

Modal Analysis Using the Signal Processing Toolbox of Matlab 2017

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Abstract The paper deals with determination of modal parameters by two software applications. Experimental modal analysis was performed by the commercial system Pulse® designed for vibration analysis of mechanical structures. The system was used to obtain excitation signals and corresponding response signals. The measurement data were subsequently processed in Pulse Reflex™ software and Matlab 2017, of which the signal processing toolbox provides the new functionalities specially aimed for modal analysis. The toolbox contains functions to calculate frequency response functions and to estimate modal parameters. The aim of the article is to compare the results achieved by both softwares when the first approach realized using commercial system is considered as reference. The subject of this work is to evaluate the validity of the obtained results and to point out the possibilities that the new Matlab toolbox provides.

Keywords: modal parameters, experimental modal analysis, vibration analysis

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1. Introduction

Modal analysis has a wide application in solving the problems of engineering practice. Its application is mainly found in the automotive and aerospace industries or in the manufacture of domestic appliances.

Modal analysis is a process of determining modal parameters of mechanical systems. Modal parameters describe their dynamic behavior. In general, modal analysis can be theoretical or numerical and experimental. Theoretical approach deals with mathematical model of mechanical system and solves the eigenvalue problem. When the modal parameters are determined by measurement it is called experimental approach [1,2,3].

The basis of experimental modal analysis is to record the time histories of excitation force and corresponding response of an investigated object. Its dynamic behavior can be described in frequency domain by the frequency response function FRF or in time domain by impulse response function IRF. These functions are calculated from measurement data. The relation between FRF and IRF is defined by inverse Fourier Transform

$$h(t) = F^{-1} \{ H(\omega) \}. \quad (1)$$

Frequency response function is defined as the ratio of output signal to input signal

$$H(\omega) = \frac{\text{output signal}}{\text{input signal}} = \frac{\text{movement}}{\text{force}}. \quad (2)$$

There are three basic forms of FRFs depending on the type of response parameter, which can be either displacement or velocity or acceleration, see Table 1. Set of all FRFs is called frequency response function matrix. Definition of one element of FRF matrix for receptance is as follows

$$\alpha_{jk}(\omega) = \frac{x_j}{F_k} = \sum_{r=1}^N \frac{\varphi_j^r \varphi_k^r}{\lambda_r^2 - \omega^2}, \quad (3)$$

where λ_r is eigenvalue of the r -th mode (natural frequency + modal damping), φ_j^r is j -th element of the r -th natural shapes' vector $\{\Phi\}$, i.e. relative displacement at the j -th point as vibrating with the r -th shape, N is number of modes. [2]

Table 1. Various Types of FRFs According to Response Parameter

Response parameter	Type of FRF
Displacement	Receptance
Velocity	Mobility
Acceleration	Inertance

The vibration modes are determined from FRF or IRF by applying special estimation procedures. Each mode is characterized by natural frequency, damping ratio and mode shape. These characteristics are commonly called modal parameters.

2. Experimental Modal Analysis with Pulse®

Modal analysis was performed by Pulse® system. The investigated object was a steel rod with diameter 22 mm and length 800 mm (Figure 1). The responses were measured by piezoelectric uniaxial accelerometer Brüel & Kjær type B4507 and object was excited by Brüel & Kjær impact hammer type 8206.



Figure 1. Investigated object

The geometric model consisting of 18 excitation points (marked by black-green hammer) and one response point (marked by red arrow) is shown in Figure 2. These points are referred as degrees of freedom (DOF). The SIMO measurement was performed. The rod was freely supported using soft foam. Frequency range was set up to 10 kHz with length of 6400 samples and sampling frequency 25.6 kHz. Only bending modes in the direction of measurement were considered.

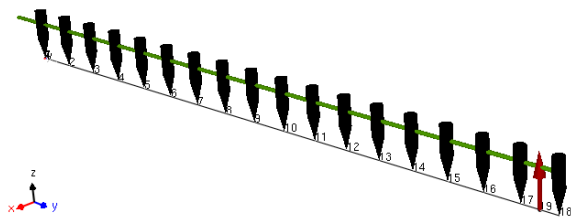


Figure 2 Geometry with DOFs

The measurement was evaluated in commercial Pulse Reflex™ software. The obtained FRFs in the form of inrtance are shown in Figure 3.

