

# The Buckling Analysis of the Composite Plates with Different Orientations of Layers

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**Abstract** This paper deals about a buckling analysis of composite plates with four different layers orientations and with three, six and twelve layers. Composite plate was modeled by the finite element method. Compression load, shear load and combination of both loads were applied to the composite plate. Then the computational buckling analysis for the all combinations of load was performed for four different orientations of layers and for plates with different numbers of layers. The computed critical buckling loads for the applied loads were compared for all orientations of layers. From the results the inappropriate orientations of layers for different loads were obtained. The critical buckling load increased almost ten times for the composite plates with two times higher thickness.

**Keywords:** buckling, composite plate, orientation of layers, finite element method, compression load, shear load

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## 1. Introduction

The increasing labor productivity in machinery puts higher demands on the strength and stiffness properties of the structural members. Since the strength and rigidity depends on the mechanical properties of the used material, the development of new materials is one way of improving the required parameters. The example of these materials are composites. These exhibit extraordinary material properties depending on the used components. The composites are often used in modern cars, ships and in others modern equipments.

The buckling of the composite plates is a very complicated subject and more details can be seen in references [1-7]. In these publications, the authors investigated pre- and post-buckling of the composite plates using various methods (experimental, numerical and computational). For analyzing the buckling behavior of laminated composite plates, a recently proposed smoothed finite element method named the cell-based smoothed discrete shear gap method (CS-DSG3) is employed in [8]. Researchers in publication [9] introduced an explicit procedure in designing laminated composite plates for maximum buckling load subjected to strength and stiffness constraints using the polar formalism, which relates stiffness to the critical buckling load in order to simplify the calculation and optimization of the buckling load. Design for maximum flexural stiffness as well as laminate effective elastic modulus was developed in order to achieve the optimal orientation of the layers giving the maximum buckling strength. In the paper [10], the thickness of laminated composite plates is minimized by

optimizing the fiber orientation angles for different load cases. The influence of material properties was investigated in papers [11,12].

The strength and stiffness of composites can be evaluated using experimental, analytical or computational methods. In this paper the linear buckling analysis of the composite plates with different three, six and twelve layers and four orientations of layers is investigated by computational method. The composite plate is modeled as shell model and then it is loaded by compression load, shear load and combination of these two loads. Then the obtained critical loads are compared for all orientations of layers and for all numbers of layers.

## 2. Buckling Analysis

The buckling analysis of the square composite plate (Figure 1) with 3, 6 and 12 plies, and four different layouts of plies is performed. The material properties of composite plate are:  $E_x = 50$  GPa,  $E_y = 15$  GPa,  $E_z = 15$  GPa,  $G_{xy} = 5$  GPa,  $G_{xz} = 5$  GPa,  $G_{yz} = 4.5$  GPa,  $\mu_{xy} = 0.3$ ,  $\mu_{yz} = 0.4$  and  $\mu_{xz} = 0.3$ . The thickness of one ply is  $t = 0.5$  mm. The orientations of plies  $\alpha$  in composite with three plies are defined as [0 45 0], [45 0 45], [0 90 0] and [90 0 90], for more plies than three are used symmetric orientations. The composite layout for orientation of plies defined as [0 45 0] is shown in Figure 2.

### 2.1. Boundary Conditions and Applied Load

The boundary conditions applied to composite plate are considered to be zero and they are defined as follows:

- on the edges parallel with  $x$  axis:  $u_z, r_x, r_y$ ;

- on the edges parallel with y axis:  $u_y, u_z, r_x, r_y, r_z$ .  
Then the buckling analysis for three different load states is performed for:

- plate loaded by compression load  $p$  (Figure 3),
- plate loaded by shear load  $q$  (Figure 4),
- plate loaded by combination of compression  $p$  and shear load  $q$  (Figure 5).

The magnitude of all loads is  $p = q = 1 \text{ N/mm}$ .

