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Optimization of the Geometric Design of Silicon Solar Cells under Concentrated Sunlight

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Abstract The optimization of silicon solar cells is presented. The desired performance characteristics of a solar cell include a high conversion efficiency and a large power output through optimal grid contact design with minimal power losses under concentrated sunlight. Both square and rectangular shapes of linear grid contact patterns are considered for silicon solar cells with constraints on design parameters. The optical and ohmic losses are noted in computing the conversion efficiency and power output, which influence the structure and size of the solar cell and the grid contact design. In the optimization of solar cell performance, the various power losses caused by grid contact that influence the solar cell size, geometry of the fingers, busbars, and spacing between the metal grid lines are also considered. The proposed methodology permits the design of the solar cell for optimal cell performance even when some of the parameters, such as the size of the cell or the geometry of the fingers, are prespecified.

Keywords: *solar cell, optimization, geometric design parameters, genetic algorithm*

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1. Introduction

The purpose of a photovoltaic concentrator solar cell is to collect as much solar energy as possible from the sun's rays. Two steps are involved in the design of a solar cell. The first one is to optimize the performance of the solar cell by adjusting its thickness using the given data, especially the doping level. The second step is to minimize the power losses for maximizing the conversion efficiency (η_c) of the solar cell under concentrated sunlight. Therefore, in order to achieve the maximum conversion efficiency, it is necessary to optimize the structure of the solar cell as well as the collecting grid contact design under concentrated sunlight. Arturo (1985) [1] described a method for the optimization of the concentration factor in terms of the size and nominal efficiency (at intensity of 1sun), by assuming practical values of the specific resistance between the grid contact patterns and the semiconductor. However, factors such as the properties of materials, the metallic value of geometric grid contact factors, and the interactions among these factors were not considered; only the relationship between the conversion efficiency (η_c) and concentrated sunlight (*C*) to the length of cell is taken into account. Arturo (1985) [1] did not indicate a procedure to optimize solar cells. Gessert and Coutts (1992) [2] reviewed the models and techniques utilized to design and optimize metal contacts and antireflective coatings and indicated the differences between grid metallization of cells used under electrical resistivity by using a computer program. A limitation of Gessert's study is that it did not examine the design in terms of constraints on design variables. Liu, Li, Chen, Wang, and Yang (2010) [3] showed the influence of metal grid lines and power losses under concentrated sunlight (*C*) in the optimization of grid contact design of a solar cell by using computer simulations. Unfortunately, the study failed to establish how to obtain individual optimal design values when geometric grid contact values are fixed as constrained parameters based on the variation of metal grid properties. In addition, the thickness of the solar cell was not factored into power output, even though these thicknesses are related to sheet resistivity due to the doping level. Kulushich, Zapf-Gottwick, Baxer-Bahci, and Werner (2013) [4] presented a method to optimize the front geometric parameters with considerations of power losses. In this work, the solar cell structure and concentration ratio were not considered. In addition, although the optimization was performed by adjusting the values of the geometric grid contact parameters through a trial and error process, the resulting value of maximum conversion efficiency (η_c) is expected to be lower compared to the value obtainable through the application of mathematical programming techniques.

The aim of this work is to demonstrate how to determine grid contact design parameters and the thickness of a solar cell for any specified set of cell structure parameters. In addition, consideration of the interactions among the grid contact design parameters, cell size and structural parameters under concentrated sunlight, and constrained design parameters is emphasized. This work seeks to optimize the design of solar cells under concentrated sunlight to find the best solar cell structure and grid contact design parameters.

2. Theoretical Model

The performance of a solar cell can be described using the fundamental equations of semiconductor devices. There are three parts in a solar cell – the structure, cell size, and grid contact design for the optimization of solar cell performance.

The total current density in the three regions can be computed using Eq. (1) with details indicated in Appendix A.

$$J_L = J_E + J_{SCR} + J_B \quad (1)$$

The maximum operating power density (P_m) from a solar cell can be determined with the short-circuit current density and open-circuit voltage. The details of the computational procedure for finding the short-circuit current density and open-circuit voltage are presented in the Appendix A.

