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The Influence of Preload on Modal Parameters of a Cantilever Beam

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Abstract The paper is focused on a determination of dynamic characteristics of a cantilever beam with a free end and dynamics characteristics of the same beam when the free end is pushed down by using a rubber string. This leads to bend of the beam and causes its preloading. The modal parameters are determined by experimental modal analysis and the results of the tests with and without preload are compared each other. The numerical simulation of non-loaded beam is the part of the study.

Keywords: *modal parameters, cantilever beam, preload*

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1. Introduction

Structural Dynamics analysis has irreplaceable space in engineering practice. Generally, the modal parameters depends of structure geometry, the boundary contion and material properties [1-6]. If the one of these parameters is changed, the modal parameters are changed, too. Modal analysis is tool to deterimate modal parameters. The modal analysis can be divided to experimental and theoretical.

The aim of experimental analysis is studying relations between excitation and response. These relations present dynamic behavior of structure. It can be expressed as:

$$H(\omega) = \frac{\text{output signal}}{\text{input signal}} = \frac{movement}{force}. \qquad (1)$$

Behavior can be analysed in time domain (then we say about Impulse Response Function) or in frequency domain (than we say about Frequency Response Function). The response can be measured in 3 forms, as displacement, velocity and acceleration. The name of Frequency Response Function with respect to the response parameter is given in Table 1.

Table 1. Frequency response function respectively to response parameter

Parameter of response	Frequency response function
Displacement	Receptance
Velocity	Mobility
Acceleration	Inertance

2. The Modal Analysis of the Beam without Preload

Cantilever beam was used to investigate. The beam was fixed to the heavy freame. Dimensions of the beam are 430 x 40 x 2 mm (Figure 1). The measurement points representing the location of output and input DOFs were marked on the top face of the beam. There were 27 points in which the structure was excited and 2 points in which the responses were measured.

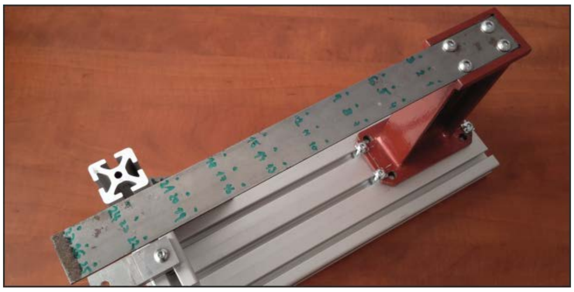

Figure 1. The analyzed beam

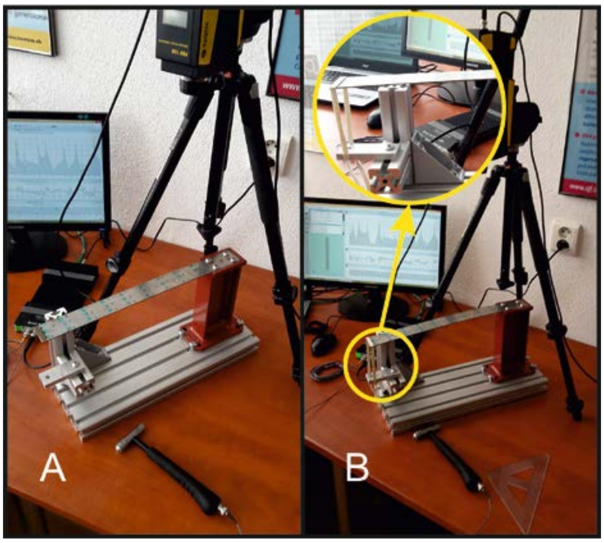

Figure 2. Experiment setup

Two experiments were carried out to review the influence of preload on the dynamic beavior of the beam. In the first case (Figure 2A), the modal parameters were determined for the free-fixed beam. In the second case (Figure 2B), the free end of the beam was bended by using preloaded rubber string.

2.1. The Setup of the First Experiment

System Pulse produced by Brüel & Kjær company was used for measuring. The measure chain consisted of:

- impact hummer,
- laser Doppler vibrometer,
- measuring modul LAN-XI
- notebook with software Pulse LabShop and Pulse Reflex®.

The impact hammer type 8206 with an aluminium tip was used for structure excitation. Responses were measured by laser vibrometer PDV 100 by Polytec. The response parameter was velocity, it means, that FRFs were measured in form of mobility.

