

Dynamic Analysis of Lathe Machine Tool

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Abstract The thesis presents a theoretical analysis of the task, while practical knowledge of selected parts of the dynamics is used to solve individual involved transfers. Further for the two types of programs includes detailed diagrams that is rev/min (rpm) increasing and rev/min reduction. The centre lathe is the object of exploring. Specifically transmission parts such as shafts, gears and pulleys then it is electric drive device, and finally driven parts such as spindle and chuck. Analyze of angular velocities, moments of inertia, kinetic energy, performance and labor of each part of the kit is performed using reduction method, and the system is reduced to a first member, which is an electric motor. We have developed integration of individual gears for the machining modes. Specifically for conventional turning 2800 rpm and for winding springs 14 rpm. In technical practice it is not possible to measure two same results of performance parameters, for reason of passive resistance.

Keywords: dynamical equations of the motion, kinetic energy, dynamic analysis of lathe, reduction method

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1. Introduction

Machining method of turning humanity is already known from the times of ancient Egypt and the ancient world. At this time, however, they were very primitive lathes and rotating the workpiece was secured by man. In later times, was replaced human labor power or horsepower water wheel. With the advent of the industrial revolution and the invention of the steam engine began to receive a similar lathe closer today, which helped the invention of the slide of lathe.

Thus turning tools are clamped firmly in the cutter head and workpiece accuracy nezáležala only the stability of the worker's hands. Nowadays lathes are powered by electric motors that the main parameters of speed and torque are transformed through a set of gears. The main priority of enterprises is the quantitative and qualitative production, and it is therefore necessary to know the speed of all processes in the working cycle of production. Using different methods of dynamic analysis we can determine the necessary times to reach the desired speed since it is on the spindle lathes. Or on the other hand, what we will have the necessary power to ensure if we want to output speed at a specified time.

2. Lathes mechanism

Lathes are devices that are used primarily for machining rotating parts, where the main cutting rotary motion carries the workpiece clamped in the chuck and side cutting movements infeed movement and provides a tool clamped in the cutter head [1]. These machines can be machined outer and inner rotary or conical surface, the

outer spherical surface, tapping and threading, drilling holes, or we can create keyway on the shaft. In addition to cutting operations with these machines can twist robust spring diameter.

Lathes are machines with a spindle that turns along with the workpiece. The workpiece may be supported between a pair of points called centres, or it may be bolted to a faceplate or held in a chuck. A chuck has movable jaws that can grip the workpiece securely. The lathe knife or other cutting tool changes the shape of the workpiece. Lathes are primarily used to manufacture rotary parts. A lathe can be used to produce parts made of ferrous and non-ferrous metals, plastics and wood [2]. The workpiece is clamped in the lathe either in a chuck, between a chuck and a centre or between two centres. Metal lathes are among the most basic and common of metal milling machines. CNC lathes were first manufactured in 1952.

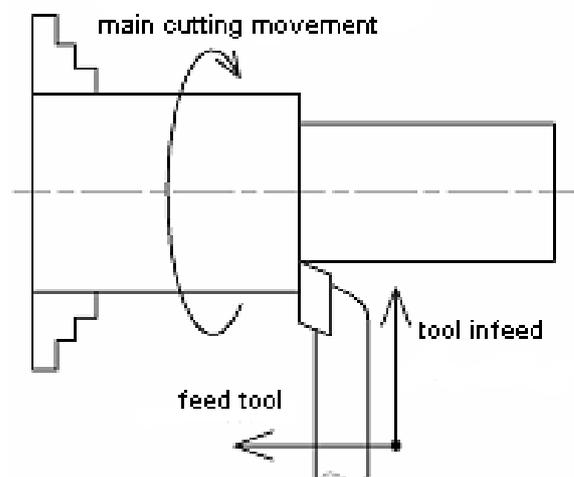


Figure 1. The principle of turning



Figure 2. View of the lathe SV 18 Ra

3. Dynamic Analysis of Lathe

Objects Dynamic analysis will center lathe SV 18 RA (Figure 2), which is still widely used for the manufacture of parts in small series and piece production, which would be the use of NC and CNC machines wasteful. It also has a strong presence in vocational and technical schools to prepare students to practice. Production of this type dates back to the 50s of the 20th century.

3.1. The Basic Parameters for Lathe SV 18 RA

The basic parameters for lathe are motor power at 50 Hz 6 kW, main engine speed at 50 Hz 2800 rev/min, swing over bed 380 mm, swing over cross slide 215 mm, chuck diameter 160-250 mm, spindle speed 31 degrees in the range 14-2800 rev/min, spindle bore 41 mm, maximum mass of workpiece 300 kg.

3.2. Determination of Kinematic Schema of the Main Drive Lathe

The Figure 3 shows a system which is composed of the actuator, the transmission and the driven mechanism.

Part of the scheme is the diagram of the casting of gear ratios (Figure 4). From the figure it shows that the drive will be provided from the electric motor by belt drive (V-belts) transfers constant gear ratio (1:2) to speed gear input shaft.

This is a two-stage gearbox with front teeth. Output speed of the gear we get by belt drive (flat belt) on the spindle pulley ratio (1:1). The pulley is not stored on two ball bearings, not on the spindle. Therefore, the vibrations do not transfer to the spindle, and then the workpiece. Gear ratio 1:8 which is necessary for connection lathe with output speed 14 rev/min. They are elaborated below.

