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Application of Polynomial Regression Models in Prediction of Residual Stresses of a Transversal Beam

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Abstract Residual stresses significantly affect the behavior of a structure or its individual components. Residual stresses can be determined experimentally. It should be noted that careful procedure and preparation of the experiment are necessary in order to assess the effects on a specific component. The transversal beam of a casting structure underwent a series of measurements as a result of frequent operational faults due to cracks which used to occur in the vicinity of welds and other areas with high concentration of stresses. The measurement was done with a mechanical method of drilling with the use of a tensiometric rosette. Since the hole was drilled into the material to a specific depth, it was necessary to predict stresses in different depth levels of the hole. The prediction of local deformations was based on methods of mathematical statistics supported by MATLAB.

Keywords: *hole-drilling method, residual stresses, regression analysis, MATLAB*

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1. Introduction

Residual stresses are stresses which occur in a material even if the object is not loaded by external forces. The analysis of residual stresses is hence necessary when determining actual stress state of structural elements. The output information is explicitly represented by mechanical and physical properties which affect functionality and life of the component. Residual stresses occur as early as in the stage of technological processes and may be of different nature with respect to direction, magnitude, depth or planar gradient. They cause various failures of structural parts, knots, machines or devices. Clients often require that the manufacturer has the knowledge of residual stress states in components or structures. In order to determine gradients or magnitudes of residual stresses it is appropriate to carry out the hole-drilling-based analysis which belongs to the most common methods. The experiment is done with the use of a drilling tool kit. This tool kit allows us to lead the drill right into the center of the tensiometric rosette mounted on the analyzed object. While drilling a hole into the center of the tensiometric rosette residual stresses inside the structural element are being released. These stresses cause deformations around the hole, which can be measured on the surface of the object by electrical strain gauges. Scientific studies imply that residual stresses allow us to formulate only a general conclusion that tensile residual stresses affect fatigue strength, whereas compressive residual stresses increase it [1]. It is necessary to note that the effect of these residual stresses is not equal. However, such statement needs to be proved and requires a careful approach so that it can be applied to a specific component. The hole-drilling method was used to identify residual stresses in the area of frequent fatigue failures which occur on the structure of the casting ladle during operation. Residual stresses and stresses, which occurred after the structure was loaded with external loads, caused cracks around welds on the transversal beam of a casting structure.

The experimentally gained data required further evaluation and were the foundation for recommendations regarding structural changes to the equipment in question. When analyzing the collected data we found out that polynomial regression models are suitable for data processing.

2. Object of Experiment

The experiment was carried out on a bearing element. This element is a component of a massive engineering structure which is used in metallurgical industry. This component is the transversal beam of a casting ladle (Figure 1). As there were frequent failures of the transversal beam, the metallurgical company made a request for structural measures to eliminate these failures.

One of the ways in which relevant data on the transversal beam can be collected is a set of measurements which aim to define stress state in critical areas of the structure (Figure 2). We found out that residual stresses in the material of the structure have a significant effect on the failure. A series of drilling-based experiments was done in order to identify residual stresses. Prior to this procedure, critical areas with maximum residual stresses had to be found on the structure. Based on the preliminary analysis we found mounting points of strain gauges on the transversal beam. These mounting points are depicted in Figure 2.

3. Experimental Procedure and Measured Strains

The experiment provided us with a large number of information. Therefore we have decided to demonstrate only the measurement in position 5 on the transversal beam as depicted in Figure 3. Strain gauges Vishay 1-RY21-3/120 were mounted in pre-selected positions according to prescribed procedure. Drilling was carried out gradually by the increment of 0.5 mm and, at the same time, strain changes were being recorded. Measured strain changes were recorded with tensiometric apparatus Vishay P3. The hole was drilled with RS-200 up to the depth of 5 mm with the increment of 0.5 mm (Figure 3). The hole of 5 mm was sufficient to determine residual stresses in the material of the transversal beam. Hole diameter was 3.2 mm which corresponds with the diameter of the drill. Directions 1, 2 and 3 on the tensiometric gauge grid 1-RY21-3/120 coincide with strain changes ε_a, ε_b, ε_c (Figure 4).

Table 1 lists strains which were measured around the hole in individual drilling steps in position 5. The output strain values help us determine principal normal stresses in the point of drilling.