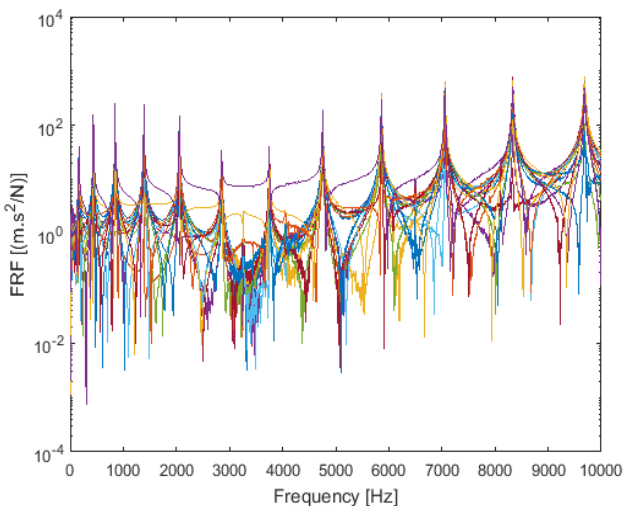


Figure 3. FRFs obtained by Pulse Reflex™

The Rational Fraction Polynomial method was used to estimate modal parameters. The natural frequencies and damping ratios for each identified mode are listed in Table 2. Figure 4 – Figure 15 show the corresponding mode shapes. It is clear that the mode shapes are more complicated as the natural frequency increases. Therefore, more DOFs are needed for a better shape representation of higher modes.

Table 2. Modal parameters estimated by Pulse Reflex®

Mode	Frequency [Hz]	Damping ratio [%]
1	164.80	1.7536
2	435.91	0.3313
3	845.64	0.1356
4	1388.00	0.1183
5	2055.03	0.1065
6	2849.16	0.1208
7	3749.44	0.0619
8	4753.31	0.0520
9	5859.11	0.0454
10	7054.13	0.0436
11	8333.33	0.0640
12	9685.42	0.0995



Figure 4. First mode shape

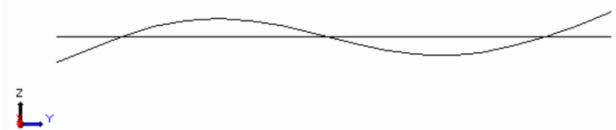


Figure 5. Second mode shape



Figure 6. Third mode shape

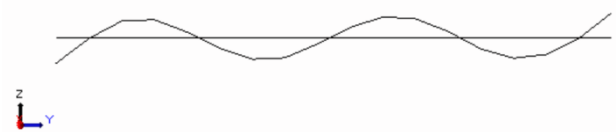


Figure 7. Fourth mode shape

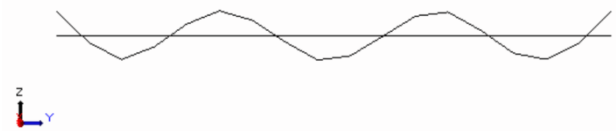


Figure 8. Fifth mode shape

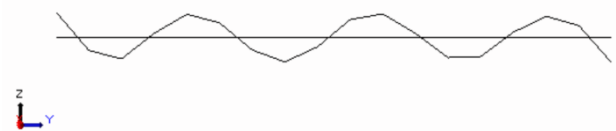


Figure 9. Sixth mode shape

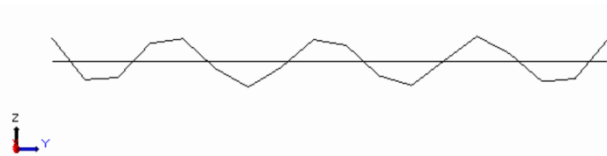


Figure 10. Seventh mode shape

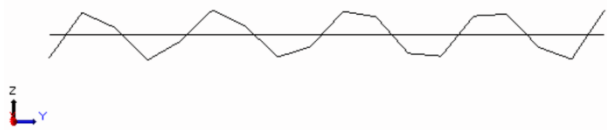


Figure 11. Eighth mode shape

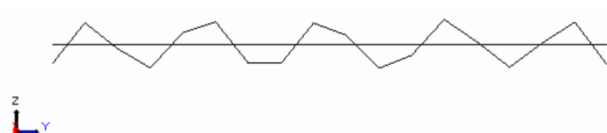


Figure 12. Ninth mode shape

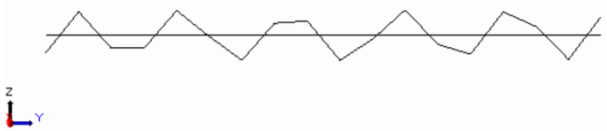


Figure 13. Tenth mode shape

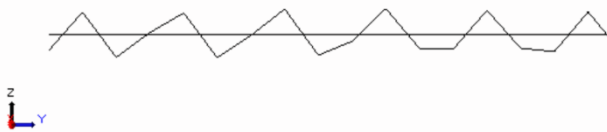


Figure 14. Eleventh mode shape

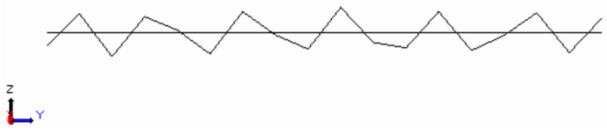


Figure 15. Twelfth mode shape

The above results were considered to be references. The excitation time signals and response time signals were subsequently exported to wav format. The exported data was then imported to Matlab 2017.

