

Simulation of Economic Production Quantity Model for Deteriorating Item

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Abstract A manufacturing business may be affected due to disruption produced either in production system or in raw material supply. The disruptions in production system occur due to appearance of uncertainty and unplanned events. For example, skilled labor problem, machine failure problem, system repair problem etc. Inventory managers need to monitor the raw material supply and anticipate the shortage before the system gets disrupted. The deterioration among produced items is also a worry factor for managers. In this paper, we suggest a production inventory model subject to condition of occurrence of disruption at input level and deterioration at output level. A comparative approach has been adopted between models with and without disruption. The main focus is on specific type disruption like sudden reduction in supply of raw material. Its effect has been examined in comparative approach. Parametric simulation is used to generate graphs and useful suggestions are made for inventory managers.

Keywords: *disrupted production system, inventory control, shortage, deterioration*

1. Introduction

There are many reasons that a production system gets disrupted like machine breakdown, unexpected events or emergency crises. An oil-drilling company may be disrupted due to failure in electricity supply, failure of drilling machines whereas an oil refining company faces problems of uneven crude oil supply, non-availability of other raw materials or earthquake and strike etc. It is assumed that a process is in control at the beginning of a production run. Suddenly the supply of raw material reduction by a constant factor, in the whole time cycle, production rate reduced. The modified production run ends at the same time as was before disruption. The change in optimal parameters before and after disruption has been examined in this paper.

Lin and Kroll (2006) one pioneer to solve the production problem under an imperfect production system subject to random breakdowns. Liao (2007) established an EPQ model by giving permission to delay in payment for the buyer to manufacturers.

A single vendor and multi-buyer inventory policy for a deteriorating item was due to Yang and Wee (2002). Teng and Chang (2005) presented an economic production quantity model for deteriorating items when the demand rate depends on not only on-display stock, but also on the selling price per unit of an item which may influence by economic policy, political scenario or agriculture productivity or both get affected. A similar approach has been followed by Hou and Lin (2006) on the deterministic economic order quantity model by taking into account the inflation and the time value of money for the deteriorating items with price and stock-dependent selling rate. By

dividing the demand rate into multiple segments, Shukla and Khedlekar (2010a) have introduced three-component demand rate for the newly launched deteriorating item.

Joglekar (2003) used a linear demand function with price sensitiveness and allowed retailers to use a continuous increasing price strategy in an inventory cycle. He derived the retailer's optimal profit by ignoring all the inventory costs. His results are restricted to growing market only, neither for stable market nor for a declining market. Expenditure sources like ordering cost, safety features, lead time and numbers of lots are the integral parts of decision making.

A number of structural properties of the inventory system are studied analytically by Samanta and Roy (2004) by determination of production cycle time and backlog for deteriorating item, which follows an exponential distribution. Qi *et al.* (2004) analyzed the supply chain-coordination with demand disruption in a deterministic scenario. Giri *et al.* (1996) who computed the optimal policy of an EOQ model with dynamic costs. The model they proposed is very basic though, since they have considered the very special case where the holding and ordering costs are linear functions of time. The other shortcoming of that paper is that the deterioration rate is also a linear function of time, and the algorithm they proposed in order to solve the problem is only valid as long as the demand rate is a linear function of time.

A central policy presented by Benjaafar ElHafsi (2006) specify a single product assemble-to-order system for my components, an end-product to serve and customer classes and problem solved as a Markov decision process and characterize the structure of an optimal policy.

We refer some useful contribution to reader Shukla and Khedlekar (2012), Lo (2007) and Khedlekar and Shukla (2013), Shukla *et al.* (2009, 2010b, 2010c). He *et al.*

(2010) obtained optimal production time to facilitate the manufacturer sell the item in multiple markets by considering constant demand rate, but they do not readjust the production system. Due to above contribution we incorporate the deteriorating factor with constant demand and adjust the disrupted production system with shortages and when it occurs an optimal time of placing an order is obtained along with order quantity from the spot market.