Table 1. Measured strain values in particular drilling stages

drilling stage *h* [mm]	strain values in particular directions [μm/m]		
	ε_a	ε_b	ε_c
0.50	−55.00	−29.00	−5.00
1.00	−88.00	−44.00	−4.00
1.50	−113.00	−61.00	−5.00
2.00	−134.00	−75.00	−4.00
2.50	−151.00	−86.00	−4.00
3.00	−172.00	−102.00	−9.00
3.50	−178.00	−108.00	−10.00
4.00	−182.00	−113.00	−10.00
4.50	−186.00	−117.00	−8.00
5.00	−192.00	−120.00	−6.00

Figure 1. Casting structure

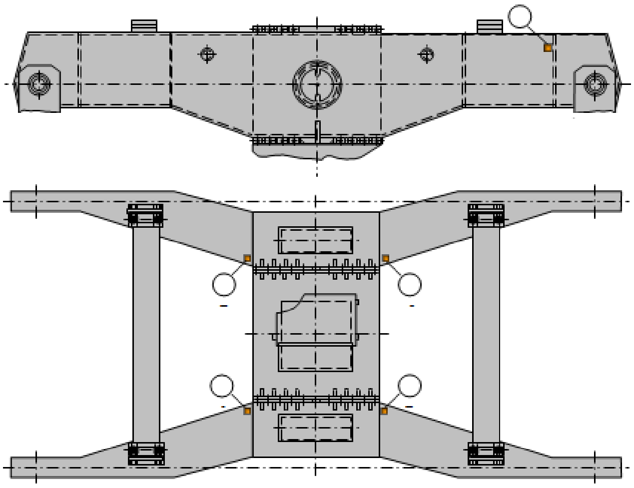

Figure 2. Strain gauge mounting positions on the transversal beam

Figure 3. Hole drilling in position 5 on the transversal beam

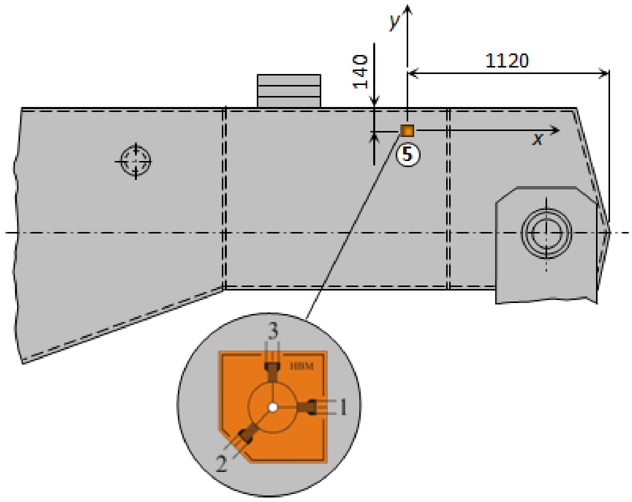

Figure 4. Strain gauge in position 5

4. Application of the Polynomial Regression Models

The purpose of this analysis was to determine the relationship between strains ε_a, ε_b, ε_c in particular directions marked as *a*, *b*, *c* and hole depth *h*. The statistical analysis of the measured data was performed with using classical least squares theory and software MATLAB.

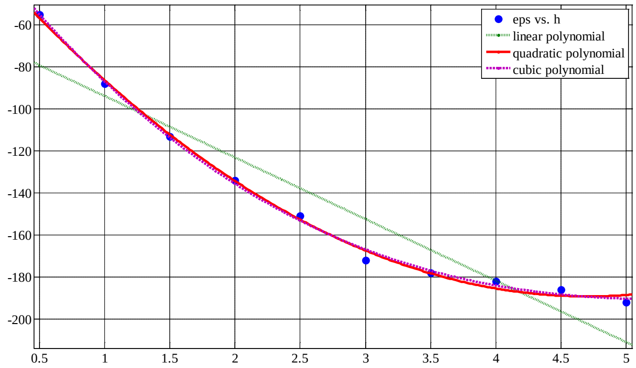

Figure 5. Comparison of three polynomial regression models with measured data – direction *a*

