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Finite Difference Technique for Unsteady MHD Periodic Flow of Viscous Fluid through a Planer Channel

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Abstract In the present paper we investigate an unsteady periodic flow of a viscous incompressible fluid through a porous planer channel in the presence of transverse magnetic field. The governing equations have been solved by finite difference method. The analytical expressions for the velocity profiles of the fluid have been obtained and numerical computations involved in the solution have been shown by tables. The effects of various parameters on the flow field have been discussed with the help of graphs and tables.

Keywords: *magnetohydrodynamics, planer channel, periodic flow, viscous fluid, Hartmann number, Finite difference technique*

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1. Introduction

The fluid flow through porous medium has been of great interest for a long time. In the history of fluid flow through channels, Navier investigated a boundary condition of fluid slip at solid surface such that $u = h\left(\frac{\partial u}{\partial y}\right)$, where h is the slip coefficient and u the velocity along x axis. In a case γ =0 indicates that there is no slip at the boundary. Sparrow and Cess (1961) investigated the effect of a magnetic field on a free convective heat transfer. Raptis et al. (1982) have been discussed hydro magnetic free convection flow through a porous medium between two parallel plates. Pal et al. (1984) investigated a longitudinal dispersion of tracer particles in a channel bounded by porous media using slip condition. Singh and Kumar (1993) investigated a free convective fluctuating flow through a porous medium with variable permeability. Aldoss et al. (1995) depicted a mixed convection flow from a vertical plate embedded in porous medium in the presence of magnetic field. Makinde (1995) investigated a laminar flow in a channel of varying width with permeable boundaries. Soltani and Yilmazar (1998) studied a slip velocity and slip layer thickness in flow of concentrated suspensions. Jha (1998) discussed an effect of applied magnetic field on transient free convective flow in a vertical channel.

Helmy (1998) investigated MHD unsteady free convective flow past a vertical porous plate.Hossain et al. (1998) investigated heat transfer response of MHD free convective flow along a vertical plate to surface temperature oscillation. Kim (2000) discussed an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Yu and Ameel (2002) investigated a slip low heat transfer in rectangular micro channels. Derek et al. (2002) established an apparent fluid slip at hydrophibic microchannel walls. Sharma et al. (2002) investigated the flow between annular spaces surrounded by coaxial cylindrical porous medium. Steinruck (2003) investigated about the physical relevance of similarity solution of the boundary layer flow equation describing mixed convection flow along a vertical plate. Khaled and Vafai (2004) discussed the effect of slip condition on Stokes and Coutte flows due to an oscillating wall. Makinde and Mhone (2005) investigated a heat transfer to MHD Oscillatory flow in a channel filled with porous medium. Makinde and Osalusi (2006) discussed a MHD steady flow in a channel with slip at permeable boundaries. Kumar et al. (2007) investigated a viscous flow coaxial cylinder in the presence of magnetic field. Mahmood and Ali (2007) established the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Mishra et al. (2008) investigated a flow and heat transfer of a MHD viscoelastic fluid in a channel with stretching walls. Kumar et al. (2010) investigated Finite difference technique for reliable MHD steady flow through channels permeable boundaries. Kumar et al. (2013) investigated a Finite difference technique for reliable MHD steady flow through channels permeable boundaries. Kumar et al. (2013) made a Perturbation Technique of MHD Free Convection Flow through Infinite Vertical Porous Plate with Constant Heat

Flux. Kumar and Agrawal (2013) profounded Mathematical and Simulation of lid driven cavity flow at different aspect ratios using single relaxation time lattice Boltzmann technique. Kumar et al. (2013) investigated Finite Element Galerkin's Approach for Viscous Incompressible Fluid Flow through a Porous Medium in Coaxial Cylinders.

2. Formulation of the Problem:

We are considering an unsteady periodic flow of viscous incompressible fluid through a planer channel with saturated porous medium in the presence of transverse magnetic field. The governing equations of the motion are given below:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} + \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\nu}{K^*}u^* - \frac{\sigma B_0^2}{\rho}u^* + g\beta\left(T^* - T_0^*\right) \quad (1)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_P}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_P}\frac{\partial q^*}{\partial y^*} \quad (2)$$

Boundary conditions are:

$$u^* = h\frac{\partial u}{\partial y^*}, T^* = T_w^* \quad at \quad y^* = 1 \quad (3)$$

where u^* is the velocity, ρ the fluid density, P^* the pressure, t^* the time, ν the kinematics viscosity coefficient, K^* the porous medium permeability coefficient, σ_e the conductivity of the fluid, B_0 the electromagnetic induction, g the acceleration due to gravity, β the coefficient of volume expansion due to temperature, T^* the fluid temperature, κ the thermal conductivity, C_p the specific heat at constant pressure and q is the radiative heat flux. Consider the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q^*}{\partial y^*} = 4\alpha^2(T^* - T_0^*) \quad (5)$$