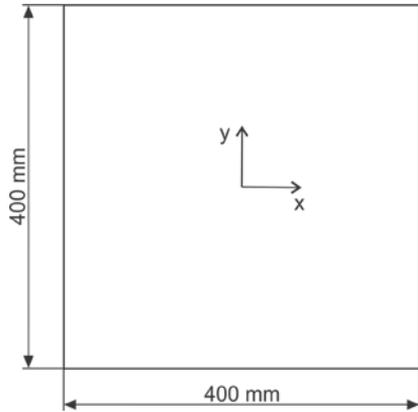


Figure 1. Model of composite plate

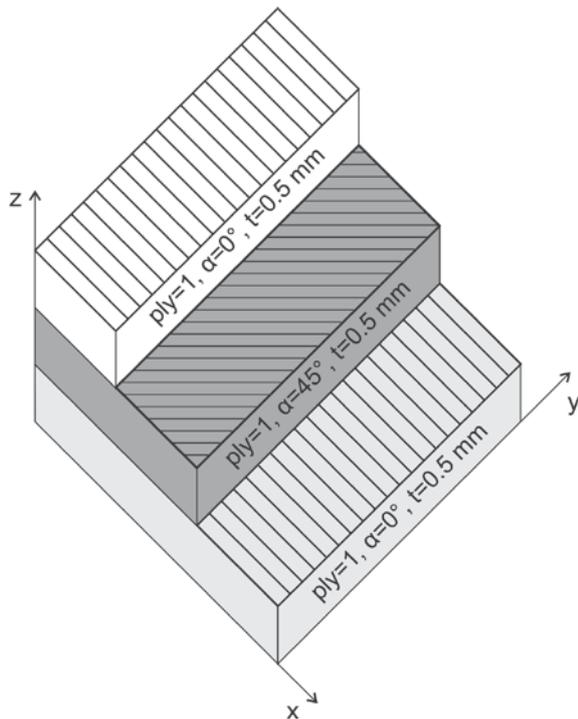


Figure 2. Composite layout for [0 45 0]

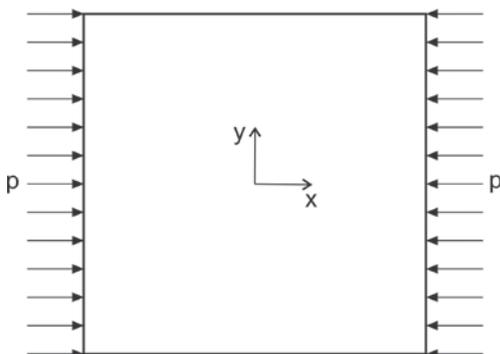


Figure 3. Composite plate loaded by compression load

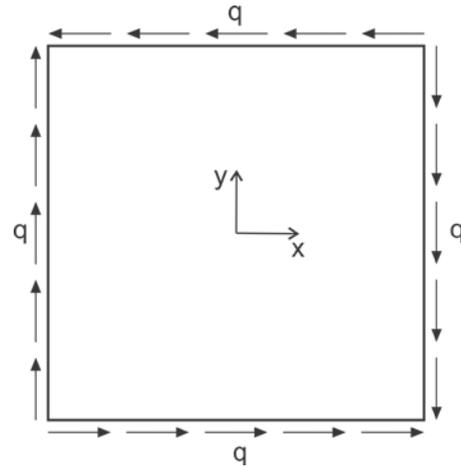


Figure 4. Composite plate loaded by shear load

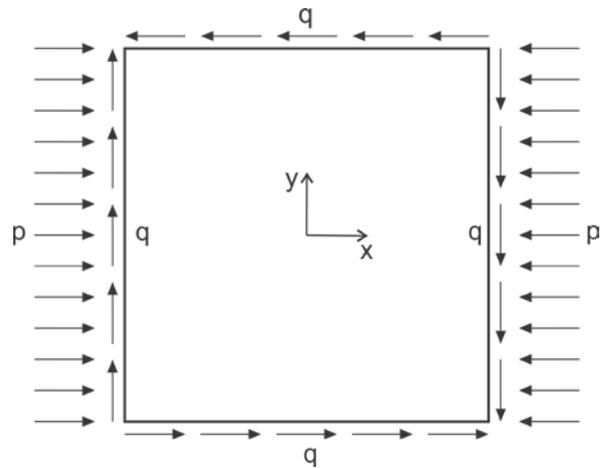


Figure 5. Composite plate loaded by combination of compression and shear load

## 2.2. Finite Element Model

For a linear buckling analysis, the eigenvalue problem is solved (below) to get the buckling load multiplier  $\lambda_i$  and buckling modes  $\psi_i$