The maximum operating power density (P_m) at one sun intensity can be found as

$$P_m = J_m V_m \quad (2)$$

For a sunlight concentration with intensity *C*, the equations for P_m, J_m and V_m can be obtained as

$$P_m(C) = J_m(C) V_m(C) \quad (3)$$

where $J_m(C)$ and $V_m(C)$ can be expressed by

$$J_m(C) = CJ_m \quad (4)$$

$$V_m(C) = V_m + V_T \log(C) \quad (5)$$

where the intensity of sunlight *C* can vary in the range of 1 to 100 suns and $V_T = \frac{KT}{nq}$ and the ideality factor (*n*), which is chosen to lie between 1 and 2 for simplicity, is a measure of how closely the diode follows the ideal diode equation. When a load is connected to the diode, a current will flow in the circuit as shown in Figure 1.

Djeffal, Bendib, Arar, and Dibi (2012) [5] described an optimum grid metal design with the combined effect of the four loss mechanisms associated with the front metal grid contact design.

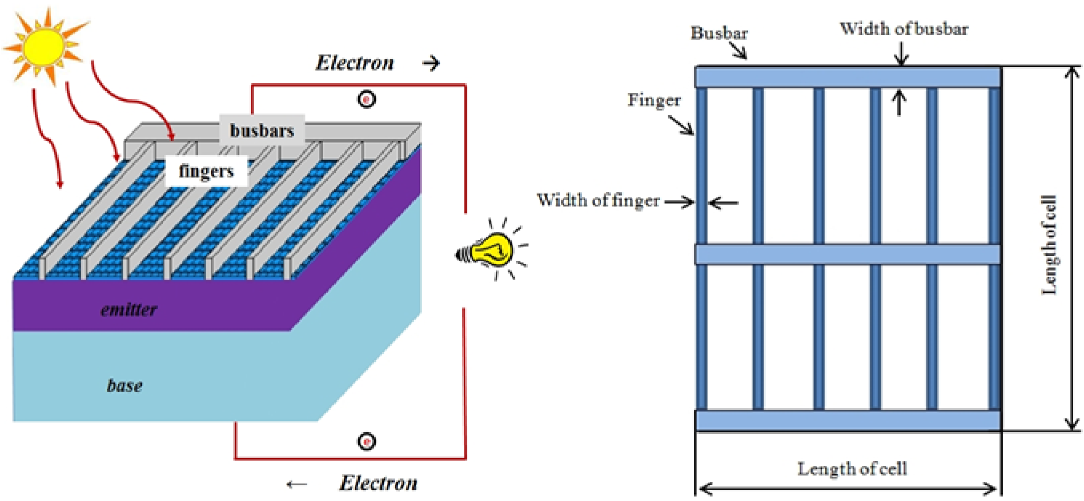

Figure 1. Simple solar cell structure with grid lines and top view of contact grid structure

After calculating the theoretical performance of a solar cell, it is essential to find the solar power without considering any of power losses from the blockage of concentrated sunlight caused by the influence of geometric parameters crucial to the extraction of voltage and current characteristics. In this work, shadowing and grid contact losses are considered in the optimization of solar cell design. Thus, the total power loss (F_{sum}) can be expressed in terms of the individual fractional power losses (Appendix A):

$$F_{sum} = F_{sr} + F_f + F_b + F_s + F_c \quad (6)$$

Arturo (1985) calculated the conversion efficiency of a solar cell under concentrated sunlight (*C*) as

$$\text{Conversion efficiency } (\eta_c), \% = \frac{J(C)_m V(C)_m}{P_{in}.C}(1 - F_{sum}) \times 100 \quad (7)$$

where P_{in} is the incident power density at 1 sun intensity and is equal to 1 Kw/m^2. In this work, the solar cell performance is optimized with consideration of fractional power losses and concentrated sunlight.