2. Model Description

Suppose a deteriorating item, manufactured by a manufacturer, sold to customers. It has demand rate μ production rate $p > \mu$ in each cycle. Since $p - \mu > 0$ so inventory accumulates at manufacture's level and stops production at time T_p due to excess stock of that item. Now inventory reduces constantly until entire stock vanishes at time H ($T_p < H$), it is normal phenomenon for any production cum storage system.

Now assume that disruption occurs in this process at time T_d ($T_d < T_p$) due to assignable causes (like strike, lake of raw material) and due to random causes (like failure of power, fatigue) etc. We consider the disruption in production as sudden cut off in the supply of raw material by a fixed amount say $\alpha\%$. This reduces p into $p + \Delta p$ (Δp may be positive or negative) at time T_d .

Now this cut off remain maintained constantly throughout the production cycle time H . the inventory curve (Figure 2) bears a shift in shape due to this region of disruption. The time of maximum inventory shifts to T_d^p ($T_d^p > T_p$) and maximum level of inventory reduces accordingly so as to finish up the production cycle at same time H (or H). This disruption may minor or major. In

case of minor level, the inventory vanishes at time H , but in case of major level, the stock condition (or supply) may be lesser to the demand rate μ and so the inventory reduces to zero before the time H .

3. Production System Without Disruption

To compare the model output first, management optimizes the production system run without disruption with production rate p (per unit time) stopped at production time T_p and there after till time H , inventory depicted due to demand rate μ and deterioration rate θ of items (see Figure 1). The presentations in differential equations for two periods $[0, T_p]$ and $[T_p, H]$ satisfy throughout its domain.

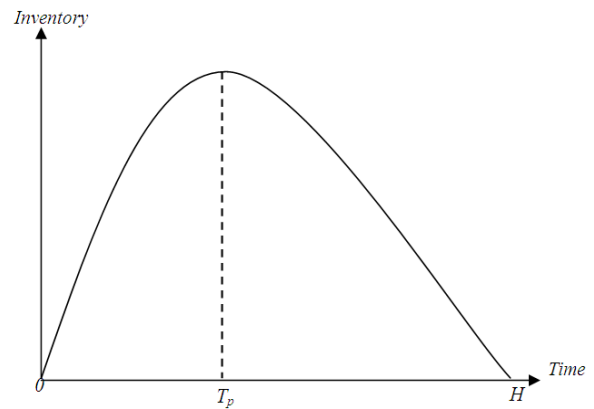


Figure 1. Production System without disruption

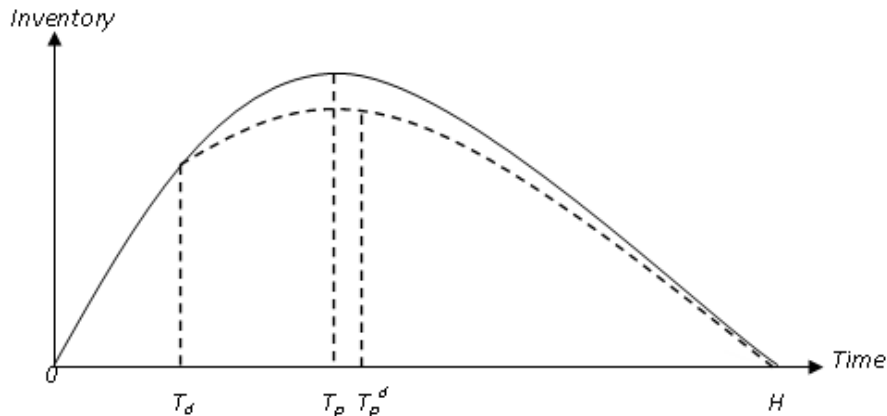


Figure 2. Production System after disruption $0 \leq T_d \leq T_p \leq T_d^p \leq H$