$$([K] + \lambda_i [S]) \{\psi_i\} = 0, \quad (1)$$

where  $[K]$  is the stiffness matrix and  $[S]$  is the stress stiffness matrix. This matrix includes the effects of the membrane loads on the stiffness of the structure. The stress stiffening matrix is assembled based on the results of a previous linear static analysis. The buckling solution uses an algorithm that extracts firstly the load multiplier  $\lambda_i$  and secondly the buckling modes  $\psi_i$  that define the corresponding mode shape. The buckling load multiplier  $\lambda_i$  is defined as

$$\lambda_i = \text{Buckling Load} / \text{Applied Load}. \quad (2)$$

The finite element model is created using shell elements with defined composite layout (Figure 6). The finite element mesh is created with element of size 10 mm. The finite element mesh consists of 4961 nodes and 1600 elements. For the study effect of element size on the magnitude of the critical buckling load, the finite element model is meshed using shell elements with size 20 mm and 5 mm, respectively. The results show that the differences between critical buckling loads are less than 2 %.

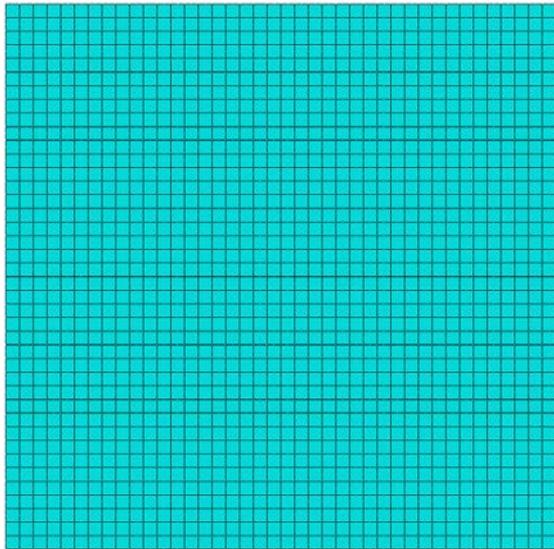


Figure 6. Finite element mesh

The applied boundary conditions on the finite element model are shown in Figure 7.

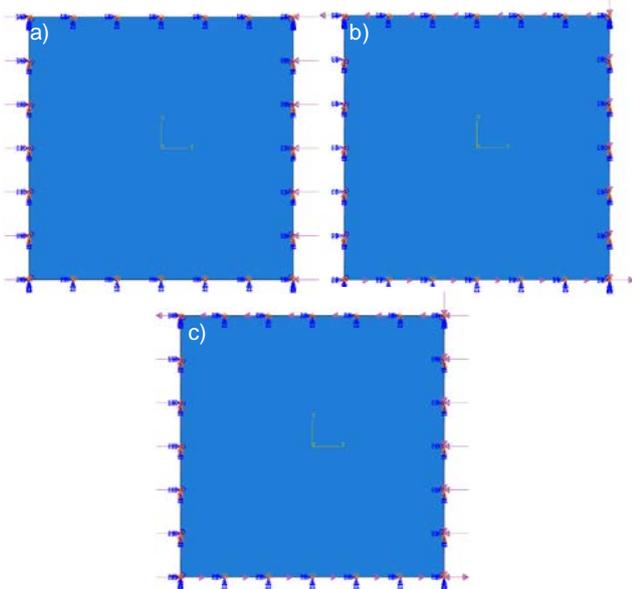


Figure 7. Applied boundary conditions and load for a) compression, b) shear, c) combination of compression and shear

Table 1. Values of the critical buckling load

Nr.	$\alpha$ [°]	Critical buckling load [N/mm]		
		C	S	C & S
3	[0 45 0]	9.79	25.31	19.93
3	[45 0 45]	8.02	20.61	23.09
3	[0 90 0]	10.21	27.36	21.44
3	[90 0 90]	7.66	27.35	18.58
6	[0 45 0] <sup>S</sup>	76.91	218.1	170.51
6	[45 0 45] <sup>S</sup>	68.542	183.63	179.98
6	[0 90 0] <sup>S</sup>	80.77	222.95	170.48
6	[90 0 90] <sup>S</sup>	76.46	222.29	164.19
12	[0 45 0] <sup>S</sup>	607.7	1687.72	1369.62
12	[45 0 45] <sup>S</sup>	551.84	1489.24	1414.32
12	[0 90 0] <sup>S</sup>	639.76	1763.68	1347.72
12	[90 0 90] <sup>S</sup>	614.82	1761.52	1311.72

Nr. – number of the composite layers,  $\alpha$  – orientations of the composite layers, C – compression load, S – shear load.