4. Computation of Dynamic Variables and Construct the Kinematic Scheme of the Lathe

Dynamic analysis for individual schemes selected lathe will be done using reduction methods [3]. The reduction

will be applied to the motion (angular velocity) ω_M electric motor. Followed by all other ω_i on each shaft (and with them converging components) we expressed using ratios to the angular velocity ω_M :

$$E_K = E_{Kred} \tag{1}$$

$$E_K = \frac{1}{2} I_{red} \omega_M^2 \tag{2}$$

$$I_{red} = \frac{2E_K}{\omega_M^2} \tag{3}$$

$$A = A_{red} \tag{4}$$

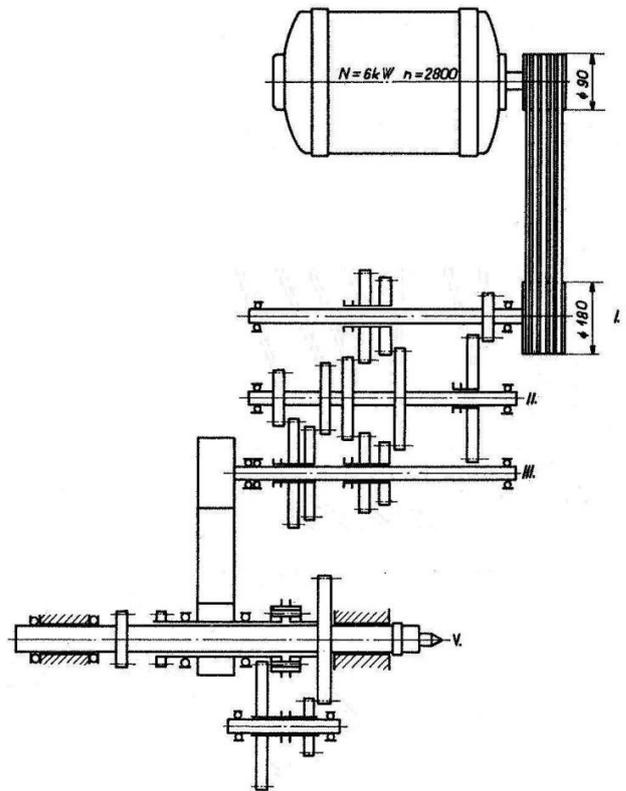


Figure 3. The scheme of the main drive lathe

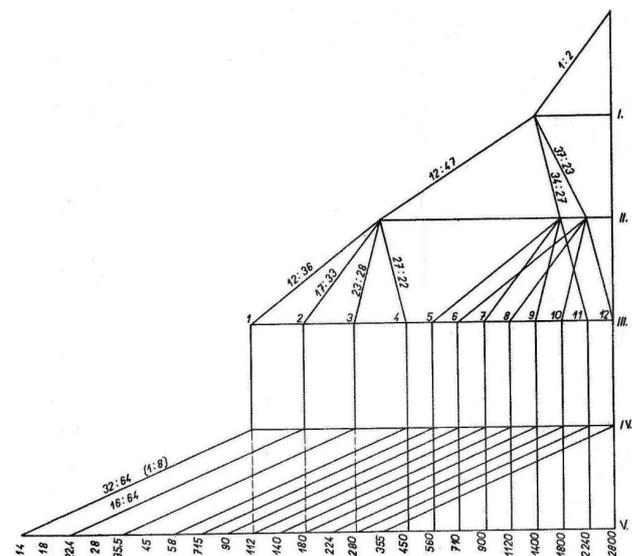


Figure 4. Kinematics scheme of the main drive lathe

$$P = P_{red} \quad (5)$$

$$P = M_{red} \omega_M \quad (6)$$

$$M_{red} = \frac{P}{\omega_M} \quad (7)$$

$$\alpha_M \cdot \frac{2 E_K}{\omega_M^2} = \frac{P}{\omega_M} \quad (8)$$

where:

E_k – kinetic energy of the system,

E_{kred} – kinetic energy of the reduced mass,

I_{red} – moment of inertia of the reduced mass,

A – work of the whole system,

A_{red} – work of the reduced mass,

M_{red} – moment of external forces of the reduced mass,

P – performance of the whole system,

P_{red} – performance of the reduced mass,

ω_M – angular velocity of the motor.

The basic dynamic equation of motion for rotational motion [4]:

$$\alpha \cdot I = M \quad (9)$$

where:

α - instantaneous angular acceleration.

Adaptation of the dynamic equation of motion for the selected method [5]:

$$\alpha_M \cdot I_{red} = \sum M_i = M_{red} \quad (10)$$

substituting equations (3) and (7) into (10) get:

$$\alpha_M \cdot \frac{2 E_K}{\omega_M^2} = \frac{P}{\omega_M} \quad (11)$$

The equation (4) is the dynamic equation of motion of the reduced mass computed on the motion of the motor.

4.1. Expression of the Angular Velocity as a Function of Angular Velocity ω_M

When the engine speed is $n_M=2800$ rev/min then the angular velocity of the motor [6]:

$$\omega_M = \frac{2 \pi n_M}{60} = \frac{280 \pi}{3} \text{ rad.s}^{-1}. \quad (12)$$

Angular velocity we expressed for the two operating modes the minimum and maximum possible output speed:

a) for the lowest spindle speed (14 rev/min), as shown in Figure 5.