Where α is the mean radiation absorption coefficient,

Here non- dimensionless quantities are:

$$x = \frac{x^*}{a}, y = \frac{y^*}{a}, u = \frac{u^*}{V}, \theta = \frac{T^* - T_0^*}{T_w^* - T_0^*},$$

$$t = \frac{t^* V}{a}, P = \frac{aP^*}{\rho\nu V}, Da = \frac{K^*}{a^2}, N^2 = \frac{4\alpha^2 a^2}{\kappa},$$

$$Gr = \frac{g\beta(T_w^* - T_0^*)a^2}{\nu V}\left(\text{Grashoff number}\right),$$

$$Pe = \frac{Va\rho C_P}{\kappa}\left(\text{Peclet number}\right)$$

$$R_e = \frac{Va}{\nu}\left(\text{Reynolds number}\right),$$

$$M = \sqrt{\frac{a^2\sigma_e B_0^2}{\rho\nu}}\left(\text{Hartmann number}\right)$$

where V is the mean of flow velocity

Using the above dimensionless quantities into equations (1) to (4), we have

$$R_e\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (k^2 + M^2)u + Gr\theta \quad (6)$$

$$P_e\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - N^2\theta \quad (7)$$

Subject to condition

$$u = h\frac{\partial u}{\partial y}, \quad \theta = 0 \quad at \quad y = 0 \quad (8)$$

$$u = 0, \quad \theta = 1 \quad at \quad y = 1 \quad (9)$$

where $k^2 = \frac{1}{Da} = \frac{a^2}{K^*}$

$$R_e\frac{u_{i+1,j} - u_{i,j}}{\Delta t} = -\frac{P_{i+1,j}}{\Delta x} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta y)^2} - (k^2 + M^2)u_{i,j} + Gr\theta_{ij} \quad (10)$$

$$Pe\frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2} - N^2\theta_{ij} \quad (11)$$

The corresponding boundary conditions are

$$u_{ij} = h\frac{u_{i+1,j} - u_{i,j}}{\Delta y}, \quad \theta = 0 \quad at \quad y = 0 \quad (12)$$

$$u = 0, \quad \theta = 1 \quad at \quad y = 1 \quad (13)$$

The system of algebraic equations can be solved by Gauss-Siedal method or any other similar method. We use the Thomas algorithm.

The shearing stress is given by $\tau = \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y}\right)_{y=0}$.

3. Computational Algorithm:

We solve these coupled non linear partial differential equations by standard Finite difference technique [Kumar et al. (2013)]. The spetially discrete form of the governing equations can be obtained by Finite difference technique. A computational iteration procedure for the equations (10)-(11) with the boundary conditions (12) and (13) has been followed which takes care of highly non linear coupled set of algebraic equations. The computational iteration process may be divided into the following steps:

Step 1. Simplify equation (10) for j = 2, 3,…n-1 and i = 2,3,….m-1. Calculate the velocity profile using boundary conditions (12) and (13).

Step 2. The algebraic equations are solved by using Guass-Seidal method

Step 3. Update u_{ij} from the previous step.

Step 4. Repeate steps (1)-(3) until convergence are reached.

4. RESULT AND DISCUSSION

The effects of various parameters like Hartmann number M, Grashoff number G_r, Peclet number P_e, Reynolds number R_e, permeability of porous medium k, Radiation parameter N, time t and wall slip parameter h on velocity and temperature profiles have been shown in tables and graphs. The brief description is as given below:

Table 1. Velocity profiles for different values of parameter $G_r = 1, R_e = 1, M = 1, N = 1, P_e = 0.7, n = 1, \lambda = 1, t = 0, k = 1$

y	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 2.0$	$\gamma = 2.5$
0	0.0000008	10.639726	15.205435	17.743453	19.359122	20.477912
0.2	0.913037	8.106225	11.545581	13.457473	14.674559	15.517346
0.4	0.141078	5.871913	8.331123	9.698166	10.568409	11.171018
0.6	0.146172	3.43654	5.420148	6.300455	6.860848	7.248896
0.8	0.101535	1.908315	2.683639	3.114632	3.388999	3.518983
1	0.0000014	0.0000001	0.0000004	0.0000008	0.0000041	0.00000037