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - \mu, 0 \leq t \leq T_p \text{ boundary condition}$$

$$I_1(0) = 0 \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -\mu, T_p \leq t \leq H \text{ boundary condition}$$

$$I_2(H) = 0 \tag{2}$$

On solving Equation (1) and Equation (2) with boundary conditions we get

$$I_1(t) = \frac{p}{\theta} (1 - e^{-\theta t}) - \frac{\mu}{\theta} (1 - e^{-\theta t}) \tag{3}$$

$$I_2(t) = \frac{\mu}{\theta} (e^{\theta H - \theta t} - 1) \tag{4}$$

As per Figure 1 inventory level $I_1(t)$ and $I_2(t)$ are equal at time T_p .

i.e. $I_1(T_p) = I_2(T_p)$ yields

$$T_p = \frac{1}{\theta} \log \frac{\theta p - \theta \mu + \theta \mu e^{\theta H}}{p \theta} \tag{5}$$

If $\theta \ll 1$, then

$$T_p = \frac{\mu(1+\theta H) - \mu}{p\theta - \mu\theta + \mu} \tag{6}$$

Proposition 1. If $\theta \ll 1$ then T_p is in increasing in θ .
By Equation (6) one can write

$$\begin{aligned} \frac{dT_p}{d\theta} &= \frac{\mu H - (p - \mu)}{(p\theta - \mu\theta + \mu)^2} \\ &\geq \frac{\mu H - \mu p}{(p\theta - \mu\theta + \mu)^2} \text{ for } p > p - H \\ &\geq \frac{\mu H - 1}{(p\theta - \mu\theta + \mu)^2} \geq 0 \end{aligned} \tag{7}$$

This proved the result*

As θ increases optimal production time T_p increases that is more product required to producing. One can conclude that to keep low deterioration is effective way to keep lower cost of production of items.

4. Production System with Disruption

As previous section production rate remains unchanged but in practice production system is always disruption due to unplanned and thus we consider the production system little changed by ΔP and disruption time is T_d . If $\Delta P < 0$, then production rate decreases and, if $\Delta P > 0$ then production rate increases.

Proposition 2.

$\Delta p \geq (p\theta e^{-\theta H} - p\theta + \mu\theta(1 - e^{-H\theta})) / \theta(1 - e^{\theta T_d - H\theta})$ then manufacturer still satisfies the demand even production system has been disrupted, otherwise If $-p \leq \Delta p \leq (p\theta e^{-\theta H} - p\theta + \mu\theta(1 - e^{-H\theta})) / \theta(1 - e^{\theta T_d - H\theta})$ then production system unable to satisfy demand that is there will be shortages due to production disruption.

Proof: Suppose the production system disrupted at time T_d as (see Figure 2) and there after the production rate will be $P + \Delta P$ thus presentations of two differential equations for intervals $[0, T_d]$ and $[T_d, H]$ are

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - \mu, 0 \leq t \leq T_d \text{ with } I_1(0) = 0 \tag{8}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = p + \Delta p - \mu, T_d \leq t \leq H \tag{9}$$

with boundary condition

$$I_1(T_d) = I_2(T_d) = \frac{p}{\theta}(1 - e^{-\theta T_d}) - \frac{\mu}{\theta}(1 - e^{-\theta T_d})$$

On solving Equation (9) with boundary condition we get

$$I_2(t) = \frac{p}{\theta}(1 - e^{-\theta t}) + \frac{\Delta p}{\theta}(1 - e^{\theta T_d - \theta t}) + \frac{\mu}{\theta}(e^{-\theta t} - 1) \tag{10}$$

If $I_2(H) \geq 0$ this means production system satisfy the demand of items.

That is $\Delta p \geq \frac{p\theta e^{-\theta H} - p\theta + \mu\theta(1 - e^{-H\theta})}{\theta(1 - e^{\theta T_d - H\theta})}$ then still

satisfy the demand.

If $I_2(H) < 0$ this means production system does not satisfy the demand of items.