3. Deterministic Optimization of Solar Cell Performance

The performance of a solar cell can be expressed in terms of two characteristics, namely, the conversion efficiency (η_c), which does not depend on the cell area, and the power output (P_o), which depends on the cell area. The goal of optimization is to obtain the best performance of the system under given restrictions. The MATLAB program, *ga*, is used for the optimization of solar cell performance. The program, *ga*, is based on genetic algorithm (GA) and, most likely, can find the global minimum of a function involving mixed-integer variables starting from a set of initial design vectors.

3.1. Formulation of Optimization Problem

An optimization problem can be stated in mathematical form as

$$\text{Find } \vec{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{Bmatrix} \quad (8)$$

to minimize or maximize the objective function $f\left(\vec{X}\right)$ subject to the constraints

$$\begin{aligned} & g_i\left(\vec{X}\right) \le 0, i = 1,2,...,m \\ & l_i\left(\vec{X}\right) = 0, i = 1,2,...,p \\ & a_j \le x_j \le b_j,\ j = 1,2,\ldots,l \end{aligned} \quad (9)$$

where $g_i\left(\vec{X}\right)$ and $l_i\left(\vec{X}\right)$ are the inequality and equality constraints, respectively, x_j is the j^{th} design variable, and a_j and b_j are the lower and upper bounds on the j^{th} design variable, respectively.

3.2. Maximization of Solar Cell Performance

The objective is to find the optimal vector $\vec{X}$ for maximization of conversion efficiency (η_c) and power output (P_o) through minimization of the power losses under a solar intensity factor of C sums. The conversion efficiency can be expressed as

$$f_1\left(\vec{X}\right) = \text{Conversion efficiency} \quad (10)$$

To remove the dependence of power output of the solar cell on its area, it is more common to express the short-circuit current density as J_{sc} in mA/cm^2. Thus, the conversion efficiency of the solar cell will be related to short-circuit current (J_{sc}), open-circuit voltage (V_{oc}), incident power density (P_{in}) at 1 sun, concentrated sunlight of intensity of C suns, and the total fractional power loss (F_{sum}). The design vector of the problem, for a rectangular solar cell, is:

$$\begin{aligned} \vec{X}^{\circ} &= \left\{T_e\ T_b\ W_c\ H_c\ W_f\ H_f\ N_f\ W_b\ H_b\ N_b\ C\right\}^T \\ &\equiv \left\{x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ x_{11}\right\}^T \end{aligned} \quad (11)$$

The optimization problem is solved by placing lower and upper bounds on the design variables as $x_i^{(l)} \le x_i \le x_i^{(u)}; i = 1\ to\ 11$, with the bounds indicated in Table 1:

Table 1. Lower and upper bounds on the design variables

i	1	2	3	4	5	6	7	8	9	10	11
$x_i^{(l)}$	0.1µm	100µm	0.5cm	0.5cm	20µm	4.6µm	2	100µm	4.6µm	2	1
$x_i^{(u)}$	8µm	450µm	5cm	5cm	200µm	50µm	100	4000µm	50µm	10	100

The constraints of the optimization problem include relationships between the heights of finger (H_f) and busbar (H_b) by considering the delivery to the busbars and the shading from the busbars, the ratio of width to height of finger, and the spacing (D) between the fingers and the busbars:

$$D_f - \left(W_c - W_f \cdot N_f\right) / \left(N_f - 1\right) = 0 \quad (12)$$

$$W_f \cdot N_f - W_c \le 0 \quad (13)$$

$$D_b - \left(H_c - W_b \cdot N_b\right) / \left(N_b - 1\right) = 0 \quad (14)$$

$$W_b \cdot N_b - H_c \le 0 \quad (15)$$

$$0 \le H_f - H_b \le 1\mu\text{m} \quad (16)$$

$$0.23 \le \frac{H_f}{W_f} \le 0.25 \quad (17)$$

The power output (f_2) can be calculated from the maximum operating power densit (P_m), which corresponds to the maximum operating voltage (V_m) and current density (J_m) including the total fractional power loss (F_{sum}), and the size of the solar cell.