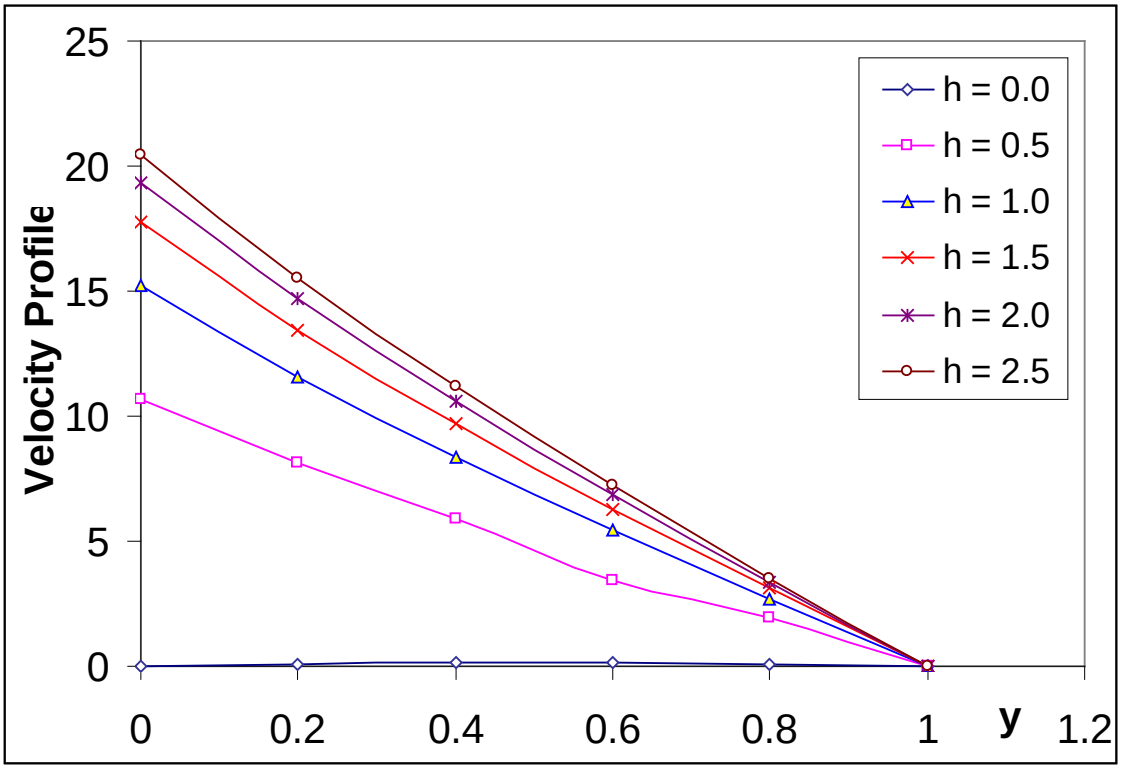

Figure 1. Velocity profiles for various values of wall slip parameter h

Table 2. Shear stress for different values of parameter $G_r = 1, R_e = 1, M = 1, N = 1, P_e = 0.7, n = 1, \lambda = 1, k = 1$

t	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 2.0$	$\gamma = 2.5$
0	35.43408	21.27945	15.20543	11.82896	9.67955	8.19116
1	35.42868	21.27621	15.20312	11.82716	9.67808	8.18991
2	35.41249	21.26649	15.19617	11.82176	9.67366	8.18617
3	35.38552	21.25029	15.18459	11.81275	9.66629	8.17993
4	35.34776	21.22761	15.16839	11.80015	9.65597	8.171211
5	35.29924	21.19847	15.14757	11.78395	9.64272	8.15999
6	35.23997	21.16288	15.12213	11.76416	9.62653	8.14629
7	35.16996	21.12084	15.09209	11.74079	9.6074	8.1301

Figure 1 shows that the velocity is everywhere increases corresponding to every increase in the wall slip parameter h in the region $y \in [0,1]$, which clearly tallies the natural phenomenon.

Figure 2 depicts the tendency of shear stress distribution corresponding to different values of wall slip parameter h. Hence for every smaller values of h the value of shear stress is seen much more than that of larger one.