The next step was to define a geometry model of the beam. The model consisted of 29 degrees of freedom. Two of them were the output reference DOFs, all other were the input DOFs. These DOFs were used to create the triangular faces approximing the surface of the beam (Figure 3). The faces (marked gray) enabled the better animation of mode shapes. Black-green markers represent the locations and driections of excitation and red arrows shows the place and directions of response measurement. The measurement frequency range was set to 0-2000 Hz with the spectrum resolution of 0.625 Hz.

2.2. The Evaluation of the Measurement

Frequency response functions obtained by the measurement were exported from Pulse LabShop software into Pulse Reflex® for the postprocessing and the extraction of modal parameters. Complex mode indentificator function (CMIF), based on singular value decomposition of FRF matrix, was used for an initial estimation. CMIF magnitude spectrum is shown in Figure 4. Every one of thirteen peaks can indicate a one mode of vibration. Two peaks at the frequency about 210 Hz represetnts very close modes. The same situation can be seen at the frequency about 640 Hz.

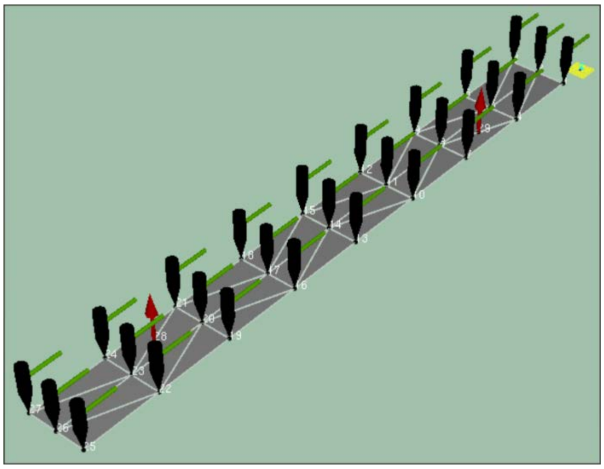

Figure 3. Geometry

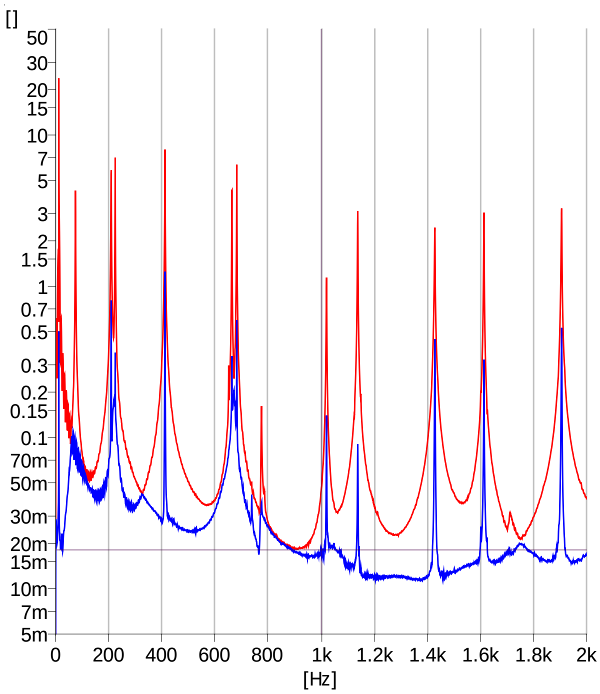

Figure 4. CMIF plot of the non-preloaded beam

Table 2. Modal parameters of the beam without preload

Mode	Damped Frequency (Hz)	Damping (%)	Complexity
1.	11.92	0.2502	0.0236
2.	74.57	0.44656	0.00803
3.	208.95	0.26299	0.01766
4.	224.28	0.11057	0.01255
5.	411.91	0.08685	0.06532
6.	663.77	0.02641	0.10194
7.	682.38	0.03979	0.12217
8.	1020.29	0.06157	0.08943
9.	1138.23	0.0246	0.75522
10.	1427.17	0.116	0.02918
11.	1613.64	0.03727	0.03934
12.	1905.77	0.04284	0.10714

Rational fraction polynomial method was used to estimate the modal parameters more precise. The twelve modes were identified by this method. The peak at the frequency circa 780 Hz was not identified as mode. This peak may be a mode corresponding to the frame.