$$\begin{aligned} & \text{Maximize } f_2\left(\vec{X}\right) \\ & = \text{Power density}\left(\frac{W}{cm^2}\right) \times \text{area of solar cell } (cm^2) \end{aligned} \quad (18)$$

The problem of maximization of conversion efficiency (η_c) is investigated for two cases – one by maximizing the theoretical conversion efficiency of the solar cell (with no front contact material using only 8 design variables) and the other by maximizing the practical conversion efficiency of the solar cell (with front contact material which causes ohmic and optical losses). In the case of maximization of power output, the problem is addressed by including a constraint that the conversion efficiency be at least a specified percentage of the maximum conversion efficiency and is investigated for different percentages of the maximum conversion efficiencies.

3.3. Solution of Optimization Problems: Genetic Algorithm (GA)

Deb (2001) [6] and Rao (2009) [7] showed that the optimization problems stated above are mixed integer programming problems and are solved using Genetic Algorithm (GA). For the GA, the population size is chosen as 20 and the initial population is generated randomly within the specified lower and upper bounds of the design variables using uniform distribution for the random numbers. The fitness of the individuals (design vectors) is based on their ranks rather than their scores. The rank of an individual is its position in the scores. In other words, the rank of the fittest individual is one, that of the next fittest is two, and so on. This type of fitness scaling removes the effect of the spread of the raw scores. The selection of parents is based on uniform random number generation with each parent corresponding to the selection of the line of length proportional to its expected fitness value. The algorithm moves along the line in steps of equal size, one step for each parent. At each step, the algorithm allocates a parent from the section it lands on. The reproduction process specifies the number of individuals that are guaranteed to survive to the next generation as two. The crossover process is implemented by specifying the fraction of the next generation that crossover produces as 0.8. Mutation produces the remaining individuals in the next generation. The mutation function makes small random changes in the individuals in the population, which provide genetic diversity and enables the genetic algorithm to search in a broader space. The crossover process selects the genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form a child. The penalty parameter for violating a constraint is taken as 10 initially and gradually increased to 100 depending on the observed rate of convergence. For the convergence of the algorithm, a function tolerance of 1e-20 and a constraint tolerance of 1e-6 (for nonlinear constraints) are used.

3.4. Maximization of Conversion Efficiency of the Solar Cell

The maximization of the conversion efficiency (η_c) of a solar cell can be considered in two stages - theoretical and practical conversion efficiency. Figure 2 shows the variations of short-circuit current density in the emitter, base, space-charge regions (SCR) and the total current density with the thickness of the emitter over the range 0-8μm (by solving a number of optimization problems by fixing the thickness of the emitter at one specific value at a time). The thickness of the base is fixed as 250 μm. The current density in the emitter is found to increase steeply from 0 to 2 μm with a slow variation beyond a value of 2μm while the current density in the base is decreased within the same range. This means that the thickness of the emitter between 2 μm and 8 μm corresponds to a small variation of the total current density between 39.88 mA and 40.01 mA. As a result, the variations in the thicknesses of the emitter and base will have a large influence on the total current density in specific ranges between 0.1 μm and 2 μm for the emitter of a solar cell. On the other hand, the range between 2 μm and 8 μm for the emitter a solar cell will have much less impact on the total current density.

To maximize the practical conversion efficiency of a solar cell, the grid variations of the contact design variables, namely, the length of the cell, the width, height and number of the fingers, and the width, height and number of the busbars are to be considered. These variables lead to optical and ohmic losses that cause a reduction in the theoretical conversion efficiency. Two shapes- square and rectangular- are considered for the maximization of the practical efficiency of the solar cell. In the case of the square solar cell configuration, the width and height of the cell will be same; as such only ten design variables are considered by eliminating x_3 in the design vector of Eq. (11). A parametric study is conducted to find the influence of the concentrated sunlight on the open-circuit voltage of the solar cell. For this, the optimization problem stated in section 3.2 is solved several times by using only the first nine variables in the design vector, Eq. (11), with the value of the concentrated sunlight fixed at a different value each time.

