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Effective Material Moduli for Composites

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Abstract In this article we deal with deriving the elastic modulus of composite materials. Modulus values in each direction are various, for example in parallel direction and the perpendicular direction. It depends on the material properties for fibers from material for matrix, density of fibers in the composite material, as well as on whether it is a single or multi-layer composite material and from the direction the filaments in the matrix.

Keywords: *fibers, matrix, composite material, effective modulus, transverse modulus, improved formulas, in-plane sher modulus, poison´s ratio, mixture rules*

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1. Introduction to Composite Material

Composite materials have at least two different material components which are bonded. The material response of a composite is determined by the material moduli of all constituents, the volume or mass fractions of the single constituents in the composite material, by the quality of their bonding, i.e. of the behavior of the interfaces, and by the arrangement and distribution of the fiber reinforcement, i.e. the fiber architecture.

The basic assumption made in material science approach models of fiber reinforced composites are:

- The bond between fibers and matrix is perfect
- The fibers are continuous and parallel aligned in each ply, they are packed regularly, i.e. the space between fibers uniform.
- Fiber and matrix materials are linear elastic, they follow approximately Hooke´s law and each elastic modulus is constant
- The composite is free of voids

Composite materials are heterogeneous, but in simplifying the analysis of composite structural elements in engineering applications, the heterogeneity of the material is neglected and approximately overlayed to a homogenous material. The most important composite in structural engineering applications are laminates and sandwiches. Each single layer of laminates or sandwich faces is in general a fiber reinforced lamina [2].

2. Elementary Mixture Rules for Fiber – Reinforced Laminate

One of the most important factor which determines the mechanical behavior of a composite material is the proportion of the matrix and the fibers expressed by their volume or their weight fraction. These fractions can be established for two phase composite in the following way. The volume V of the composite is made from a matrix volume V_m and the fiber volume V_f. In a similar way the weight or mass fraction of fiber and matrices can be defined. The mass M of the composite is made from M_m and M_f. The rule of mixture is based on the statement that the composite property is the sum of the properties of each constituent multiplied by its volume fraction. For density of the composite material we can write [1]:

$$\rho = \frac{M}{V} = \frac{M_f + M_m}{V} = \frac{\rho_f V_f + \rho_m V_m}{V} = \rho_f v_f + \rho_m v_m = \rho_f v_f + \rho_m v_m (1 - v_f). \qquad (1)$$

In a actual lamina the fibre are randomly over the lamina cross-section and the lamina thickness is about 1mm and much higher than the fibre diameter (about 0,01mm).

3. Longitudial Moduls of Elasticity

When an unidirectional lamina is acted upon by either a tension or compression load parallel to the fibers, it can be assumed that the strains of the fibers, matrix and composite in the loading direction are the same.

$$\varepsilon_{Lf} = \varepsilon_{Lm} = \varepsilon_L = \frac{\Delta l}{l} \qquad (2)$$

Figure 1. Mechanical model to calculate the effective Young´s modules E_L [2]

The mechanical model has a parallel arrangement of fibres and matrix and the resultant axial force F_L of the composite is sheared by both fibre and matrix so that:

$$F_L = F_{Lf} + F_{Lm}$$

or

$$F_L = \sigma_{Lf} A_f + \sigma_{Lm} A_m. \tag{3}$$

With Hooke´s law it follows that

$$\sigma_L = E_L \varepsilon_L$$
$$\sigma_{Lf} = E_{Lf} \varepsilon_{Lf} \tag{4}$$
$$\sigma_{Lm} = E_{Lm} \varepsilon_{Lm}$$

or

$$E_L \varepsilon_L A = E_f \varepsilon_f A_f + E_m \varepsilon_m A_m. \tag{5}$$

If we divide with area A we get

$$E_L = E_f \frac{A_f}{A} + E_m \frac{A_m}{A} \tag{6}$$

with

$$\frac{A_f}{A} = \frac{A_f l}{Al} = \frac{V_f}{V} = v_f \tag{7}$$

$$\frac{A_m}{A} = \frac{A_m l}{Al} = \frac{V_m}{V} = v_m \tag{8}$$

and the effective modulus E_L can be written as follows [3]