The results of the measurement are listed in Table 2, where the values of natural frequencies and damping ratio are written. The corresponding modes shapes are shown in Figure 6 and Figure 7 on the left side.

2.3. Finite Element Analysis

The numerical simulation was performed for the verification and comparing of results from the first experiment.

The simulation was performed in Siemens NX 10.0, where the CAD model of structure has been created. The beam cross section with dimensions of 40x2 mm was sketched as the first. This cross section area was extruded to the length of 430 mm. The holes were created on the base of dimensions of the physical model. Subsequently, the finite element mesh with 8-node linear brick elements type C3D8 was created. Global element size was defined to 2.5 mm. The finite element model is shown in Figurr 5.

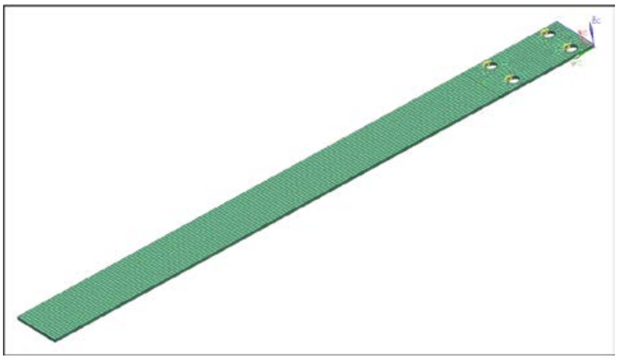

Figure 5. Finite element model of the beam

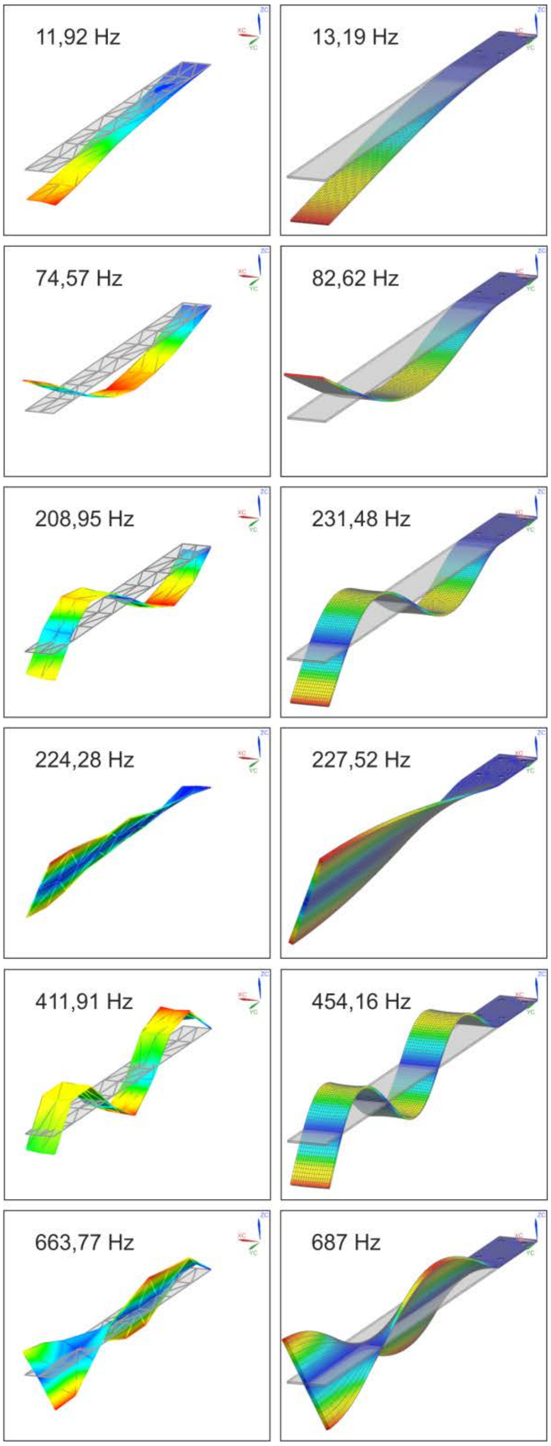

Figure 6. The 1. – 6. mode shapes of the beam without preload obtained by experiment (left) and by FE analysis (right)