That is $-p \leq \Delta p < \frac{p\theta e^{-\theta H} - p\theta + \mu\theta(1 - e^{-\theta H})}{\theta(1 - e^{\theta T_d - H\theta})}$ then

there will be shortages in the system.

This proved the lemma*

Again if $I_2(H) \geq 0$ then we find optimal production time (with disruption) T_p^d such that at time H entire stock will be sold-out and inventory level will be zero.

If $I_2(H) < 0$ there will be shortages in the system and in this situation we will find the optimum time T_r of placing the order and respective order quantity Q_r .

Proposition 3. If $I_2(H) \geq 0$ then production time with disruption T_p^d is obtained by.

$$e^{\theta T_p^d} = \frac{p\theta - \mu\theta + \Delta p\theta e^{\theta T_d} + \mu\theta e^{H\theta}}{(p + \Delta p)\theta}$$

Proof: If $I_2(H) \geq 0$

or $\Delta p \geq \frac{p\theta e^{-\theta H} - p\theta + \mu\theta(1 - e^{-\theta H})}{\theta(1 - e^{\theta T_d - H\theta})}$ that is on hand

inventory is $I_2(H)$.

Therefore we will find out the optimal time T_p^d (see Figure 2) when we stopped the production after disruption in such a manner that stock remains zero at time H . the presentations of two differential equations for intervals $[T_d, T_p^d]$ and $[T_p^d, H]$ are

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = p + \Delta p - \mu, T_d \leq t \leq T_p^d \tag{11}$$

Boundary condition

$$I_1(T_d) = I_2(T_d) = \frac{p}{\theta}(1 - e^{-\theta T_d}) - \frac{\mu}{\theta}(1 - e^{-\theta T_d})$$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -\mu, T_d \leq t \leq H \text{ boundary condition}$$

$$I_3(H) = 0 \tag{12}$$

On solving (11) with boundary condition we get

$$\begin{aligned} I_2(t) &= -\frac{p}{\theta}(e^{\theta T_d} - 1)e^{-\theta t} + \frac{\mu}{\theta}(e^{\theta T_d} - 1)e^{-\theta t} \\ &+ \frac{p + \Delta p}{\theta}(1 - e^{\theta T_d - \theta t}) - \frac{\mu}{\theta}(1 - e^{\theta T_d - \theta t}) \end{aligned} \tag{13}$$

$$I_3(t) = \frac{\mu}{\theta}(e^{\theta H - \theta t} - 1) \tag{14}$$

Using condition $I_2(T_p^d) = I_3(T_p^d)$

$$e^{\theta T_p^d} = \frac{p\theta - \mu\theta + \Delta p\theta e^{\theta T_d} + \mu\theta e^{H\theta}}{(p + \Delta p)\theta} \quad (15)$$

This proved the result*

4.1. An Application with Simulation

If $I_2(H) < 0$,

By Equation (10), $I_2(T_r) = 0$,

$$\frac{p}{\theta}(1 - e^{-\theta T_r}) + \frac{\Delta p}{\theta}(1 - e^{\theta T_d - \theta T_r}) + \frac{\mu}{\theta}(e^{-\theta T_r} - 1) = 0, \quad (16)$$

and thus by Equation (14), order quantity will be

$$Q_r = I_3(T_r) = \frac{\mu}{\theta}(e^{\theta H - \theta T_r} - 1) \quad (17)$$

For application we assumed a particular case when $p = 350$, $\mu = 300$, $\Delta p = -100$, $\theta = 0.01$, $H = 15$ and $T_d = 1$, on applying then we get $I_2(H) > 0$ and thus by Equation (6) and Equation (15) gives $T_p = 0.14$, $T_p^d = 13.71$ days. We simulate the application on same data in which other parameters are invariant.