$${}_{14}\omega_1 = \left(\frac{1}{2}\right) \omega_M \quad (13)$$

$${}_{14}\omega_2 = \left(\frac{1}{2} \cdot \frac{12}{47}\right) \omega_M = \left(\frac{6}{47}\right) \omega_M \quad (14)$$

$${}_{14}\omega_3 = \left(\frac{1}{2} \cdot \frac{12}{47} \cdot \frac{12}{36}\right) \omega_M = \left(\frac{2}{47}\right) \omega_M \quad (15)$$

$${}_{14}\omega_4 = \left(\frac{1}{2} \cdot \frac{12}{47} \cdot \frac{12}{36} \cdot \frac{1}{8}\right) \omega_M = \left(\frac{1}{188}\right) \omega_M \quad (16)$$

substituting:

$$2800 \cdot \frac{1}{2} = 1400 \text{ rev / min} \quad (17)$$

$$1400 \cdot \frac{12}{47} = 357,45 \text{ rev / min} \quad (18)$$

$$357,45 \cdot \frac{12}{36} = 119,15 \text{ rev / min} \quad (19)$$

$$119,15 \cdot \frac{1}{8} = 14,9 \text{ rev / min}. \quad (20)$$

With the V-belt transfer that input speed (2800 rev/min) of the pulley electric motor to the pulley input shaft through a gear transmission ratio 1:2. In Figure 6 are shown in red along meshing spur gear inside the gearbox which are stored on the splined shaft moved or fixed, in two cases, pinions made together with the shafts. Axle journals rotate in a single row ball bearings (type 62), since no forces in the axis of the shaft was not present.

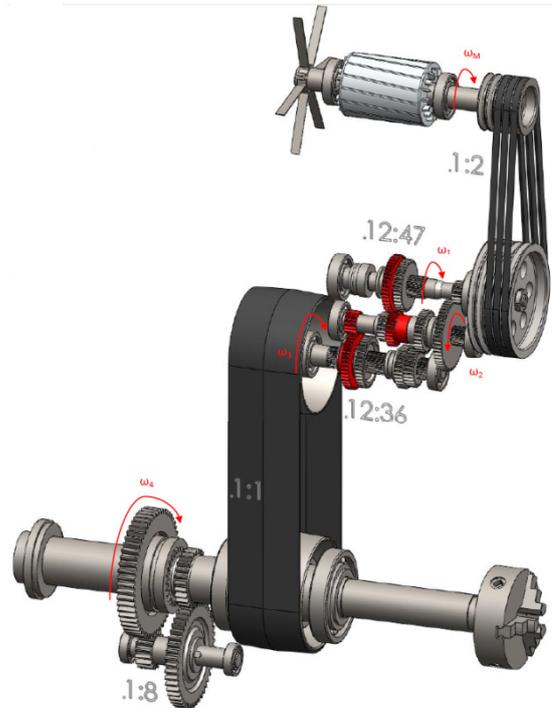


Figure 5. Determination of gear engagement system for 14 rev/min

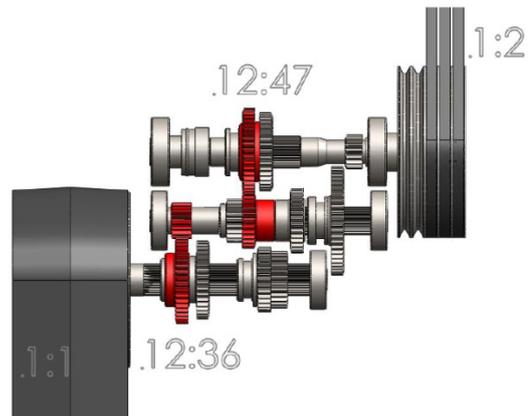


Figure 6. Model with the gear meshing gears (14rev/min)

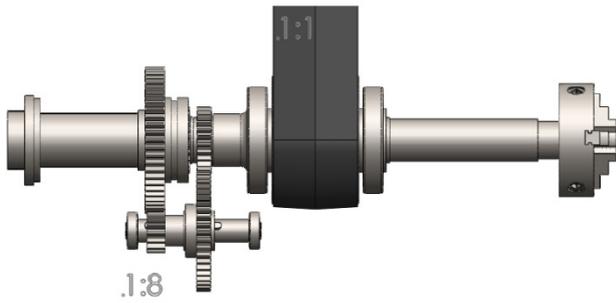


Figure 7. Models for the spindle gear to achieve a gear ratio of 1:8

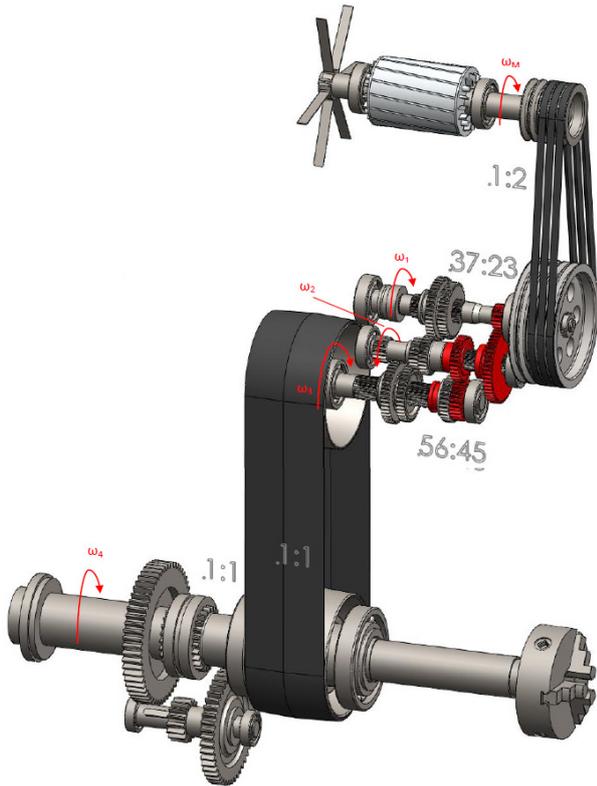


Figure 8. Determination of gear engagement system for 2800 rev/min

The reduction in speed is provided by including a further pair of toothed wheels the gear unit directly on the shaft of the spindle that shown in Figure 7. Center hole in which the pulley is not yet mounted on the spindle by bearings to avoid transmission of vibrations and the possibility of change gear ratios.