$$E_L = E_f v_f + E_m v_m = E_f v_f + E_m (1 - v_f). \tag{9}$$

The equation (9) is known as rule of mixture. The predicted values of E_L are in good agreement with experimental results. The stiffness in fiber direction is dominated by the fiber modulus. The ratio of the load taken by the fiber to the load taken by the composite is a measure of the load shared by the fiber.

$$\frac{F_{Lf}}{F_L} = \frac{E_{Lf}}{E_L} v_f. \tag{10}$$

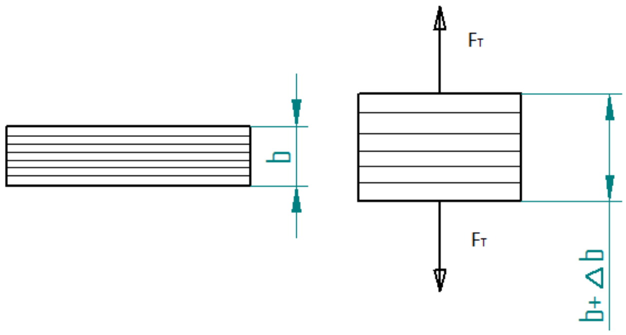
Since the fiber stiffness is several times greater than the matrix stiffness, the second term in (7) may be neglected [2]

$$E_L \approx E_f v_f. \tag{11}$$

4. Efective Transferse Modulus of Elasticity

In the following Figure we can see transverse load composite. The stress resultant F_T respectively the stress σ_T is equal for all phase

$$F_T = F_{Tf} = F_{Tm}$$
$$\sigma_T = \sigma_{Tf} = \sigma_{Tm} \tag{12}$$



Figurr 2. Mechanical model to calculate the effective transverse modulus E_T

From Figure 2 it follows that

$$\Delta b = \Delta b_f + \Delta b_m$$

$$\varepsilon_T = \frac{\Delta b}{b} + \frac{\Delta b_f + \Delta b_m}{b}. \tag{13}$$

end with

$$b = v_f b + (1 - v_f) b = b_f + b_m \tag{14}$$

and

$$\varepsilon_T = \frac{\Delta b_f}{v_f b} \frac{v_f b}{b} + \frac{\Delta b_m}{(1 - v_f) b} \frac{(1 - v_f) b}{b} = v_f \varepsilon_{Tf} + (1 - v_f) \varepsilon_{Tm} \tag{15}$$

with

$$\varepsilon_{Tf} = \frac{\Delta b_f}{v_f b}$$

$$\varepsilon_{Tm} = \frac{\Delta b_m}{(1 - v_f) b}. \tag{16}$$

Using Hooke´s law for the fiber, the matrix and the composite [4]

$$\sigma_T = E_T \varepsilon_T$$
$$\sigma_{Tf} = E_{Tf} \varepsilon_{Tf} \tag{17}$$
$$\sigma_{Tm} = E_{Tm} \varepsilon_{Tm}.$$

Substituting Eqs. (17) in (16) and considering (5) gives to formula of E_T

$$\frac{1}{E_T} = \frac{v_f}{E_f} + \frac{1 - v_f}{E_m} = \frac{v_f}{E_f} + \frac{v_m}{E_m} \tag{18}$$

or

$$E_T = \frac{E_f E_m}{(1 - v_f) E_f + v_f E_m}. \tag{19}$$

Equation (19) is referred to Reuss estimate. The predicted value of E_T are seldom in good agreement with experimental results [2].