## 2.2. Results and Discussion

We investigated buckling behavior of rectangular composite plate under different load using the finite element method. From computation we get the load multiplier  $\lambda_i$ . This parameter has to be premultiplied by applied load in order to get buckling load. Accordingly, the critical buckling load for four orientations of the layers and for three numbers of the layers are obtained and are shown in Table 1.

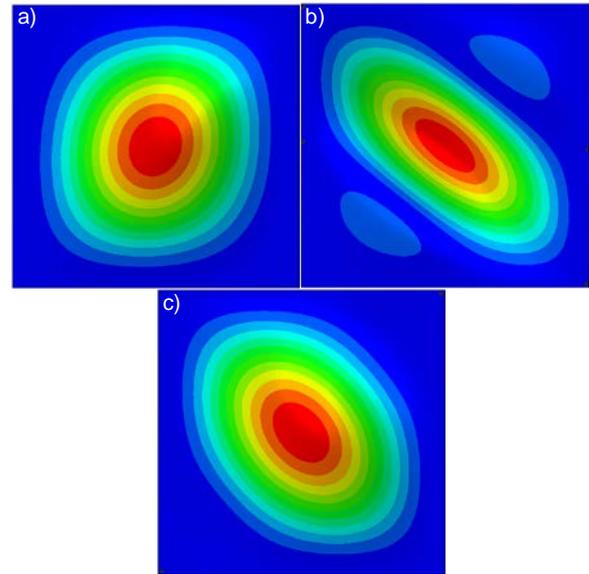


Figure 8. The 1<sup>st</sup> buckling mode of the composite plate with twelve layers and orientation of layers [0 45 0] and loaded by a) compression load, b) shear load, c) combination of compression and shear load

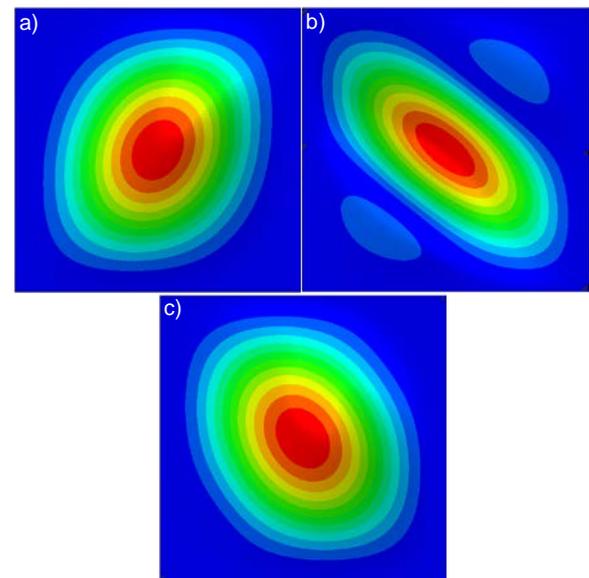
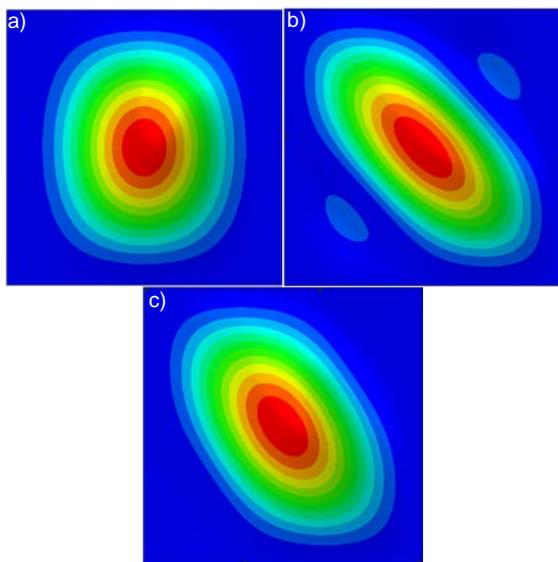