b) for maximum spindle speed (2800 rev/min), as shown in Figure 8.

$$2800 \omega_1 = \left(\frac{1}{2}\right) \omega_M \tag{21}$$

$$2800 \omega_2 = \left(\frac{1}{2} \cdot \frac{37}{23}\right) \omega_M = \left(\frac{37}{46}\right) \omega_M \tag{22}$$

$$2800 \omega_3 = \left(\frac{1}{2} \cdot \frac{37}{23} \cdot \frac{56}{45}\right) \omega_M = \left(\frac{1036}{1035}\right) \omega_M \tag{23}$$

$$2800 \omega_4 = 2800 \omega_3 \tag{24}$$

substituting:

$$2800 \cdot \frac{1}{2} = 1400 \text{ rev / min} \tag{25}$$

$$1400 \cdot \frac{37}{23} = 2252,2 \text{ rev / min} \tag{26}$$

$$2252,2 \cdot \frac{56}{45} = 2802,7 \text{ rev / min} \tag{27}$$

$$2802,7 \cdot \frac{1}{1} = 2802,7 \text{ rev / min.} \tag{28}$$

In Figure 9 we demonstrated the system and transfer it to V-belt from the inlet via a gear (where red means meshing with gears mode for 14 rev/min to chuck) to the flat belt that transmits the resulting transformed speed on the spindle pulley. The picture is further marked by numbers corresponding ratios for couples engaging wheels.

Figure 10 shows the connection of the spindle of the pulley to the hub by means of the gear with internal teeth, this charge has in the picture at its left end of the gear teeth of the same nature as that is pressed on the spindle, therefore, we speak of gear ratio 1:1.

4.2. Computation of the Moment of Inertia and Kinetic Energy

Using SolidWorks (function-mass properties), we found mass values m_e and the moment of inertia to their own axes for engaging all components and they are: rotor, pulleys, shafts, gears and chuck [8]. To simplify the calculation, and a small effect on the results, we decided to ignore posts from bearings, V-belts and flat belts, rings and lip of the distance to the rotor shaft. The resulting values are shown in the figures.

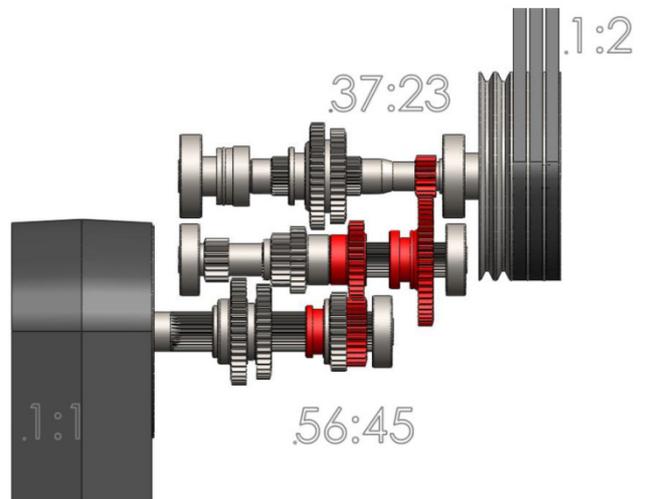


Figure 9. Model with meshing gears gearboxes (2800 rev/min)

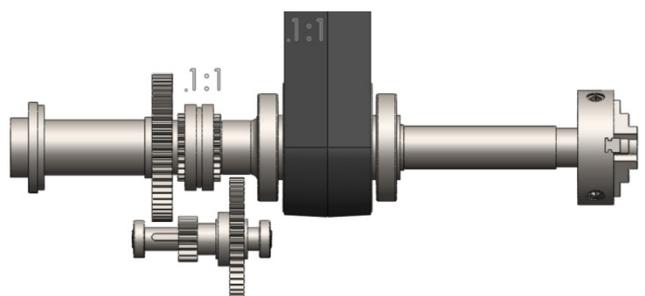


Figure 10. Models for the spindle gear to achieve a gear ratio 1:1

As shown in Figure 11 the shaft of the electric motor mounted in a pair of rolling bearings, one of which is a rotor. At the right end of the shaft is mounted pulley with spring-loaded connections from that transmits the speed gear input shaft. The kinetic energy of bearings and impeller do not consider because of negligible value. On this basis, we calculate moments of inertia and kinetic energy as the shaft of the rotor and pulley.

In Figure 12 and Figure 13 are given the mass m , moments of inertia I , and the kinetic energy of the model for the modes 14 rev/min and 2800 rev/min for the parts of the electric motor with the angular velocity ω_M .