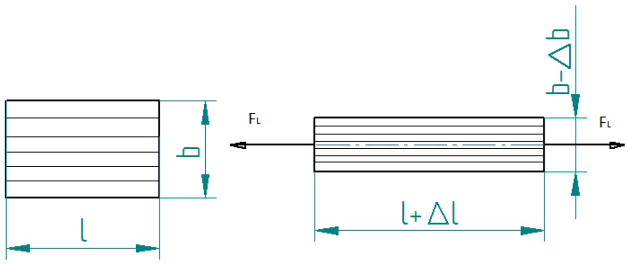


Figure 3. Mechanical model to calculate the major Poisson´s ratio μ_{LT}

5. Longitudial Moduls of Elasticity

Assume a composite is loaded in the on-axis direction as show Figure 3. The major Poison´s ratio is defined as the negative of the ratio of the normal strain in the transverse direction to the normal strain in the longitudinal direction

$$\mu_{LT} = -\frac{\varepsilon_T}{\varepsilon_L}. \tag{20}$$

With

$$\begin{aligned} -\varepsilon_T &= \mu_{LT}\varepsilon_T \\ &= -\frac{\Delta b}{b} = \frac{\Delta b_f + \Delta b_m}{b} \\ &= -\left[v_f \varepsilon_{Tf} + (1-v_f)\varepsilon_{Tm}\right] \end{aligned} \tag{21}$$

$$\mu_f = -\frac{\varepsilon_{Tf}}{\varepsilon_{Lf}}$$

$$\mu_f = -\frac{\varepsilon_{Tm}}{\varepsilon_{Lm}} \tag{22}$$

it follows that

$$\varepsilon_f = -\mu_{LT}\varepsilon_L = -v_f \mu_f \varepsilon_{Lf} - (1-v_f)\mu_m \varepsilon_{Lm}. \tag{23}$$

The longitudinal strain in the composite, the fiber and the matrix are assumed to be equal and the equation for the major Poisson´s ratio reduces to [5]

$$\mu_{LT} = v_f \mu_f + (1-v_f)\mu_m = v_f \mu_f + v_m \mu_m. \tag{24}$$

The major Poisson´s ratio μ_{LT} obeys the rule of mixture. The minor Poisson´s ratio μ_{LT} can be derived with the symmetry condition or reciprocal relationship.

$$\frac{\mu_{TL}}{E_T} = \frac{\mu_{TL}}{E_L} \tag{25}$$

$$\begin{aligned} \mu_{TL} &= \mu_{LT}\frac{E_T}{E_L} \\ &= (v_f \mu_f + v_m \mu_m)\frac{E_f E_m}{(v_f E_m + v_m E_f) + (v_f E_f + v_m E_m)} \end{aligned} \tag{26}$$

The values of Poisson´s ratio for fiber or matrix rarely differ significantly, so that neither matrix nor fiber characteristic dominate the major or the minor elastic constant μ_{LT} and μ_{TL} [3].

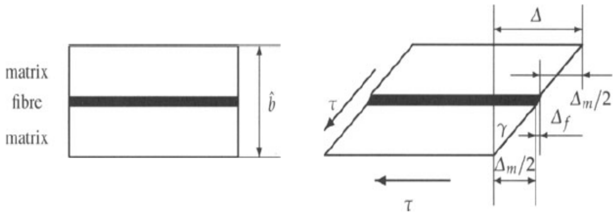


Figure 4. Mechanical model to calculate the effective in-plane shear modulus G_{LT} [2]