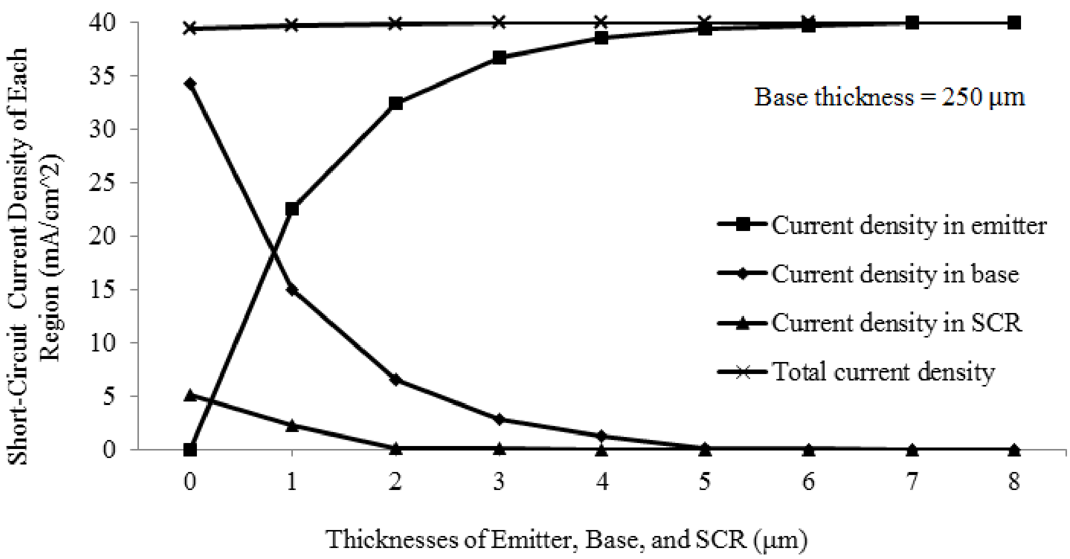

Figure 2. Relationships between short-circuit current density and thicknesses of emitter

3.4.1. Theoretical Conversion Efficiency of the Solar Cell Under a Concentration Ratio of One

The efficiency of a solar cell depends on a range of factors, such as materials, dopant concentrations (N_a and N_d), thickness of a solar cell (T_e and T_b), recombination velocity, and the variations associated with the fabrication process. An efficient solar cell consists of a thin emitter formed by low energy, low recombination velocity at the front surface, and enough base thickness for proper absorption from a wide range of wavelengths. However, if the thickness is thin enough, there is a limited recombination velocity of minority carriers toward the p-n junction. The emitter allows a solar cell to generate power. Heavily doped materials are the most important factors to determine the cell performance, because those minority-carrier diffusion coefficients (D_p and D_n), minority-carrier lifetime, and minority diffusion lengths (L_p and L_n) influence the open-circuit voltage $\left(V_{oc}\right)$ and short-circuit current $\left(J_{sc}\right)$. Once the design factors of a solar cell's performance are determined, there are a number of potential methods that could be utilized based on a study of the performance characteristics at various concentrations of sunlight. The optimal design factor of cell thickness reduces material costs because it results in a thin solar cell, which is competitively priced to have significant impact on the size of large-scale power systems. If a solar cell has a smaller thickness, material is saved. At the same time, its performance is maximized. The objective function for the maximization of the theoretical conversion efficiency of a solar cell can be expressed as

$$\text{Minimize } f_1\left(\vec{X}\right), \% = -\frac{J(C)_m V(C)_m}{P_{in} \cdot C} \times 100 \qquad (19)$$

All the constraints stated in section 3.2, except those associated with the grid contact materials, are considered in the solution process. Note that the first three design variables in Eq. (11), which relate to the grid contact materials, are excluded from the design vector. The theoretical conversion efficiency is determined to find the characteristics of the solar cell using the values of structural parameters and pre-specified values of the cell thickness and concentrated sunlight. The thickness of base $\left(T_b\right)$ is assumed as 250 µm, same as the value recommended in the literature under a concentrated sunlight of 1 sun intensity. The input design factors used in this study are given by : dopant concentration of emitter $=2\times10^{17}$cm^{-3}, dopant concentration of base = 5×10^{16}cm^{-3}, recombination velocity of front surface = 1 $\times10^{2}$cm/s , and recombination velocity of back surface = 1 $\times10^{4}$cm/s. The charateristics of the solar cell at optimal design, including the optimal values of the thicknesses of the emitter and base and the optimal conversion efficiency, are shown in Table 2. Next, a parametric study is conducted to find the influence of emitter thickness on the short-circuit current density.