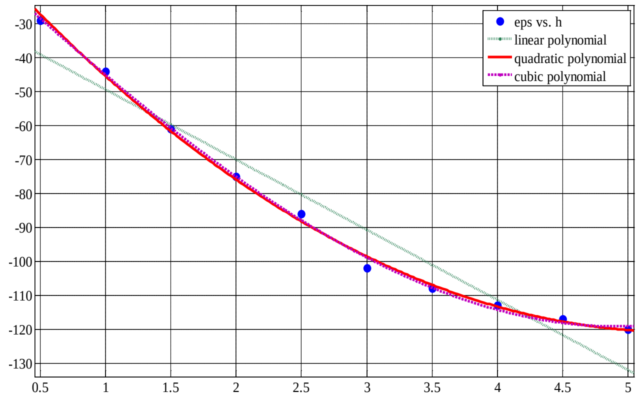

Figure 6. Comparison of three polynomial regression models with measured data – direction *b*

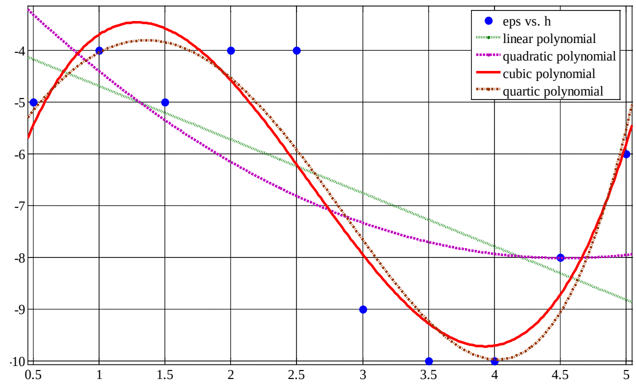

Figure 7. Comparison of four polynomial regression models with measured data – direction *c*

When applied polynomial regression in this example, we fit a linear, quadratic, cubic, maybe a quartic polynomial, and then see if can reduce the model by a few terms. In this case, the polynomial may provide a good approximation of the relationship.

Figure 5 – Figure 7 presents a scatter diagrams of the strain values ε_a, ε_b, ε_c in particular directions marked as *a*, *b*, *c* versus drilling stage *h* with fitted regression polynomial models.

The basic statistical outputs for particular directions *a*, *b*, *c* are, respectively, shown in Table 2 – Table 4

The root mean squared error *RMSE* is an estimator of the standard deviation σ of the random error term. *RMSE* is measure of the size of the errors in regression and do not give indication about the explained component of the regression fit.

Table 2. Regression statistics for direction *a*

	polynomial model		
	linear	quadratic	cubic
RMSE	15.4281	2.9982	2.8451
R^2	0.9029	0.9968	0.9975
R^{*2}	0.8907	0.9959	0.9963
MAPE	10.4619	1.5241	1.1310
DW	0.4761	2.0661	2.0907

Table 3. Regression statistics for direction *b*

	polynomial model		
	linear	quadratic	cubic
RMSE	8.1650	1.8952	1.7658
R^2	0.9429	0.9973	0.9980
R^{*2}	0.9358	0.9965	0.9970
MAPE	9.5102	1.9598	1.3403
DW	0.4750	2.0944	2.5637

Table 4. Regression statistics for direction *c*

	polynomial model			
	linear	quadratic	cubic	quartic
RMSE	2.0798	2.1338	1.2791	1.3497
R^2	0.3875	0.4359	0.8262	0.8388
R^{*2}	0.3109	0.2747	0.7394	0.7098
MAPE	26.2340	27.0337	14.8676	13.3011
DW	1.1084	1.1522	2.1686	2.3417

The *R*-squared R^2 (coefficient of determination) measures percentage of variation in the dependent variable explained by the independent variable. The value of R^2 is always between zero and one. An R^2 value of 0.9 or above is very good, a value above 0.8 is good, and value of 0.6 or above may be satisfactory in some applications, although we must be aware of the fact, in such cases errors in prediction may be relatively high. When the R^2 value is 0.5 or below, the regression explains only 50 % or less of the variation in the data; therefore, prediction may be poor. Adjusted *R*-squared R^{*2} is adjusted for the number of variables included in the regression equation.

Mean absolute percentage error *MAPE* is the most useful measure to compare the accuracy of forecasts between different items or products since it measures relative performance. If *MAPE* calculated value is less than 10%, it is interpreted as excellent accurate forecasting, between 10 − 20 % good forecasting, between 20 − 50% acceptable forecasting and over 50% inaccurate forecasting [2].

The Durbin-Watson (*DW*) statistic is used in a test for serial correlation of residuals (i.e., error terms) in simple or multiple regression models, and time series trend models. The Durbin-Watson statistic is always between 0 and 4. The value *DW* of 2 means, that there is no autocorrelation in the sample. A value close to 0 indicates strong positive correlation, while a value of 4 indicates strong negative correlation. An acceptable range is 1.4 − 2.6 [3,4].