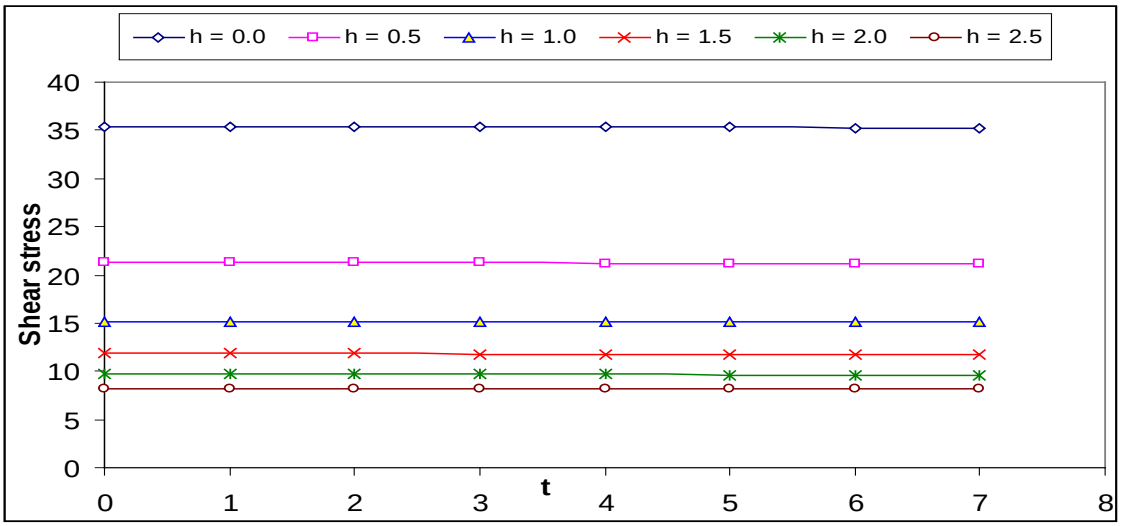

Figure 2. Shear stress for various values of wall slips parameter h

Table 3. Velocity profiles for different values of parameter $G_r = 1, R_e = 1, M = 1, N = 1, P_e = 0.7, n = 1, \lambda = 1, k = 1, \gamma = 1$

y	t=0	t=π/4	t=π/2	t=3π/4	t=π	t=5π/4
0	15.20543	10.75186	0	-10.7519	-15.2054	-10.7519
0.2	11.54558	8.16395	0	-8.16395	-11.5455	-8.16395
0.4	8.33112	5.89099	0	-5.89099	-8.33112	-5.89099
0.6	5.42014	3.83262	0	-3.83262	-5.42014	-3.83262
0.8	2.68363	1.89761	0	-1.89761	-2.68363	-1.89761
1	0	0	0	0	0	0

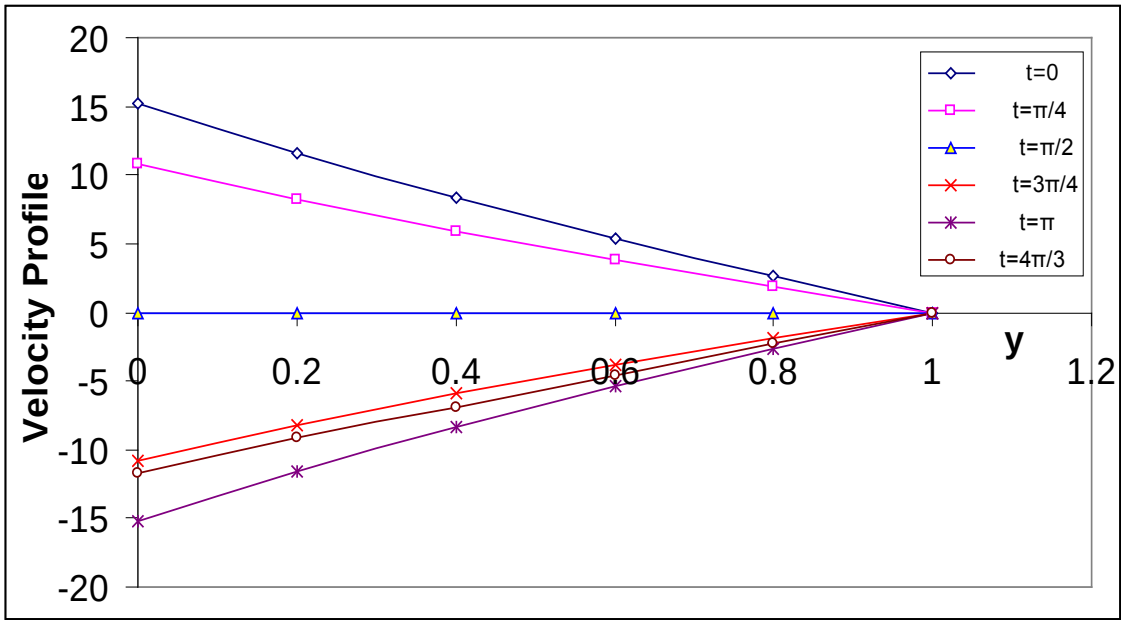

Figure 3. Velocity profile for various values of time t

Figure 3 depicts that the velocity profile of the fluid is periodic with period π.