3. Modal Analysis Using Matlab 2017

Signal Processing Toolbox available in Matlab 2017 includes modal analysis functions that have been used to process the data exported from Pulse.

The function **modalfrf** was used to compile FRF matrix. Its syntax is as follows:

```
[frf,f,coh] = modalfrf(x, y, fs, window,...
    noverlap, 'Estimator', 'H1', 'Measurement',...
    'rovinginput', 'Sensor','acc');
```

The function **modalfrf** calculates a FRF matrix from the excitation signals (x) and the response signals (y), all sampled at a rate fs. The data of signal x and signal y must

have the same number of rows. If x or y is a matrix, each column represents a signal. The signals can be multiplied by weighting window function (window). Matlab has a wide offer of window functions.

The output, frf, is an H1 estimate computed using Welch's method. Available are other method to estimate FRF, H2 and Hv. H1 method is used when there is noise at the input signal. H2 method is used when there is noise at the output signal. Hv method is used when there is noise at the input and output signal. The estimate method is determined by a comma-separated pair, 'Estimator', 'H1' or 'H2' or 'Hv'. The default is H1 method. The comma-separated pair 'Sensor', 'acc' or 'vel' or 'dis' is used to specify the response parameter [1,4].

The function **modalfit** is used for the estimation of modal parameters from FRFs. Its syntax is as follows

```
[fn,dr,ms] = modalfit(frf,f,fs,mnum,...
    'FitMethod', 'lsce', ...
    'FreqRange', '[100 200]',...
    'DriveIndex', '[2,3]);
```

The function **modalfit** estimates the natural frequencies, damping ratios and mode shapes vectors of modes (mnum = number of modes expected in the frequency range) of a system having measured frequency response functions (frf) defined at frequencies (f) and for a sample rate (fs). Least-Square Complex Exponential Methods and Peak Picking Methods are available to estimate modal parameters. The estimation method is determined by a comma-separated pair 'FitMethod', 'lsce' or 'pp'. The estimation in a specific frequency range, e.g. from 100 to 200 Hz, is adjust by a comma-separated pair 'FreqRange', '[100 200]'. [4]

The **modalfrf** function was used to create a frequency response function for all measured combination excitation/response. The obtained FRFs are shown in Figure 16.

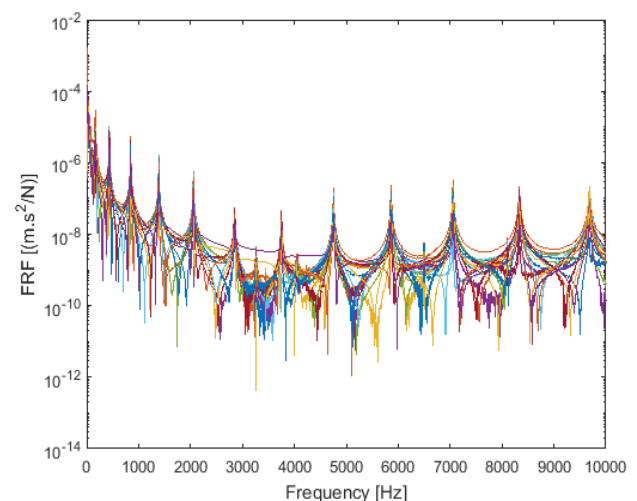


Figure 16. FRFs obtained from measured signals using modalfrf function

As can be seen, the functions are different from the references. Therefore, the function **tfestimate** that uses Welch's averaged periodogram method was used to compute new FRFs from measured data. Its syntax is as follows:

```
[frf, f] = tfestimate(x, y, window,...
noverlap, nfft, fs);
```

where (nfft) defines the number of sampling points to calculate the discrete Fourier transform. The FRFs obtained such way are shown in Figure 17. An increased noise level may be observed, but sufficient match is achieved in the resonance areas.