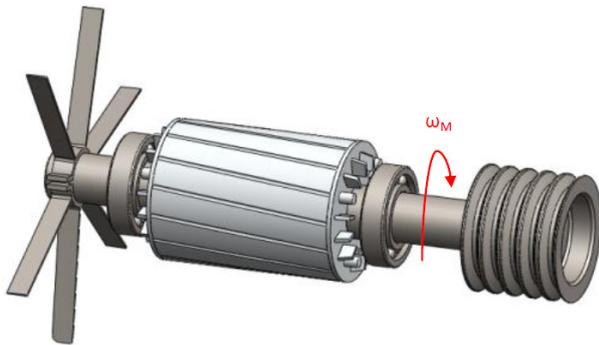


Figure 11. Shaft electric motor

	$m = 5,755 \text{ kg}$
	$I = 7,558126 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
	ω_M
	ω_M
	${}_{14}Ek = 3,77906 \cdot 10^{-3} \omega_M^2$
	${}_{2800}Ek = 3,77906 \cdot 10^{-3} \omega_M^2$

Figure 12. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_M

	$m = 2,082 \text{ kg}$
	$I = 6,787849 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
	ω_M
	ω_M
	${}_{14}Ek = 3,39393 \cdot 10^{-3} \omega_M^2$
	${}_{2800}Ek = 3,39393 \cdot 10^{-3} \omega_M^2$

Figure 13. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_M

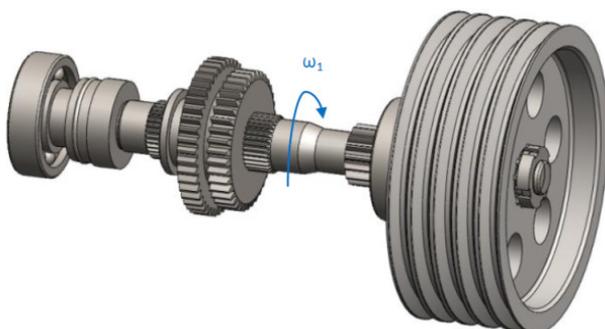


Figure 14. Gear input shaft

Figure 14 describes the input shaft to the gearbox mounted by ball bearings at the free end of the shaft is positioned on the grooved pulley shaft which is driven by motor speed and for insurance against axial displacement of the nut and washer. The smallest sprocket is made together with the shaft whereas the other two are stored in the teeth of the splined shaft.

In Figure 15 and Figure 16 are listed the dynamic characteristic parameters of the components of Figure 14 which the angular speed ω_1 in the transmission ratio 1:2 on the angular velocity ω_M to 14 rev/min for the 2800 rev/min.

The layshaft in Figure 17 is the only one in the whole assembly whose direction of rotation is opposite to the other, for the most part is made as splines. The layshaft is typical that most of the gears is stored securely. In our case, only the largest wheel displaceably stored. As in the previous two cases, the deposit are also used as the ball bearings of gears with straight teeth are no axial forces are applied to.

	$m = 1,976 \text{ kg}$
	$I = 0,310658 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
	${}_{14}\omega_1 = (1/2)\omega_M$
	${}_{2800}\omega_1 = (1/2)\omega_M$
	${}_{14}Ek = 3,88322 \cdot 10^{-5} \omega_M^2$
	${}_{2800}Ek = 3,88322 \cdot 10^{-5} \omega_M^2$
	$m = 0,69 \text{ kg}$
	$I = 0,371007 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
	${}_{14}\omega_1 = (1/2)\omega_M$
	${}_{2800}\omega_1 = (1/2)\omega_M$
	${}_{14}Ek = 4,63758 \cdot 10^{-5} \omega_M^2$
	${}_{2800}Ek = 4,63758 \cdot 10^{-5} \omega_M^2$
	$m = 4,415 \text{ kg}$
	$I = 0,0282035111 \text{ kg} \cdot \text{m}^2$
	${}_{14}\omega_1 = (1/2)\omega_M$
	${}_{2800}\omega_1 = (1/2)\omega_M$
	${}_{14}Ek = 3,52544 \cdot 10^{-3} \omega_M^2$
	${}_{2800}Ek = 3,52544 \cdot 10^{-3} \omega_M^2$

Figure 15. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_1

	$m = 0,054 \text{ kg}$
	$I = 1,11764 \cdot 10^{-5} \text{ kg} \cdot \text{m}^2$
	${}_{14}\omega_1 = (1/2)\omega_M$
	${}_{2800}\omega_1 = (1/2)\omega_M$
	${}_{14}Ek = 1,39705 \cdot 10^{-6} \omega_M^2$
	${}_{2800}Ek = 1,39705 \cdot 10^{-6} \omega_M^2$
	$m = 0,465 \text{ kg}$
	$I = 0,304934 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
	${}_{14}\omega_1 = (1/2)\omega_M$
	${}_{2800}\omega_1 = (1/2)\omega_M$
	${}_{14}Ek = 3,81167 \cdot 10^{-5} \omega_M^2$
	${}_{2800}Ek = 3,81167 \cdot 10^{-5} \omega_M^2$

Figure 16. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_1

In the Figure 18 and Figure 20 are described the considered parameters of all components of Figure 17, namely the weight, moment of inertia and kinetic energy which vary according to the different angular speed ω_2 at 14 rev/min and 2800 rev/min.

In the Figure 19 is interpreted the output shaft of the gearbox where the speed transmitted transformed using a flat belt onto the spindle. The splined shaft carries the two pairs of gears of interference, the lower wheel of each pair of the grooved and, if necessary, to make a joint movement of the sliding grooves. At the end of the cylindrical shaft is arranged by means of a pen-joint pulley, and at this point the shaft is mounted in two ball bearings to compensate for the rigidity of the shaft end.