Figure 4 depicts velocity profile for various values of magnetic parameter M. It shows that the flow decelerates

on the imposition of magnetic field i.e. on the increase of magnetic parameter M.

Table4. Velocity profiles for different values of parameter $G_r = 1, R_e = 1, N = 1, P_e = 0.7, n = 1, \lambda = 1, k = 1, \gamma = 1, t = 0$

y	M=0	M=0.5	M=1.0	M=1.5	M=2.0	M=2.5
0	22.40381	20.18969	15.20543	10.3673	6.89356	4.64272
0.2	17.39652	15.59154	11.54558	7.65592	4.90775	3.16674
0.4	12.76581	11.39433	8.33112	5.41026	3.3748	2.11022
0.6	8.39547	7.4736	5.42014	3.47412	2.13198	1.31034
0.8	4.17562	3.71282	2.68363	1.71199	1.04614	0.64212
1	0	0	0	0.000001	0.000001	0.000001

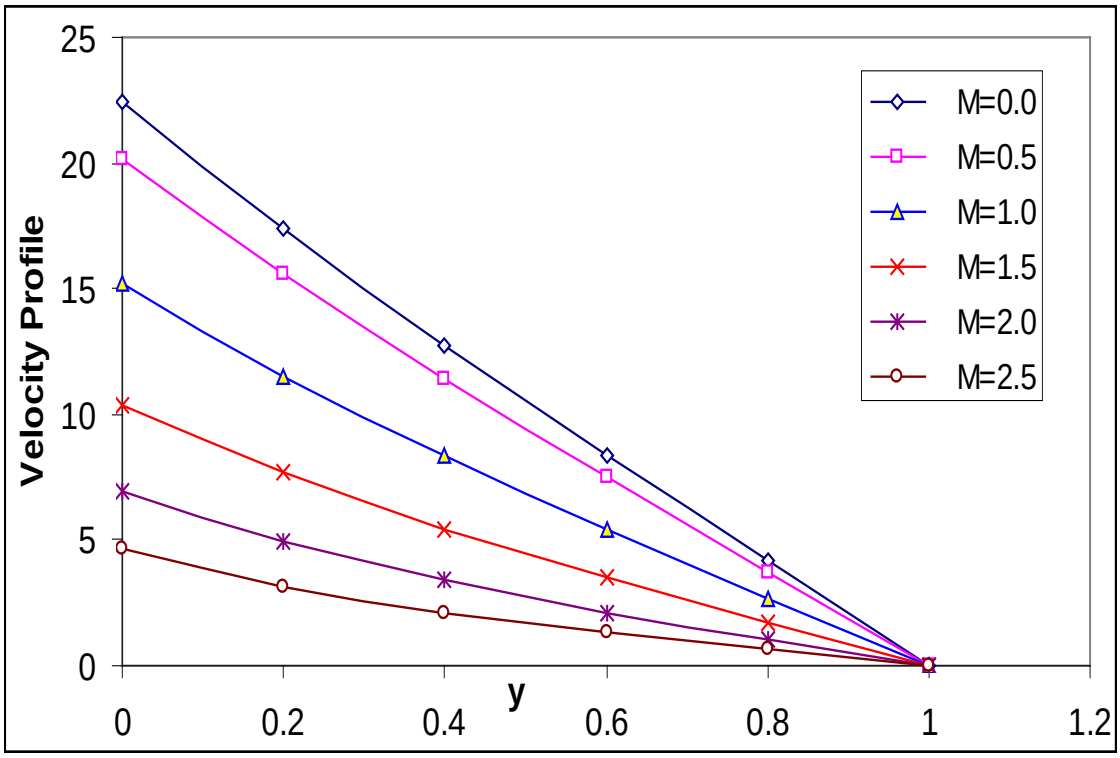

Figure 4. Velocity profile for various values Hartmann number M

Table 5. Velocity profiles for different values of parameter $G_r = 1, R_e = 1, M = 1, N = 1, P_e = 0.7, n = 1, \lambda = 1, \gamma = 1, t = 0$

y	k=0	k=0.5	k=1.0	k=1.5	k=2.0	k=2.5
0	22.40381	20.18969	14.20543	10.3673	6.89356	4.64272
0.2	17.39652	15.59154	10.54558	7.65592	4.90775	3.16674
0.4	12.76581	11.39433	8.33112	5.41026	3.3748	2.11022
0.6	8.39547	7.4736	5.42014	3.47412	2.13198	1.31034
0.8	4.17562	3.71282	2.68363	1.71199	1.04614	0.64212
1	0	0	0	0.000001	0.000001	0.000001

Figure 5 depicts that velocity profile for different values of porosity parameter k.

