the system for three cases at time t

t	Availability of the system with ρ and ρ_c	Availability of the system with ρ	Availability of the system without repair
0	1.00	1.00	1.0
10	0.9655	0.957	0.86071
20	0.94735	0.93018	0.63665
30	0.93626	0.91003	0.43588
40	0.92766	0.89199	0.28473
50	0.92025	0.87477	0.18020
60	0.9136	0.85802	0.11148
70	0.90756	0.84162	6.7790×10^{-2}
80	0.90206	0.82555	4.0675×10^{-2}
90	0.89705	0.80978	2.4144×10^{-2}
100	0.89247	0.79432	1.4206×10^{-2}

Setting $t = 0,1,2,............$, in equations (7.1.2), (7.2.2) and (7.3.2), one gets Table 2. Variation of expected operational time w.r.t. time in three cases is shown in Figure 2.

Table 2. Expected operational time of the system for three cases at time t

t	Expected operational time of the system with ρ and ρ_c	Expected operational time of the system with ρ	Expected operational time of the system without repair
0	0.0	0.0	0.0
10	9.8202	9.7777	9. 5255
20	19.375	19.204	17.135
30	28.79	28.402	22.566
40	38.108	37.411	26.207
50	47.347	46.245	28.552
60	56.515	54.908	30.022
70	65.621	63.406	30.925
80	74.668	71.742	31.471
90	83.663	79.918	31.797
100	92.611	87.938	31.99

Setting $h = 0, .02, .04.........$, in equations (6.1.14), (6.2.14), one gets Table 3. Variation of steady state availability for different values of human error in three cases is shown in Figure 3.

Table 3. Relationship between steady state availability and human error for three cases

h	Steady state Availability of the system with ρ and ρ_c	Steady state Availability of the system with ρ
0.0	0.78469	0.95122
.02	0.7696	0.93965
.04	0.75507	0.92154
.06	0.74108	0.90245
.08	0.7276	0.88351
10	0.7146	0.86503
.12	0.70206	0.84714
.14	0.68995	0.82988
.16	0.67825	0.81324
.18	0.66694	0.79721
.20	0.65601	0.78177

Setting $h = 0, .02, .04.........$, in equations (6.1.16), (6.2.16) and (6.3.12), one gets Table 4. Variation of MTSF for different values of human error in three cases is shown in Figure 4.

MTSF of the system with ρ and $\rho_c = \infty$, MTSF of the system with ρ = 500

Table 4. Relationship between MTSF and human error for three cases

h	MTSF of the system with ρ and ρ_c	MTSF of the system with ρ	MTSF of the system without repair
0.0	∞	500	46.485
.02	∞	500	24.093
.04	∞	500	16.259
.06	∞	500	12.269
.08	∞	500	9.8520
10	∞	500	8.2303
.12	∞	500	7.0670
.14	∞	500	6.1919
.16	∞	500	5.5096
.18	∞	500	4.9627
.20	∞	500	4.5146

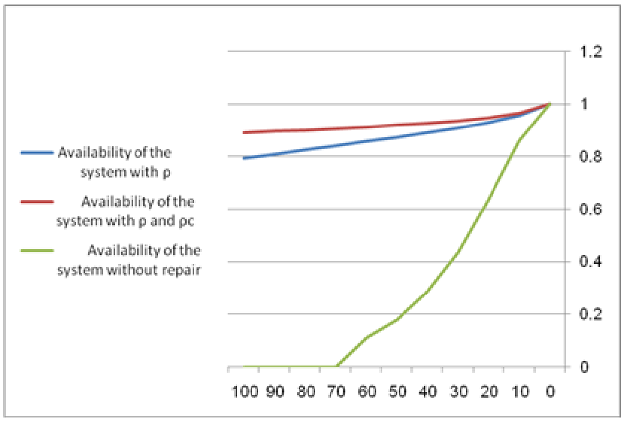

Figure 1. Variation of availability w.r.t. time in the three cases

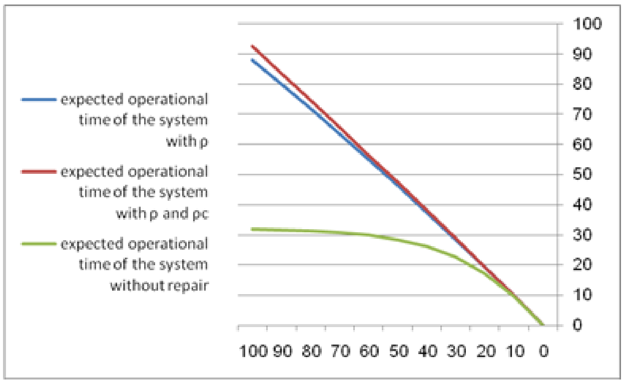

Figure 2. Variation expected operational time of system w.r.t. time in the three cases