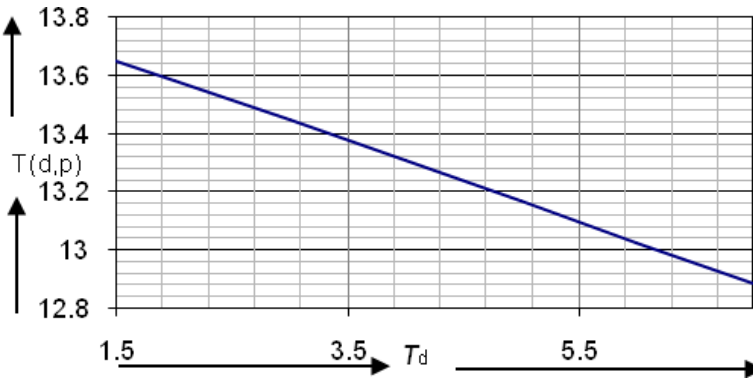


Figure 3. (Effect of T_d on T_p^d)

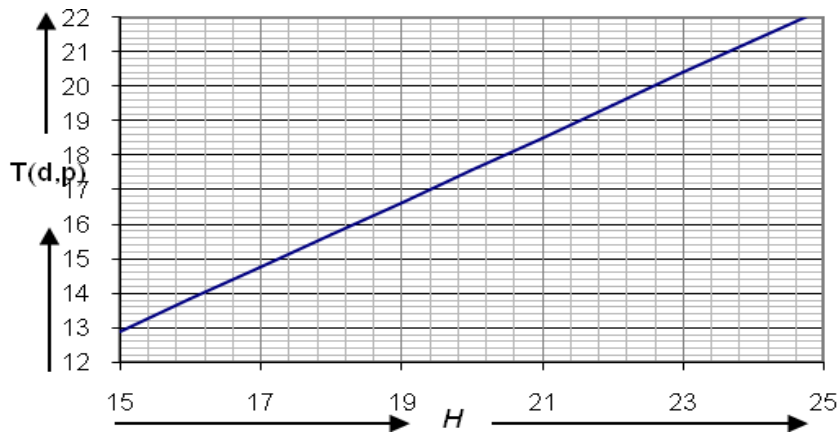


Figure 4. (Effect of H on T_p^d)

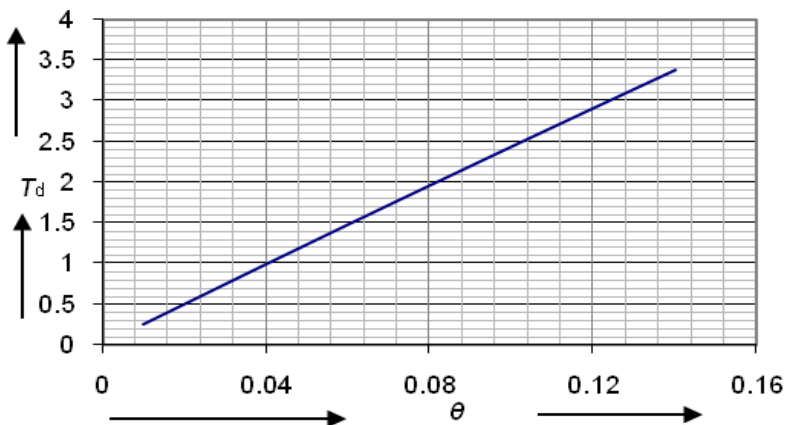


Figure 5. (Effect of θ on T_p^d)