The effect of Peclet number P_e on the velocity profiles are shown in Table 6 and Figure 6. It is found that the fluid velocity decreases as the values of Peclet number increases corresponding to the same value of y.

Table 7 & Figure 7 also show similar effects of Reynolds number R_e as the Peclet number has the flow.

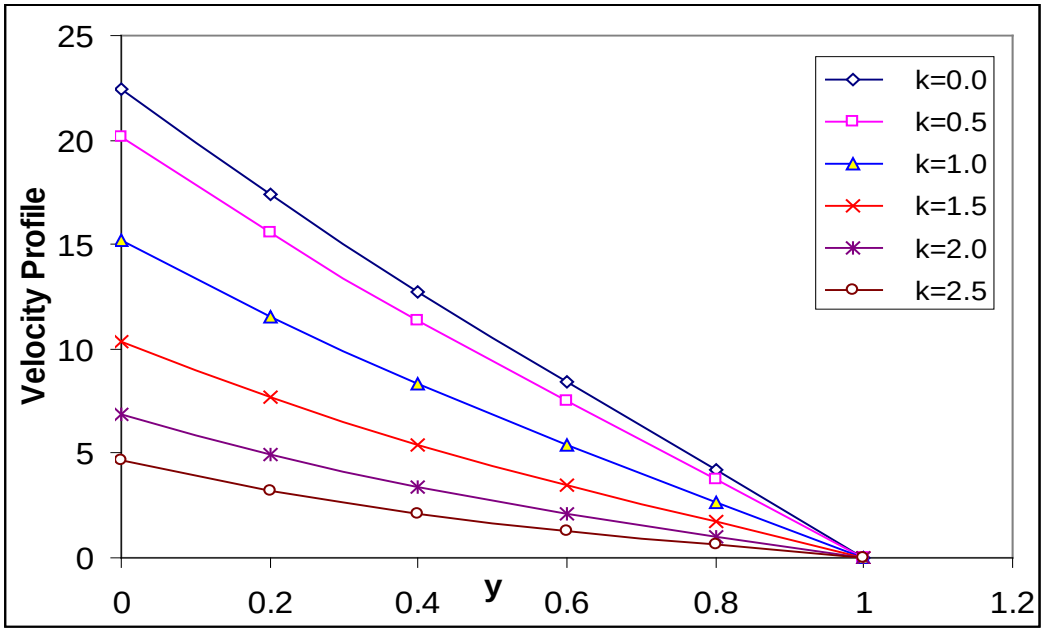

Figure 5. Velocity profile for various values permeability parameter k

Table 6. Velocity profiles for different values of parameter $G_r = 1, R_e = 1, N = 1, n = 1, \lambda = 1, \gamma = 1, t = 0$

y	$P_e = 0$	$P_e = 0.5$	$P_e = 1.0$	$P_e = 1.5$	$P_e = 2.0$	$P_e = 2.5$
0	15.73652	15.4478	14.76867	13.96437	13.07485	12.41948
0.2	11.94633	11.72845	11.21602	10.60913	9.91402	9.44345
0.4	8.61839	8.4622	8.09491	7.65992	7.12995	6.82442
0.6	5.60576	5.50482	5.26755	4.98654	4.59973	4.44679
0.8	2.77494	2.72526	2.60862	2.47049	2.21125	2.20517
1	0.000129	0.000001	0	0	-0.14157	0.000001