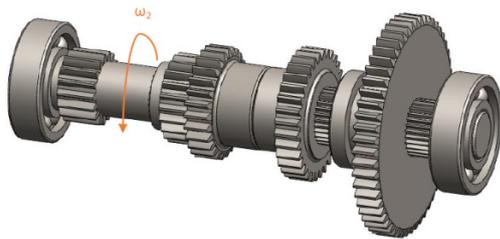


Figure 17. Countershaft gearbox shaft

	$m = 1,158 \text{ kg}$
	$I = 0,1116817 \cdot 10^{-3} \text{ kg.m}^2$
	${}_{14}\omega_2 = (6/47)\omega_M$
	${}_{2800}\omega_2 = (37/46)\omega_M$
	${}_{14}Ek = 9,14896 \cdot 10^{-7} \omega_M^2$
	${}_{2800}Ek = 3,60964 \cdot 10^{-5} \omega_M^2$
	$m = 0,112 \text{ kg}$
	$I = 3,2965 \cdot 10^{-5} \text{ kg.m}^2$
	${}_{14}\omega_2 = (6/47)\omega_M$
	${}_{2800}\omega_2 = (37/46)\omega_M$
	${}_{14}Ek = 2,70050 \cdot 10^{-7} \omega_M^2$
	${}_{2800}Ek = 1,06546 \cdot 10^{-5} \omega_M^2$
	$m = 0,389 \text{ kg}$
	$I = 0,1676714 \cdot 10^{-3} \text{ kg.m}^2$
	${}_{14}\omega_2 = (6/47)\omega_M$
	${}_{2800}\omega_2 = (37/46)\omega_M$
	${}_{14}Ek = 1,37356 \cdot 10^{-6} \omega_M^2$
	${}_{2800}Ek = 5,41927 \cdot 10^{-5} \omega_M^2$

Figure 18. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_2

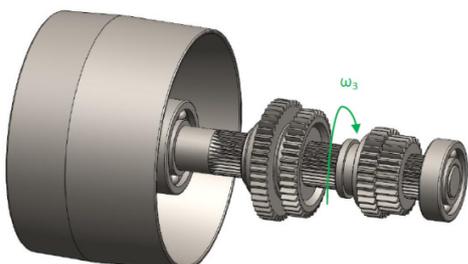


Figure 19. Gearbox output shaft

In the Figure 21 and Figure 22 are reflected our values necessary for the dynamical equations of the components in Figure 19 i.e. parts with a common axis of rotation for angular velocity ω_3 at 14 rev/min and 2800 rev/min.

In the Figure 24 is shown to transform rotation of the gear transmission ratio of 1:8 we used for our specific case for 14 rev/min on the spindle, for the 2800 rev/min was a transmission ratio 1:1 which is in Figure 23.

	$m = 0,062 \text{ kg}$
	$I = 1,5857 \cdot 10^{-5} \text{ kg.m}^2$
	${}_{14}\omega_2 = (6/47)\omega_M$
	${}_{2800}\omega_2 = (37/46)\omega_M$
	${}_{14}Ek = 2,70050 \cdot 10^{-7} \omega_M^2$
	${}_{2800}Ek = 1,06546 \cdot 10^{-5} \omega_M^2$
	$m = 0,302 \text{ kg}$
	$I = 8,8430 \cdot 10^{-5} \text{ kg.m}^2$
	${}_{14}\omega_2 = (6/47)\omega_M$
	${}_{2800}\omega_2 = (37/46)\omega_M$
	${}_{14}Ek = 7,24421 \cdot 10^{-7} \omega_M^2$
	${}_{2800}Ek = 2,85814 \cdot 10^{-5} \omega_M^2$
	$m = 1,103 \text{ kg}$
	$I = 0,9544557 \cdot 10^{-3} \text{ kg.m}^2$
	${}_{14}\omega_2 = (6/47)\omega_M$
	${}_{2800}\omega_2 = (37/46)\omega_M$
	${}_{14}Ek = 7,81890 \cdot 10^{-6} \omega_M^2$
	${}_{2800}Ek = 3,08488 \cdot 10^{-4} \omega_M^2$

Figure 20. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_2

	$m = 1,869 \text{ kg}$
	$I = 0,226665 \cdot 10^{-3} \text{ kg.m}^2$
	${}_{14}\omega_3 = (2/47)\omega_M$
	${}_{2800}\omega_3 = (1,001)\omega_M$
	${}_{14}Ek = 2,09552 \cdot 10^{-7} \omega_M^2$
	${}_{2800}Ek = 1,13559 \cdot 10^{-4} \omega_M^2$
	$m = 0,204 \text{ kg}$
	$I = 9,6287 \cdot 10^{-5} \text{ kg.m}^2$
	${}_{14}\omega_3 = (2/47)\omega_M$
	${}_{2800}\omega_3 = (1,001)\omega_M$
	${}_{14}Ek = 8,90171 \cdot 10^{-8} \omega_M^2$
	${}_{2800}Ek = 4,82397 \cdot 10^{-5} \omega_M^2$
	$m = 0,546 \text{ kg}$
	$I = 0,363245 \cdot 10^{-3} \text{ kg.m}^2$
	${}_{14}\omega_3 = (2/47)\omega_M$
	${}_{2800}\omega_3 = (1,001)\omega_M$
	${}_{14}Ek = 3,35820 \cdot 10^{-7} \omega_M^2$
	${}_{2800}Ek = 1,81986 \cdot 10^{-4} \omega_M^2$