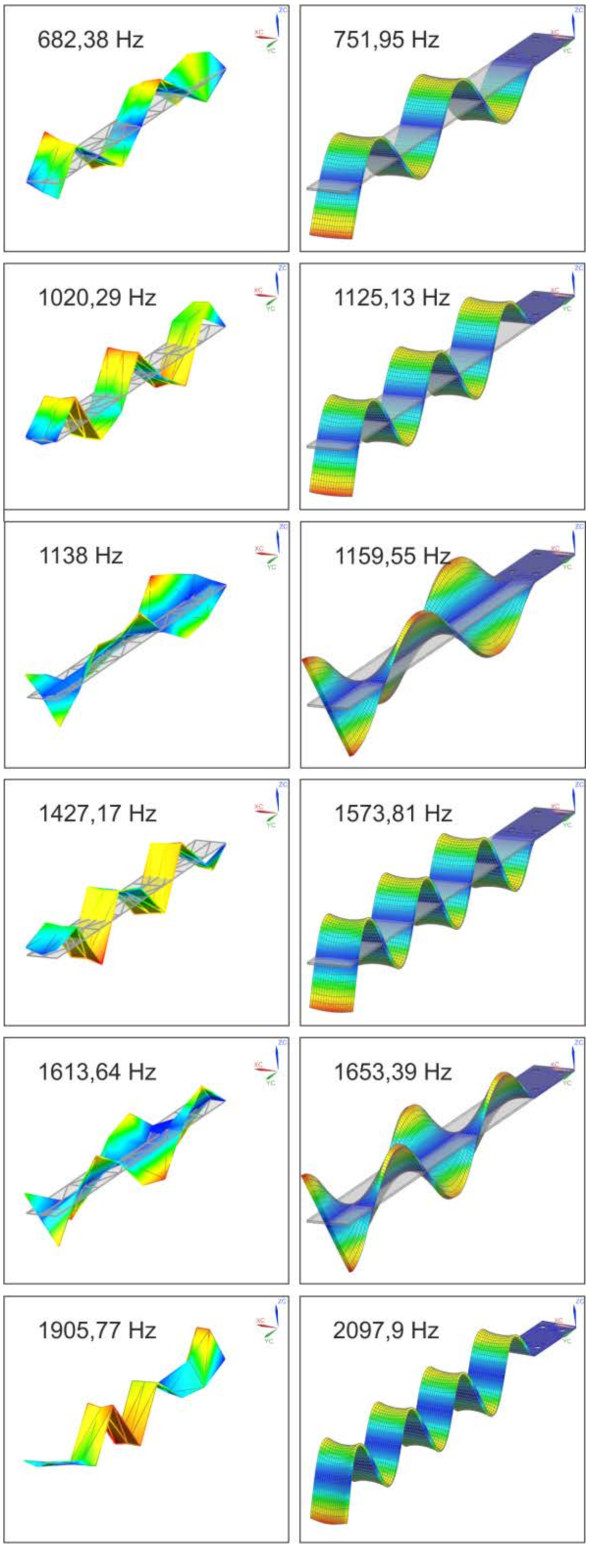

Figure 7. The 7. – 12. mode shapes of the beam without preload obtained by experiment (left) and by FE analysis (right)

Solution type SOL 103 Eigenvalues was used for the analysis. The constraints were as follows:

- all translations on the internal faces of the holes have been removed,
- the translation in the normal direction of the bottom face having a contact with the frame has been set to zero.

The natural frequencies resulting from the simulation are written in Table 3. The mode shapes are shown in Figure 6 and Figure 7 on the right side.

Table 3. The natural frequencies resulting from the simulation

Mode	Natural Frequency (Hz)
1.	13.19
2.	82.62
3.	227.52
4.	231.48
5.	239.59
6.	454.16
7.	687
8.	751.95
9.	1125.13
10.	1159.55
11.	1427.59
12.	1573.81
13.	1653.39
14.	2097.9

2.5. The Comparison of the Results

The comparison of the results from the experiment and the simulation shows relatively small differences between the measured and the computed natural frequencies. These frequency shifts are caused by differences in boundary conditions. Computed frequencies are higher than measured because the constraints considered in the simulation are totaly rigid.