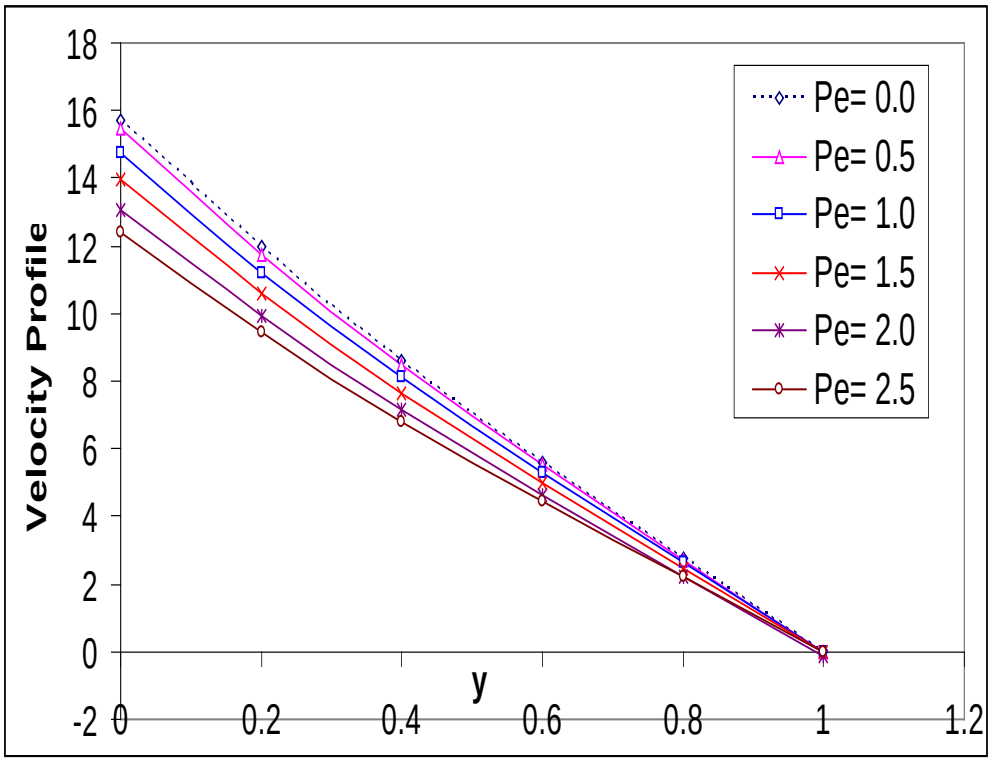

Figure 6. Velocity profile for various values Peclet number P_e

Table 7. Velocity profiles for different values of parameter $G_r = 1, M = 1, N = 1, P_e = 0.7, n = 1, \lambda = 1, \gamma = 1, t = 0$

y	Re=0	Re=0.5	Re=1.0	Re=1.5	Re=2.0	Re=2.5
0	15.8918	15.70765	15.20544	14.49766	13.69436	12.87216
0.2	12.10094	11.95187	11.54558	10.97365	10.32555	9.663441
0.4	8.750434	8.637843	8.331123	7.899777	7.411637	6.913751
0.6	5.700659	5.625317	5.420148	5.131826	4.80587	4.473808
0.8	2.824052	2.786332	2.683639	2.539392	2.376417	2.210514
1	0.000001	0.000001	0	0	0	0.000001