Figure 21. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_3

	$m = 0,405 \text{ kg}$ $I = 0,348649 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_3 = (2/47)\omega_M$ ${}_{2800}\omega_3 = (1,001)\omega_M$ ${}_{14}Ek = 3,22326 \cdot 10^{-7} \omega_M^2$ ${}_{2800}Ek = 1,74673 \cdot 10^{-4} \omega_M^2$
	$m = 0,574 \text{ kg}$ $I = 0,484854 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_3 = (2/47)\omega_M$ ${}_{2800}\omega_3 = (1,001)\omega_M$ ${}_{14}Ek = 4,48247 \cdot 10^{-7} \omega_M^2$ ${}_{2800}Ek = 2,42912 \cdot 10^{-4} \omega_M^2$
	$m = 6,011 \text{ kg}$ $I = 0,036568703 \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_3 = (2/47)\omega_M$ ${}_{2800}\omega_3 = (1,001)\omega_M$ ${}_{14}Ek = 3,38078 \cdot 10^{-5} \omega_M^2$ ${}_{2800}Ek = 0,018320938 \omega_M^2$

Figure 22. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_3

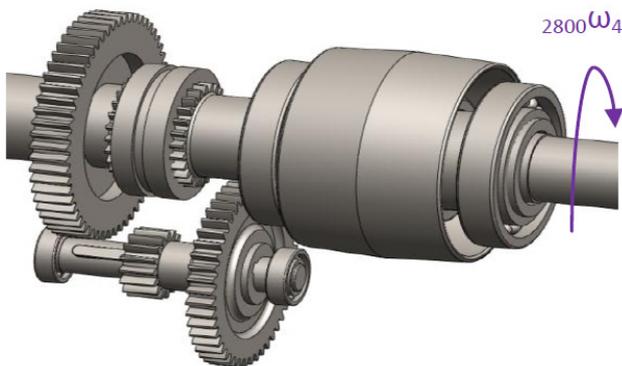


Figure 23. Spindle (2800 rev/min)

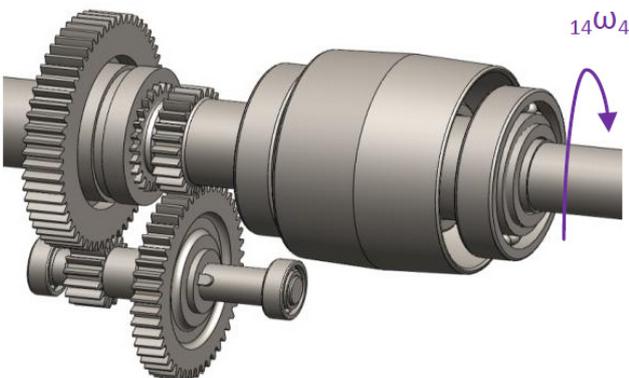


Figure 24. Spindle (14 rev/min)

In the Figure 25 to Figure 28 are listed parameters required for constructing dynamical equations in both modes machining.

	$m = 2,191 \text{ kg}$ $I = 2,211681 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_4 = (1/188)\omega_M$ ${}_{2800}\omega_4 = (1,001)\omega_M$ ${}_{14}Ek = 3,12879 \cdot 10^{-8} \omega_M^2$ ${}_{2800}Ek = 1,10805 \cdot 10^{-3} \omega_M^2$
	$m = 8,996 \text{ kg}$ $I = 0,038226776 \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_4 = (1,001)\omega_M$ ${}_{2800}\omega_4 = (1,001)\omega_M$ ${}_{14}Ek = 1,91722 \cdot 10^{-3} \omega_M^2$ ${}_{2800}Ek = 1,91722 \cdot 10^{-3} \omega_M^2$
	$m = 4,933 \text{ kg}$ $I = 7,659491 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_4 = (1,001)\omega_M$ ${}_{2800}\omega_4 = (1,001)\omega_M$ ${}_{14}Ek = 3,83741 \cdot 10^{-3} \omega_M^2$ ${}_{2800}Ek = 3,83741 \cdot 10^{-3} \omega_M^2$

Figure 25. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_4

	$m = 1,046 \text{ kg}$ $I = 0,127322 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_4 = (1/188)\omega_M$ ${}_{14}Ek = 1,8012 \cdot 10^{-9} \omega_M^2$
	$m = 0,931 \text{ kg}$ $I = 0,490294 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_4 = (1/188)\omega_M$ ${}_{14}Ek = 6,93603 \cdot 10^{-9} \omega_M^2$

Figure 26. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_4

	$m = 6,608 \text{ kg}$ $I = 0,023523278 \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_4 = (1/188)\omega_M$ ${}_{2800}\omega_4 = (1,001)\omega_M$ ${}_{14}Ek = 3,32776 \cdot 10^{-7} \omega_M^2$ ${}_{2800}Ek = 0,011785174 \omega_M^2$
	$m = 5,966 \text{ kg}$ $I = 0,028435371 \text{ kg} \cdot \text{m}^2$ ${}_{14}\omega_4 = (1/188)\omega_M$ ${}_{2800}\omega_4 = (1,001)\omega_M$ ${}_{14}Ek = 4,02266 \cdot 10^{-7} \omega_M^2$ ${}_{2800}Ek = 0,014246135 \omega_M^2$

Figure 27. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_4

	$m = 1,530 \text{ kg}$
	$I = 2,911245 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
	${}_{14}\omega_4 = (1/188)\omega_M$
	${}_{2800}\omega_4 = (1,001)\omega_M$
	${}_{14}E_k = 4,11844 \cdot 10^{-8} \omega_M^2$
	$m = 2,932 \text{ kg}$
	$I = 5,643062 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$
	${}_{14}\omega_4 = (1/188)\omega_M$
	—
	${}_{14}E_k = 7,98305 \cdot 10^{-8} \omega_M^2$

Figure 28. Moments of inertia and kinetic energy of the part on the axis with the angular velocity ω_4