Table 5. ***P*-values of *t*-tests for direction *a***

Type of polynomial	*p*-values
Linear	$[2.8222 \cdot 10^{-4};\ 2.5333 \cdot 10^{-5}]$
Quadratic	$[2.9055 \cdot 10^{-4};\ 5.7186 \cdot 10^{-8};\ 1.9341 \cdot 10^{-6}]$
Cubic	$[0.0189;\ 6.6661 \cdot 10^{-5};\ 0.0127;\ 0.23125]$

Table 6. ***P*-values of *t*-tests for direction *b***

Type of polynomial	*p*-values
Linear	$[8.8569 \cdot 10^{-4};\ 2.9728 \cdot 10^{-6}]$
Quadratic	$[0.0155;\ 8.1752 \cdot 10^{-8};\ 6.7465 \cdot 10^{-6}]$
Cubic	[0.0184 ; 0.0005 ; 0.6822 ; 0.2008]

Table 7. ***P*-values of *t*-tests for direction *c***

Type of polynomial	*p*-values
Linear	[0.0326 ; 0.0546]
Quadratic	[0.4339 ; 0.2526; 0.4637]
Cubic	[0.0094 ; 0.0331 ; 0.0137 ; 0.0104]
Quartic	[0.1731 ; 0.6746 ; 0.8842 ; 0.8076 ; 0.5603]

There is a test that uses the *t*-distribution to test single parameters of the regression model. Let's set the null hypothesis H_0: $\beta_j = 0$, and the alternative hypothesis H_1: $\beta_j \neq 0$, $j = 0, 1, \ldots, k$. [5] Software MATLAB prints all these tests and the corresponding *p*-values (see Table 5 – Table 7). We used significance level $\alpha = 0.05$.

Direction *a*: In Table 2 and Table 5 we can see that the quadratic regression polynomial $\hat{\varepsilon} = \hat{\beta}_0 + \hat{\beta}_1 h + \hat{\beta}_2 h^2$ fit the measured data very good. Symbols $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are unbiased estimators of the true regression coefficients β_0, β_1, and β_2. Least squares parameter estimates for this model:

$$\hat{\beta} = (-23.4500, -70.3803, 7.4697)^T.$$

Direction *b*: The quadratic polynomial regression model is here the best (see Table 3 and Table 6). Least squares parameter estimates for this model:

$$\hat{\beta} = (-7.0833, -42.2500, 3.9242)^T.$$

Direction *c*: We find that the cubic polynomial regression model appears to fit the data best (see Table 4 and Table 7). Least squares parameter estimates for this model:

$$\hat{\beta} = (-9.3333, 10.2471, -5.2890, 0.6760)^T.$$

Estimation of the model parameters requires the assumption that the errors are uncorrelated random variables normally distributed with mean zero and constant variance σ^2. The random error terms are estimated by the residuals – the differences between the observed values and the fitted values. The analysis of residuals plays an important role in validating the regression model. Graphical analysis is much more effective in trying to detect patterns in the residuals than looking at the raw numbers. It may also be useful to plot standardized residuals vs. fitted values (Figure 8). The residuals would have to be randomly distributed around zero. If the errors are $norm(0, \sigma^2)$, then approximately 95 % of the standardized residuals should fall in the interval $(-2, +2)$ [3].

The residuals for directions *a*, *b*, *c* in Figure 9 show no curvature or changing variance. The residual plots show a fairly random pattern that indicates models provide a good fit to the data.

An important application of a regression model is predicting new or future observations of strains ε_a, ε_b, ε_c corresponding to a specified level of the independent variable *h*. We can compute e.g. 95 % prediction interval for strains ε_a, ε_b, ε_c in particular directions marked as *a*, *b*, *c*. Figure 9 – Figure 11 show the 95 % prediction interval for strains in particular directions by using the best polynomial regression model.