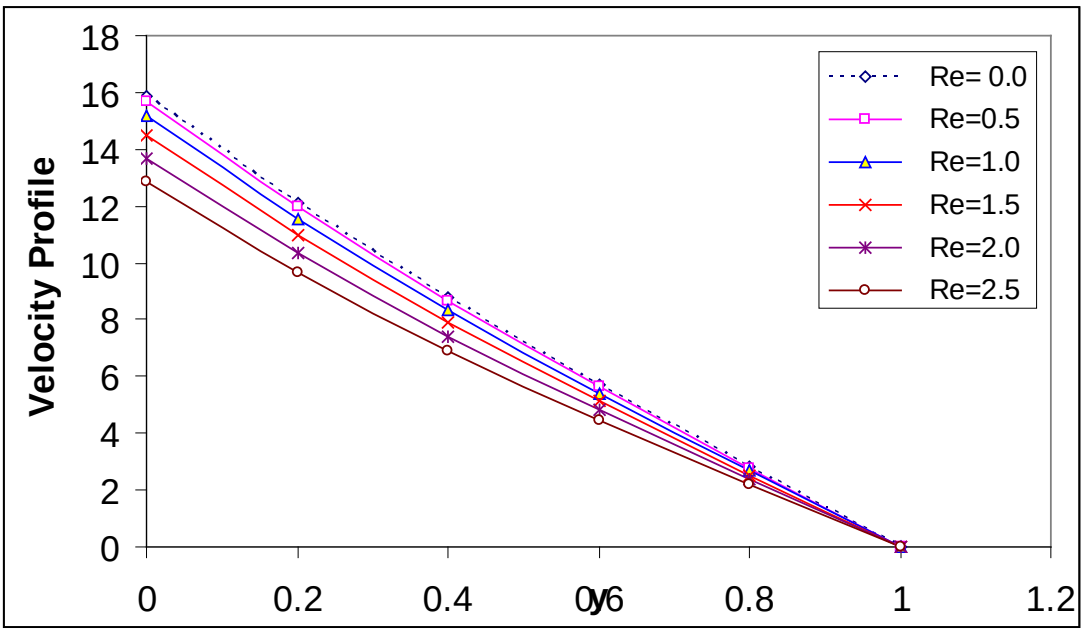

Figure 7. Velocity profile for various values Reynolds number R_e

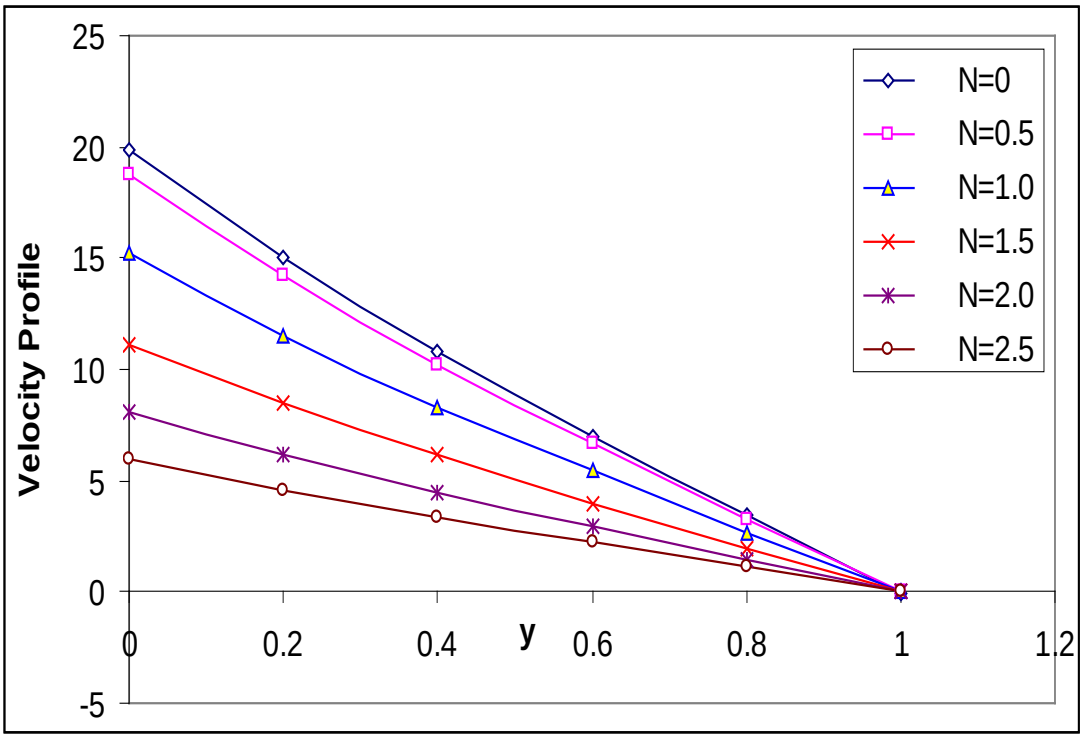

Figure 8. Velocity profile for various values radiation parameter N

Table 8. Velocity profiles for different values of parameter $G_r = 1, M = 1, R_e = 1, P_e = 0.7, n = 1, \lambda = 1, \gamma = 1, t = 0$

y	N=0	N=0.5	N=1.0	N=1.5	N=2.0	N=2.5
0	19.82857	18.727573	15.20543	11.15656	8.07829	5.97638
0.2	15.02676	14.2032	11.54558	8.49051	6.16781	4.58258
0.4	10.81671	10.23596	8.33112	6.14141	4.47662	3.34142
0.6	7.012565	6.65071	5.42014	4.00555	2.93006	2.19811
0.8	3.445713	3.28853	2.68363	1.98828	1.45961	1.10201
1	-0.04236	-0.00000002	0.0000004	0.0000026	-0.00000001	0.004504

Table 8 & Figure 8 depict the velocity profiles for various values of radiation parameter N. From Table 8 or Figure 8, it is observed that the increasing values of radiation parameter N have the similar effects on the velocity distribution as parameter P_e and number R_e have.

Nomenclature:

u : Velocity along x axis
P : Fluid pressure
ρ : Fluid density
ν : Kinematic viscosity
B_0 : Electromagnetic induction
σ_e : Fluid conductivity
H_0 : Intensity of magnetic field
β : Coefficient of volume expansion due to temperature
μ_e : Magnetic permeability
λ : A constant
C_p : Specific heat at constant pressure
g : Acceleration due to gravity
M: Hartmann number
κ : Thermal conductivity
K: Porous medium permeability
N: Radiation parameter
k: Porous medium shape factor
T: Fluid temperature
T_0 : Temperature at the boundary y = 0
T_w : Temperature at the boundary y = a
n : Frequency of the periodic flow
θ : Non dimensional temperature
R_e : Reynolds number
G_r : Grashoff number
Da : Darcy number

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