6. Effective in – plane Sher Modulus

Apply a pure stress τ to a lamina as shov in Figure 4. Assuming that the shear stresses on the fiber and the matrix are the same, but the shear strain are diferent

$$\gamma_m = \frac{\tau}{G_m}$$

$$\gamma_m = \frac{\tau}{G_m} \tag{27}$$

$$\gamma = \frac{\tau}{G_{LT}}.$$

The model is a connection in series and therefore

$$\tau = \tau_f = \tau_m$$

$$\Delta = \Delta_f + \Delta_m \tag{28}$$

$$\Delta = \hat{b}\tan\gamma = \gamma_f \hat{b}_f + \gamma_m \hat{b}_m$$

and with

$$\begin{aligned} \hat{b} &= \hat{b}_f + \hat{b}_m = \\ &= (v_f + v_m)\hat{b} = v_f \hat{b} + (1-v_f)\hat{b} \end{aligned} \tag{29}$$

it follows that

$$\Delta_f = \gamma_f v_f \hat{b}$$

$$\Delta_m = \gamma_m \left(1-v_f\right)\hat{b} \tag{30}$$

$$\Delta_f = \gamma_f v_f \hat{b},\ \Delta_m = \gamma_m\left(1-v_f\right)\hat{b}.$$

Using Hooke´s law we get [2]

$$G_{LT} = \frac{G_m G_f}{(1-v_f)G_f + v_f G_m} \tag{31}$$

$$G_f = \frac{E_f}{2(1+\mu_f)},\ G_m = \frac{E_m}{2(1+\mu_m)}. \tag{32}$$

7. Longitudial Moduls of Elasticity

Effective elastic moduli related to loading in the fiber direction, such as E_L and μ_{LT}, are dominated by the fibers. All estimations in this case and experimental results are very close to the rule of mixtures estimations. But the values obtained for transverse Young´s modulus and in-plane shear modulus with the rule of mixture which can be reduced to the two model connection of Voigt and Reuss, do not agree well with experimental results. This establishes a need for better modelling techniques based on elasticity solutions and [1]

$$\frac{M}{M_m} = \frac{1+\xi\eta v_f}{1-\eta v_f} \quad (33)$$

variational principle models and includes analytical and solution methods.

Unfortunately, the theoretical models are only available in the form of complicated mathematical equations and the solution is very limited and needs huge effort. Semi-empirical relationship have been developed to overcome the difficult with purely theoretical approaches [5].

The most useful of those semi-empirical models are those of Halpin and Tsai which can be applied over a wide range of elastic properties and fiber volume fractions.

Starting from results obtained in theoretical analysis, Halpin and Tsai proposed equations that are general and simple in formulation. The moduli of a unidirectional composite are given by the following equations [4]

- E_L and μ_{LT} by the law of mixture Equation (9) and (20)
- For the other moduli by

$$\eta = \frac{\left(M_f / M_m\right) - 1}{\left(M_f / M_m\right) + \xi} \quad (34)$$

ξ - is called the reinforcement factor and depends on

- the geometry of the fibers
- the packing arrangement of the fibers
- the loading conditions [2].

The more complicated formulas for E_L and μ_{LT} as the formulas given above by the rule of mixtures yields practically identical values to the simpler formulas and are not useful. But the elasticity solution for the modulus G_{LT} yields much better results and should be applied [5]

$$G_{LT} = G_m \frac{G_f(1+v_f) + G_m(1-v_f)}{G_f(1-v_f) + G_m(1+v_f)} \quad (35)$$

The following recommendation may be possible for an estimate of effective elastic moduli of unidirectional laminae

- E_L and μ_{LT} should be estimate by the rule of mixtures
- μ_{TL} follows from the reciprocal condition
- G_{LT} should be (a) or the Halpin/Tsai formulas (34) and (33)
- E_T may be estimate with help of the Halpin/Tsai formulas. But only when reliable experimental value of E_T and G_{LT} are available for a composite the factor ξ can be derivate for this case and can be used to predict effective moduli for a range of fiber volume ratio of the same composite [2].

It is also possible to look for numerical or analytical solutions for ξ based on elasticity theory. In generally, ξ may vary from zero to infinity, and the Reuss and Voigt models are special case, e.g.

$$E_T = \frac{1+\xi\eta v_f}{1-\eta v_f} E_m \quad (36)$$

for $\xi = 0$ and $\xi = \infty$, respectively [4].

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