Table 2. Characteristics of the optimal solar cell

Characteristic	Value
Thickness of emitter (T_e)	2 µm
Thickness of base (T_b)	250 µm
Concentrated sunlight (C)	1
Total short-circuit current (J_{sc})	40.01 mA/cm^2
Total open-circuit voltage (V_{oc})	630.56 mV
Maximum total current (J_m)	38.21 mA/cm^2
Maximum total voltage (V_m)	550.42 mV
Fill factor (FF)	83.35%
Conversion efficiency (η_c)	21.03%

3.4.2. Maximization of Practical Conversion Efficiency Considering All Ten Design Variables in Eq. (11)

The optimization problem stated in section 3.2 is solved by placing lower and upper bounds on all the design variables. In reference to the work of Sharan (1987) [8], Gessert (1992) [2] and Djeffal (2012) [5], the design data are assumed as N_d = 2 $\times10^{17}$cm^{-3}, N_a = 5 $\times10^{16}$cm^{-3} (dopant concentrations of emitter and base), S_n = 1 *cm/s* , S_p = 1 $\times10^{2}$*cm/s* (recombination velocities) to determine the thickness of the solar cell by specifying grid contact resistance, sheet resistance, and metal resistivity for silicon as ρ_c = 3 $\times10^{-3}$**Ω** cm^2, R_{sh} = 100 **Ω**/cm^2, and ρ_m = 1.6 $\times10^{-6}$**Ω**·*cm*, respectively. The solution of the optimization problem yields the optimal values of the solar cell thickness (i.e., emitter and base thicknesses), grid contact design parameters, and the intensity of concentrated sunlight.

Table 3. Optimal design variables and other characteristics of the solar cell

Quantity	Square cell	Rectangular cell
Design variables :		
Thickness of emitter (T_e)	7.3 µm	7.6 µm
Thickness of base (T_b)	244 µm	208 µm
Size of the solar cell: (L_c for square cell and $W_c \times H_c$ for rectangular cell)	0.81 × 0.81 cm^2	2.45 × 0.5 cm^2
Width of finger (W_f)	20 µm	20 µm
Height of finger (H_f)	5 µm	5 µm
Number of finger (N_f)	18	12
Width of busbar (W_b)	100 µm	100.4 µm
Height of busbar (H_b)	6 µm	6 µm
Number of busbar (N_b)	2	3
Concentrated sunlight (C)	6	6
Characteristics of solar cell :		
Short-circuit current ($J_{sc}(C)$)	240 mA/cm^2	240 mA/cm^2
Open-circuit voltage ($V_{oc}(C)$)	676.9 mV	676.9 mV
Maximum current ($J_m(C)$)	229.3 mA/cm^2	229.3 mA/cm^2
Maximum voltage ($V_m(C)$)	596.7 mV	596.7 mV
Fill factor (FF)	84.2 %	84.2 %
Conversion efficiency (without power losses (η_c))	22.80 %	22.80 %
Optimal conversion efficiency (with power losses (η_c))	20.28 %	20.54 %

If the cell size is permitted to vary along with other design variables during optimization, the optimal values of the remaining design factors are expected to be different compared to the values found in the previous cases. This study is conducted in this section. The optimal values of the design variables, the value of the geometric grid lines, short-circuit current, open-circuit voltage, and the optimal theoretical and practical efficiencies of the solar cell can be obtained as shown in Table 3.