Two more modes resulted from the simulation in the given frequency range. These modes are both inplane (Figure 8), so they could not be measured in the experiment because the excitation was perpendicular to the the oscillation plane of these modes.

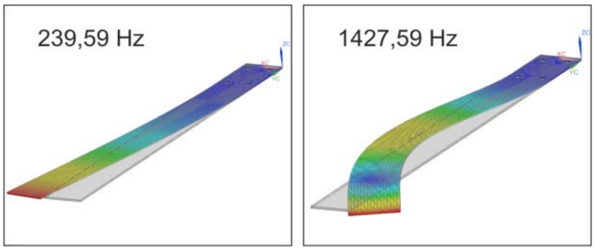

Figure 8. Inplane mode shapes

Table 4. The natural frequencies and MAC values

Mode	Experiments Frequency (Hz)	Simulation Frequency(Hz)	MAC (-)
1.	11.92	13.19	0.990
2.	74.57	82.62	0.994
3.	208.95	231.48	0.978
4.	224.28	227.52	0.977
5.	411.91	454.16	0.980
6.	663.77	686.99	0.946
7.	682.38	751.95	0.583
8.	1020.29	1125.13	0.892
9.	1138.00	1159.55	0.881
10.	1427.17	1573.81	0.812
11.	1613.64	1653.39	0.933
12.	1905.77	2097.90	0.764

The matched mode shapes were correlated by MAC criterion [4]. MAC value is a scalar quantity, acquiring values in the range of zero to one. The value one indicates that shapes are the same. The value zero indicates zero compliance of the modes. The higher MAC value is, the more similar shapes are. The linear collinearity between the measured and the computed modes is expressed in Table 4.

3. The Experimental Modal Analysis of the Preloaded Beam

Preload was realised by using rubber string. Free end was pushed down about 15 mm (Figure 9).

Figure 9. The initial bending of the preloaded beam

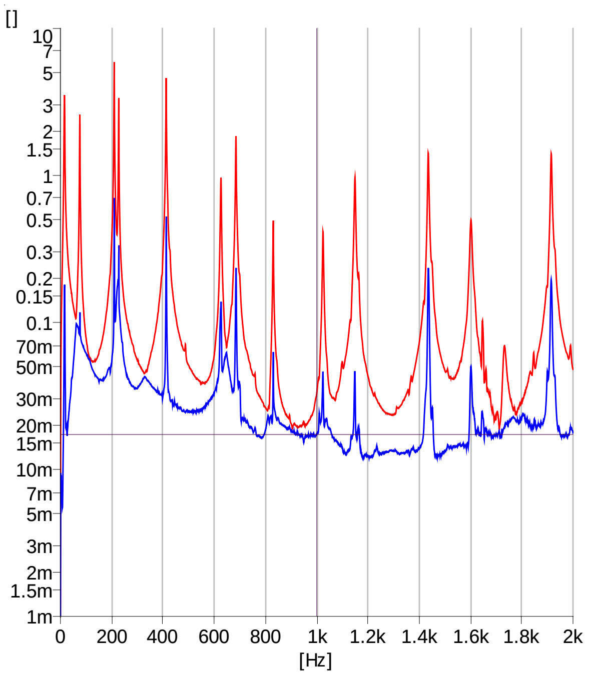

Figure 10. CMIF plot of the preloaded beam

Table 5. Modal parameters of the preload beam

Mode	Damped Frequency (Hz)	Damping (%)	Complexity
1.	15.22	5.50047	0.07209
2.	75.53	0.61505	0.06766
3.	209.77	0.27466	0.03935
4.	227.54	0.19008	0.19887
5.	412.44	0.16137	0.03736
6.	625.69	0.13526	0.197
7.	684.75	0.11531	0.08423
8.	1025.033	0.12632	0.18164
9.	1148.99	0.14316	0.09119
10.	1435.12	0.06134	0.24137
11.	1601.44	0.23032	0.31267
12.	1915.22	0.11994	0.04359