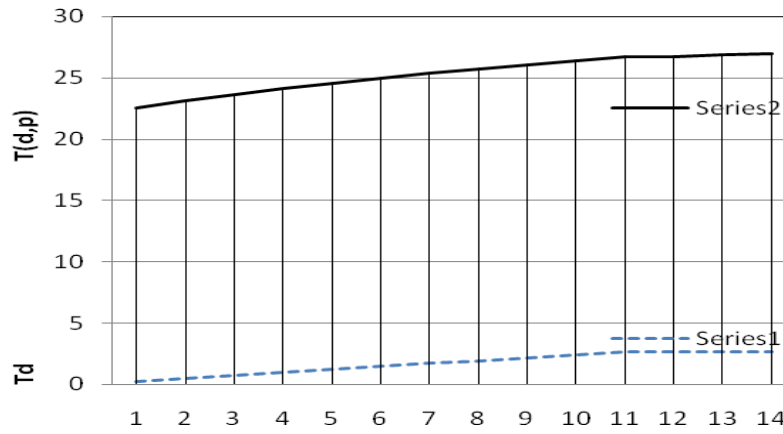


Figure 6. (Comparison between T_p^d and T_p with and without disruption w.r.t. θ)

Table 1. (Variations in parameters)

μ	θ	P	H	T_d	T_p	T_p^d
300	0.01	375	15	1	0.1496	13.719
300	0.01	375	15	2	0.1496	13.583
300	0.01	375	15	3	0.1496	13.446
300	0.01	375	15	4	0.1496	13.308
300	0.01	375	15	5	0.1496	13.167
300	0.01	375	15	6	0.1496	13.026
300	0.01	375	15	7	0.1496	12.882
300	0.01	375	15	7	0.1496	12.882
300	0.01	375	16	7	0.1596	13.820
300	0.01	375	17	7	0.1695	14.758
300	0.01	375	18	7	0.1795	15.697
300	0.01	375	19	7	0.1895	16.636
300	0.01	375	20	7	0.1995	17.576
300	0.01	375	21	7	0.2094	18.517
300	0.01	375	22	7	0.2194	19.458
300	0.01	375	23	7	0.2294	20.399
300	0.01	375	24	7	0.2394	21.341
300	0.01	375	25	7	0.2493	22.283
300	0.02	375	25	7	0.4975	22.621
300	0.03	375	25	7	0.7444	22.908
300	0.04	375	25	7	0.9900	23.152
300	0.05	375	25	7	1.2345	23.358
300	0.06	375	25	7	1.4778	23.533
300	0.07	375	25	7	1.7199	23.682
300	0.08	375	25	7	1.9607	23.808
300	0.09	375	25	7	2.2004	23.916
300	0.10	375	25	7	2.4390	24.008
300	0.11	375	25	7	2.6763	24.086
300	0.12	375	25	7	2.9126	24.154
300	0.13	375	25	7	3.1477	24.213
300	0.14	375	25	7	3.3816	24.264

4.2. Discussion

Time horizon H linearly increases the time T_p and T_p^d both (see Table 1). Production time T_p^d after disruption is linearly decreases as disruption T_d decreases (see Figure 3) that is system get disrupted later is in favor of management. Also the production time is not longer if it gets disrupted later. If deterioration increase then T_p^d is in linearly increases (see Figure 4), the same result followed by T_p with respect to deteriorations (see Figure 5). There is

a shift of stop time of production before and after disruption (see Figure 6), and both are highly sensitive on deterioration.

5. Conclusion

The deterioration factor affects negatively when disruption is present in the production system. In beginning, if production rate is higher than demand rate, then inventory managers are benefitted. It provides a little accumulation of stock to the managers. But, due to specific type of disruption, the reserve inventory helps to manage the market only up to a shorter period. It is interesting fact that when disruption appears its early occurrence may be dangerous but later occurrence is manageable.

Production model with disruption is quite different from the production model without disruption. The stop time for production after disruption is always greater than the stop time of production time without disruptions. If system get disrupted later it goes to favour of production system so, management should delay the disruptions as well as possible.

If production disrupted time is longer, then it is difficult to manage and it need to order more quantity from the spot market. The demand parameter highly affected the policy. Thus, the performance of any production system will be robust to demand variations and model uncertainty. The proposed model can be further extended by considering the more realistic assumption like time dependent production along with time dependent demand even production system gets disrupted, and deterioration may follow a probability distribution function.

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