5. Reduction Method

5.1. Computation of the Total Kinetic Energy of the System

Computation of the total kinetic energy of the system for speed 14 rev/min in function to the angular velocity ω_M :

$$\begin{aligned}
 {}_{14}E_K &= \sum {}_{14}E_{Ki} \\
 &= E_{KHRM} + \sum_{j=1}^6 E_{KHRj} + \sum_{k=1}^{12} E_{KOKk} \\
 &+ \sum_{l=1}^5 E_{KOKVRI} + \sum_{m=1}^4 E_{KREm} + E_{KSK} \\
 &= 0,01662526 \cdot \omega_M^2,
 \end{aligned} \quad (29)$$

where:

E_{KHR} - kinetic energy of the shaft,
 E_{KOK} - kinetic energy the gear wheel,
 E_{KREM} - kinetic energy the pulley,
 E_{KOKVR} - kinetic energy of the spindle gearwheels,
 E_{KSK} - kinetic energy of the chuck.

Computation of the total kinetic energy of the system for speed 2800 rev/min in function to the angular velocity ω_M :

$$\begin{aligned}
 {}_{2800}E_K &= \sum {}_{2800}E_{Ki} = E_{KHRM} + \sum_{j=1}^5 E_{KHRj} + \\
 &\sum_{k=1}^{12} E_{KOKk} + \sum_{l=1}^3 E_{KOKVRI} + \sum_{m=1}^4 E_{KREm} + \\
 &+ E_{KSK} = 0,064779103 \cdot \omega_M^2.
 \end{aligned} \quad (30)$$

5.2. Computation of the Instantaneous Angular Acceleration

By measuring, we found that the time taken to start up the machine at 14 rev/min was the 0.6second and for 2800 rev/min, it was 2.3second.

For 14 rev/min:

$$\alpha_M = \frac{\omega_M}{t} = \frac{280 \cdot \pi}{0,6} = 488,69 \text{ rad} \cdot \text{s}^{-2}, \quad (31)$$

where:

α_M – angular acceleration,
 ω_M – angular velocity of motor,
 t – time of the acceleration of the machine.

For 2800 rev/min:

$$\alpha_M = \frac{\omega_M}{t} = \frac{280 \cdot \pi}{2,3} = 127,49 \text{ rad} \cdot \text{s}^{-2}. \quad (32)$$

5.3. Computation of the Total Performance of the System

The total performance of the system we determine from the individual performance of members as follows:

$$\begin{aligned}
 P &= \sum P_i \\
 &= P_M + P_{HRM} + \sum_{j=1}^5 P_{HRj} + \sum_{k=1}^{12} P_{OKk} \\
 &+ \sum_{l=1}^3 P_{OKVRI} + \sum_{m=1}^4 P_{REm} + P_{SK} - P_{PO},
 \end{aligned} \quad (33)$$

where:

P_M – performance of motor,
 P_{PO} – passive resistors [7],
 P_{HR} - performance of shaft,
 P_{REM} – performance of pulley,
 P_{OK} - performance of the gear transmissions,
 P_{OKVR} - performance of the gear spindle,
 P_{SK} - performance of the chuck.

$$P = P_M - P_{PO} \quad (34)$$

$$P_M = M_{KM} \cdot \omega_M \quad (35)$$

$$M_{KM} = \frac{P_M}{\omega_M} = \frac{6000 \text{ W}}{\frac{280\pi}{3} \text{ rad} \cdot \text{s}^{-1}} = 20,46 \text{ Nm} \quad (36)$$

$$P_M = 20,46 \cdot \omega_M \quad (37)$$

$$P_{PO} = (0,05 \approx 0,2) \cdot P_M. \quad (38)$$

Consider the size of passive resistance 20% of the installed capacity:

$$P_{PO} = 0,2 \cdot 20,46 \cdot \omega_M = 4,092 \cdot \omega_M \quad (39)$$

$$P = P_M - P_{PO} = 20,46 \cdot \omega_M - 4,092 \cdot \omega_M \quad (40)$$

$$\frac{P}{\omega_M} = 16,368 \quad (41)$$

5.4. Dynamical Equations of the Motion

Substituting in the following dynamical equations we can verify the suitability of propulsion the lathe for the shaft speed:

$$\alpha_M \cdot \frac{2E_K}{\omega_M^2} = \frac{P}{\omega_M} \quad (42)$$

will be for 14 rev/min left side of equation in form:

$$\alpha_M \cdot \frac{2E_K}{\omega_M^2} = 488,69 \cdot \frac{2,0,01662526 \omega_M^2}{\omega_M^2} = 16,249 \quad (43)$$

Comparison with the right side of equation:

$$16,249 \cong 16,368 \quad (44)$$

it indicates, that the drive meets the lathe at an engine speed 14 rev/min.

For 2800 rev/min will be the left side of equation in form:

$$\alpha_M \cdot \frac{2E_K}{\omega_M^2} = 127,49 \cdot \frac{2,0,064779103 \omega_M^2}{\omega_M^2} = 16,517 \quad (45)$$

Comparison with right side of equation will be in form:

$$16,517 \cong 16,368 \quad (46)$$

and it shows that the lathe operator complies with the engine speed 2800 rev/min.

6. Conclusion

Derivation and calculation we have shown physical equality dynamical equations for the maximum and minimum output speed. Whereas the kinetic energy system that achieves maximum spindle speed (2800 rev/min) approximately four times greater than the kinetic energy in the system with a minimum speed of wiggle (14 rev/min) so therefore are the times required to ensure a constant speed at the output fourfold difference. For the verification procedure and calculations we can use other methods such as the method of release or Lagrange equations of second order. For the verification results would be possible doing experiment and computer simulation.

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