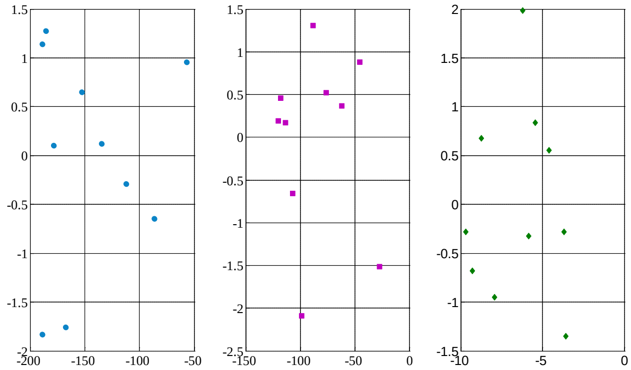

Figure 8. Scatter plots of standardized residuals vs. fitted values for directions *a*, *b*, *c*

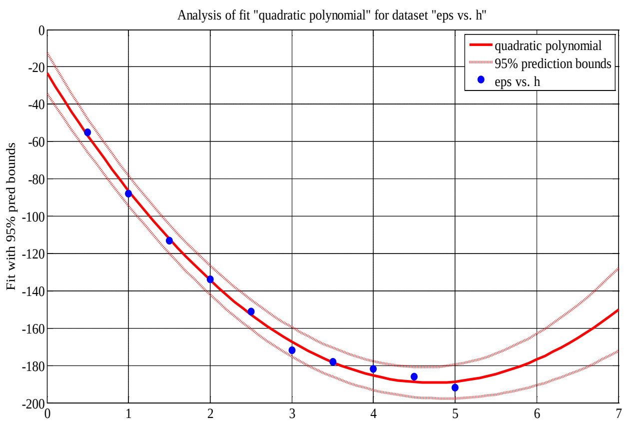

Figure 9. 95% prediction interval using quadratic polynomial–direction *a*

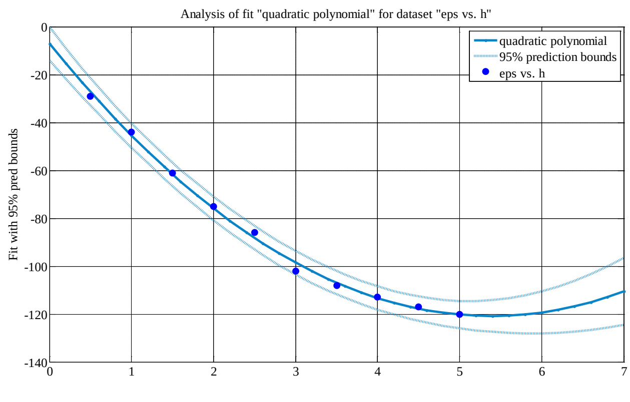

Figure 10. 95% prediction interval using quadratic polynomial–direction *b*

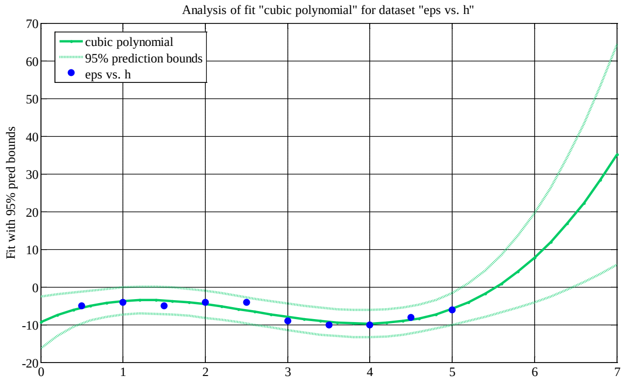

Figure 11. 95 % prediction interval using cubic polynomial – direction *c*

5. Conclusion

Measured strains in positions 1 – 5 on the structure (Figure 2), principal normal stresses, which were determined on the basis of these strains, and their gradients demonstrate that principal normal stresses are tensile stresses. With respect to occurrence and spread of cracks, such stresses indicate the most critical case. On the analyzed surface these stresses reach the limit of yield strength and cause plastic deformations. These deformations combine with stresses and superpose to operational loading. Direction offset angles of bigger residual stresses from the reference axis of strain gauges confirm the initial point of crack spread. The occurrence of cracks in positions 1, 3 and 4 was caused by high stresses and a different curvature radius of the stress concentrator which in this case was the weld.

These results point out, that such high values of residual stresses have considerably affected the life of the bearing structure. Based on the above-mentioned, structural changes have been made to decrease stresses which affect working life of the structure.

By means of mathematical statistics we were able to identify the tendency of strain changes ε_a, ε_b, ε_c relative to the depth of drilling. These changes can be seen in Figure 9 – Figure 11. The above-mentioned statistical analysis and the application of polynomial regression models were used to verify the chosen procedure. Such procedure can be used for future verification of similar tasks.

Acknowledgements

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