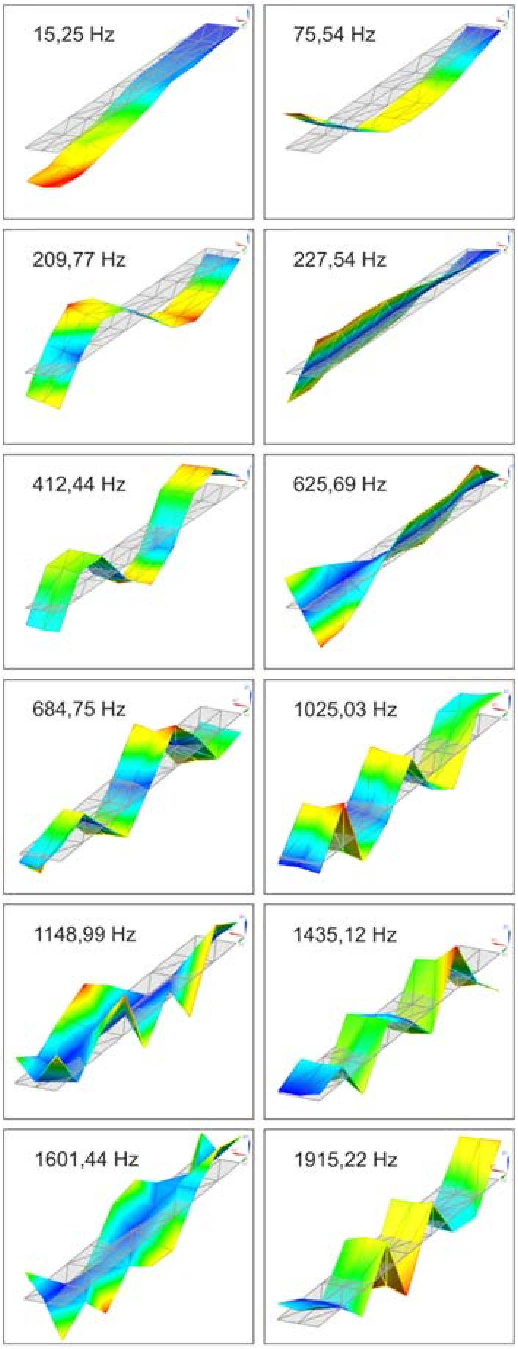

Figure 11. The mode shapes of the beam with preload

The experimental modal analysis was performed the same way as the first measurement. CMIF function from the second measurement is shown in the Figure 10. There are 14 peaks, but ony twelve of them correspond to the outplane modes of the beam. The peaks with frequencies about 830 Hz and 1710 Hz probably represent the (unmeasurable) inplane modes. The natural frequencies and damping ratios of the preloaded beam are listed in Table 5. The mode shapes are shown in Figure 11.

4. Discussion of Results

If we compare the natural frequencies and damping ratios of the beam with and without preload, we can find that both values are higher in the second case. See Table 6. An increasing of the frequencies was caused by the change of beam stiffness due to its bending and its reinforcement in the place where the rubber string was applied. The significant increase in damping ratio values was caused due to the rubber string used on the free end. We can also see that the mode shapes are influenced. Taking these facts into account, the use of electromagnet could be the better way to achieve properly beam.

Table 6. Modal parameters of the beam with and without preload

Mode	Without preload		With preload	
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)
1.	11.92	0.2502	15.22	5.50047
2.	74.57	0.44656	75.53	0.61505
3.	208.95	0.26299	209.77	0.27466
4.	224.28	0.11057	227.54	0.19008
5.	411.91	0.08685	412.44	0.16137
6.	663.77	0.02641	625.69	0.13526
7.	682.38	0.03979	684.75	0.11531
8.	1020.29	0.06157	1025.033	0.12632
9.	1138.23	0.0246	1148.99	0.14316
10.	1427.17	0.0116	1435.12	0.06134
11.	1613.64	0.03727	1601.44	0.23032
12.	1905.77	0.04284	1915.22	0.11994

5. Conclussion

In the paper, experimental modal analyses of the cantilever beam with and without preload were performed for the purpose of the assessment, how this preloading influences on the dynamic behavior of the beam. The preload was realised by the rubber string applied on the free end. The inial maximal bending was 15 mm. The results showed that the change of boundary conditions led to shift of natural frequencies and damping ratios to higher values and to distortion of mode shapes. In the future work, an electromagnet will be used for the preloading of the beam.

Acknowledgements

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