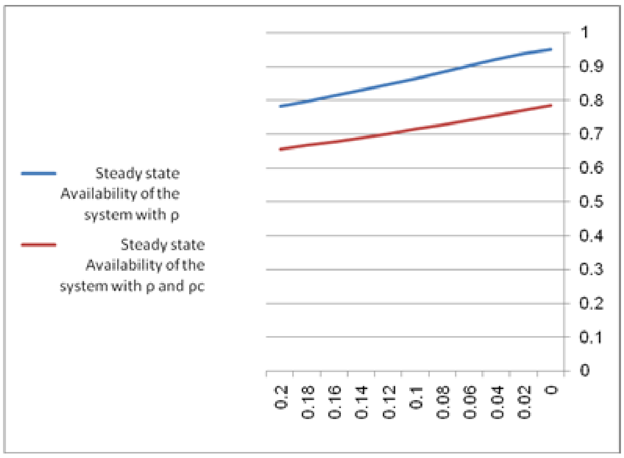

Figure 3. Variation of steady state availability for different values of human error for case II &III

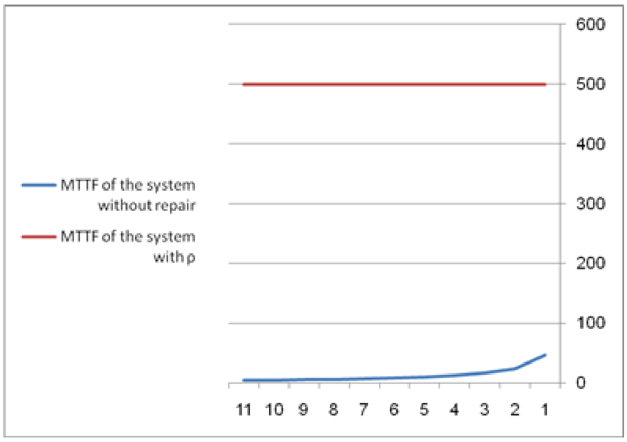

Figure 4. Variation of MTSF for different values of human error for case II & III

8. Sensitivity Analysis and Conclusive Observations

Table 1 computes availability of the system at any time. Figure 1 shows availability of the system decreases in the interval (0, t] for the three cases. By comparing the availability with respect to time for three cases with and without repair graphically, it is observed that:

- The availability of system decreases with respect to time. We conclude that the system availability with ρ and ρ_c is greater than the system with ρ, and availability with ρ is greater than without repair.

Table 2 computes expected operational time for the system at any time. Figure 2 shows expected operational time increases in the interval (0, t] for the three cases. By comparing the expected operational time with respect to time t for three cases with and without repair graphically, we observe that:

- The system expected operational time increases with respect to time. And it is also noticeable that the system expected operational time with ρ and ρ_c is greater than the system with ρ and with ρ repair is greater than the system without repair.

Table 3 compute the relationship between steady state availability and human error for three cases. Figure 3 shows variation of steady state availability with respect to human failure for the two cases of repair. By comparing the steady state availability with respect to human failure for the systems with ρ and ρ_c graphically, it is remarked that:

- The human failure rate h increases, however the steady state availability of the system decreases at constant $\lambda = .02, \lambda_c = .002, \rho = .8, \rho_c = .008$.
- The steady state availability of system with repair ρ only is greater than the system availability with ρ and ρ_c (i.e., system include common-cause failure). The availability of the system in the steady-state is equal to zero.

Table 4 computes variation of mean time of system failure (MTSF) with respect to human failure. Figure (4) displays the variation of MTSF with respect to human failure for the second and third cases. By comparing the MTSF with respect to human failure for the system with ρ and without repair graphically, we observe that:

- The mean time of system failure (MTSF) with ρ and $\rho_c = \infty$
- MTSF with ρ = 500 (constant). However, in case of MTSF without repairing, as for as the value of human failure rate h increases at constant $\lambda = .02, \lambda_c = .002, \rho = .8, \rho_c = .008$, the MTTF of the system decreases.

Finally with passing above remarks, we have succeeded to investigate significant performance measures of k-out of-n repairable system involving human and common-cause failures. It is highly expected that our present contribution for performance analysis and inference of k-out of-n repairable system involving human and common-cause failures will be useful for researchers, statisticians, mathematicians and management professionals for their future research and development in this direction, e.g. very recently Maurya (2013 c) proposed a computational approach to cost and profit analysis of k-out of-n repairable system integrating human error and system failure constraints.

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