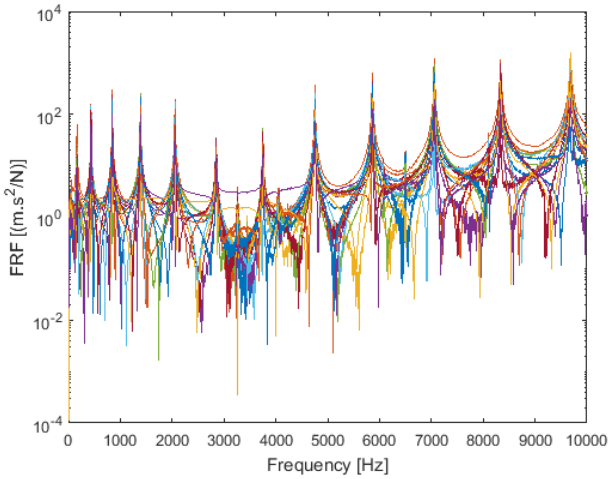


Figure 17. FRFs obtained from measured signals by the standard procedure using Fast Fourier Transform (FFT) algorithm

The estimation of modal parameters in Matlab was carried out by the function **modalfit** which was applied to four frequency ranges 0 ÷ 1 kHz, 1 ÷ 3 kHz, 3 ÷ 5 kHz, 5 ÷ 7.5 kHz, 7.5 ÷ 10 kHz, respectively. In this case, the PP method provides better estimation than the LSCE method, especially in terms of damping values. The estimated modal parameters are shown in Table 3.

Table 3. Modal parameters estimated by Matlab functions

Mode	Peak Picking method		LSCE method	
	Frequency [Hz]	Damping ratio [%]	Frequency [Hz]	Damping ratio [%]
1.	164.99	1.409	160.65	5.942
2.	435.90	0.232	436.56	0.485
3.	845.59	0.086	846.33	0.087
4.	1387.9	0.070	1389.4	0.099
5.	2054.6	0.098	2056.2	0.160
6.	2849.2	0.143	2849.7	2.133
7.	3749.6	0.078	3752.2	0.162
8.	4754.0	0.068	4754.6	0.037
9.	5858.6	0.036	5863.9	0.054
10.	7053.6	0.033	7054.8	0.060
11.	8333.4	0.059	8337.7	0.074
12.	9685.9	0.096	9679.3	0.120

The comparison of the results listed in Table 2 and Table 3 shows the sufficient match in natural frequencies and damping ratios. Mode shapes identified by **modalfit** function are shown in Figure 18 and Figure 19.

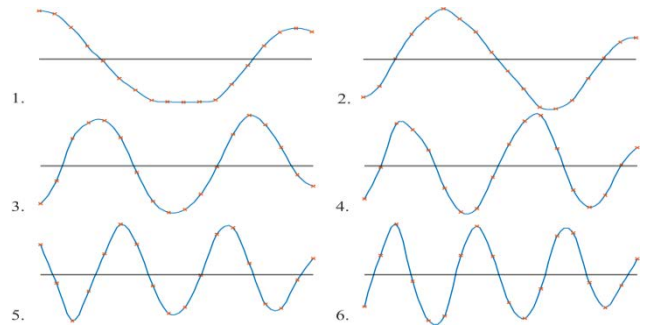


Figure 18. Mode shapes (1-6) estimated by Matlab functions

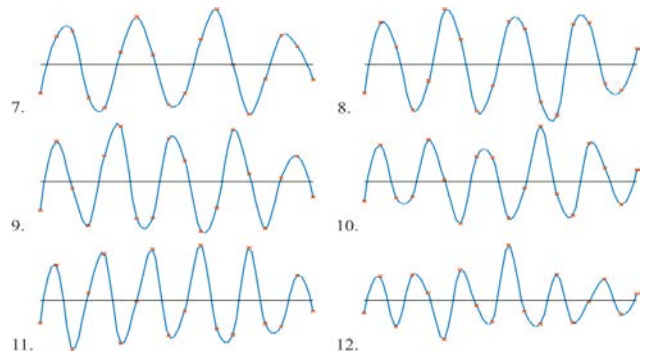


Figure 19. Mode shapes (7-12) estimated by Matlab functions.

4. Conclusion

In the paper, the experimental modal analysis of the free supported steel rod was performed for the purpose of the assessment the accuracy of the results obtained by using the new Matlab functions aimed to estimation of modal parameters. The function **modalfrf** was used to calculate frequency response functions, i.e. to built FRF matrix from time signals of excitation and response. FRFs obtained using these function were not correct compared to the references. Therefore, the older function **tfestimate** had to be used instead. It is necessary to note that the change of parameter 'Sensor' to 'dis' solved the problem, but this setting did not correspond to reality. The function **modalfit** was used for the estimation of modal parameters from FRF matrix. The function uses two methods (PP, LSCE) for the estimation. Since the both methods have its advantages and disadvantages, the emphasis must be placed on their use and interpretation of the results.

Acknowledgements

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References

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