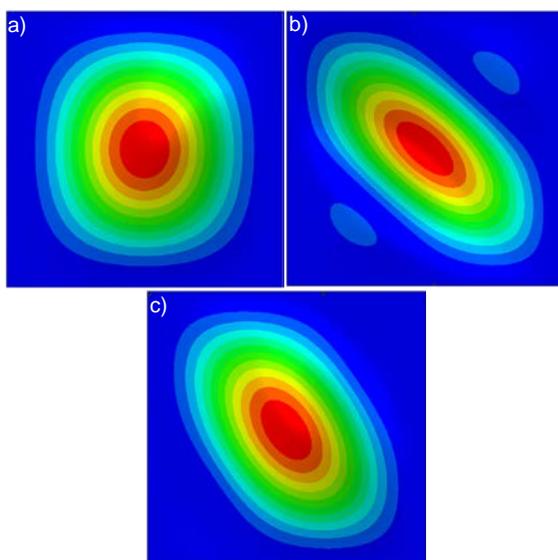
Figure 9. The 1<sup>st</sup> buckling mode of the composite plate with twelve layers and orientation of layers [45 0 45] and loaded by a) compression load, b) shear load, c) combination of compression and shear load

The resistance to the loss of stability is obtained from the computations for the three different number of layers and four orientations of layers of the rectangular composite plate. The inappropriate orientations of layers for composite plate loaded by compression load are: a) [90 0 90] for the composite plate with three layers ( $t = 1.5$  mm), b) [45 0 45]<sup>S</sup> for the composite plate with six ( $t = 3$  mm) and twelve ( $t = 4.5$  mm) layers. For composite plate loaded by shear load

the inappropriate orientations are: a)  $[45\ 0\ 45]$  for three layers, b)  $[45\text{-}0\text{-}45]^S$  for six and twelve layers. The composite plate loaded by compression and shear loads have the smallest critical buckling load for the orientation of layers: a)  $[0\ 90\ 0]$  for the composite plate with three of layers, b)  $[90\ 0\ 90]^S$  for the composite plate with six and twelve layers. The critical buckling load of the composite plate with six layers is almost ten times higher than the critical buckling load of the composite plate with three layers. For the composite plate with twelve layers is the critical buckling load approximately sixty times higher than the critical buckling load of the composite plate with three layers. The thickness of composite plate with three, six and twelve layers is 1.5mm, 3 mm and 4.5 mm, respectively. The first buckling modes for composite plates with twelve layers and for the different orientations of layers are shown in Figure 8-Figure 11. The buckling shapes are slightly effected by orientations of layers and number of layers of the composite plate.



**Figure 10.** The 1<sup>st</sup> buckling mode of the composite plate with twelve layers and orientation of layers  $[0\ 90\ 0]$  and loaded by a) compression load, b) shear load, c) combination of compression and shear load



**Figure 11.** The 1<sup>st</sup> buckling mode of the composite plate with twelve layers and orientation of layers  $[90\ 0\ 90]$  and loaded by a) compression load, b) shear load, c) combination of compression and shear load

### 3. Conclusion

The paper studied the buckling behavior of the rectangular composite plates with four different layer orientations and three, six and twelve layers. The all composite plates were modeled using finite element method. The finite element model of composite plate consisted from shell elements, which had defined the material of composite plate, the thickness of composite plate and layout of layers. The composite plates were loaded by three types of loads (compression load, shear load and combination of both). The boundary conditions on the parallel edges with  $x$  and  $y$  were applied. The computed critical buckling loads for all configurations showed that:

- critical buckling force increase with increasing numbers of layers as well as thickness of the composite plate; two times higher thickness of layers leads to almost ten times higher critical buckling load and three times higher thickness leads to sixty times higher critical buckling force,
- the orientations of composite layers effect the value of critical buckling load, for example the composite plate with three layers and orientation of layers  $[90\ 0\ 90]$  is more sensitive to compression load than shear load,
- the buckling modes are slightly effected by all orientations of layers and number of layers, the visible change is obtained at first mode shape from the critical buckling compression load.

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