As can be seen from Table 3, the length of the cell and width of the finger approached their respective lower bound values. The maximum practical efficiencies of the square and rectangular cells with minimal power losses are found to be 20.28 % and 20.54 %, respectively. The corresponding theoretical efficiencies are found to be 22.80 % in both types of cells.

3.4.3. Optimization with Pre-specified Cell Size and Width of Fingers

The practical conversion efficiency of the solar cell is maximized by specifying the area of the cell as 1×1cm^2 for a square cell and 3×2cm^2 for a rectangular cell, and the width of fingers as 80 μm. Thus there will be only eight design variables left in Eq. (11) in the optimization problem. The other data are same as the ones given in Table 2. The results obtained from the solution of the optimization problem (using the program, *ga*) are shown in Table 4.

Table 4. Results of optimization with eight design variables

Quantity	Square cell	Rectangular cell
Design variables :		
Thickness of emitter (T_e)	8 μm	8 μm
Thickness of base (T_b)	304 μm	363 μm
Height of finger (H_f)	20 μm	20 μm
Height of busbar (H_b)	21 μm	21 μm
Width of busbar (W_b)	100 μm	415.6 μm
Number of finger (N_f)	10	17
Number of bus bar (N_b)	2	2
concentrated sunlight (C)	4	3
Characteristics of the solar cell :		
Short-circuit current ($J_{sc}(C)$)	160 mA/cm^2	120 mA/cm^2
Open-circuit voltage ($V_{oc}(C)$)	666.4 mV	659 mV
Maximum current ($J_m(C)$)	152.8 mA/cm^2	114.6 mA/cm^2
Maximum voltage ($V_m(C)$)	586.3 mV	578.8 mV
Fill factor (FF)	84 %	83.9 %
Theoretical conversion efficiency (without power losses(η_c))	22.4 %	22.4 %
Optimal conversion efficiency (with power losses(η_c))	18.17 %	19.17 %

It is noticed that the specification of the geometric parameters of the cell area and the width of a finger did not affect the thickness of the solar cell, short-circuit current and open-circuit voltage. The practical conversion efficiency is decreased from 20.28 % to 18.17 % and 20.54 % to 19.17 %, while the fractional power losses are increased from 11.09 % to 18.88 % and 9.93 % to 13.32 % for the square and rectangular cells, respectively, due to the optical and resistive losses. This shows that the change in the conversion efficiency is not proportional to the change in fractional power losses due to concentrated sunlight and interactions among the various geometric parameters.

3.5. Maximization of Power Output

The conversion efficiency of the solar cell is independent of the power output. It is only a factor when the solar cell is designed for maximum power output within a specified area. Thus, in order to find the maximum power output of the solar cell, a minimum constraint of 80 % of the maximum conversion efficiencies (in both square and rectangular cells) becomes necessary to prevent unrealistic conversion efficiencies resulting from the intensity of sunlight. Table 5 shows the values of design variables and other outputs corresponding to maximum power output with a constraint of at least 80 % of the maximum conversion efficiency in square and rectangular cells.

Table 5. Optimal design variables and other characteristics of the solar cell

Quantity	Square cell	Rectangular cell
Design variables :		
Thickness of emitter (T_e)	7.81 μm	6.19 μm
Thickness of base (T_b)	218.39 μm	222.94 μm
Size of the solar cell: (L_c for square cell and W_c× H_c for rectangular cell)	$5 \times 5\ cm^2$	$5 \times 5\ cm^2$
Width of finger (W_f)	35.96 μm	37.64 μm
Height of finger (H_f)	8.99 μm	9.41 μm
Number of finger (N_f)	41	40
Width of busbar (W_b)	179.80 μm	188.21 μm
Height of busbar (H_b)	10.00 μm	10.42 μm
Number of busbar (N_b)	7	7
Concentrated sunlight (C)	1	1
Conversion efficiency (with power losses(η_c))	19.08 %	19.08 %
Maximum power output (P_o)	0.4770 W	0.4770 W

In the case of a square cell, the maximum power output is 0.4770 W and the conversion efficiency is 19.08 %, which is 94 % of the maximum conversion efficiency. Since the power output is related the size of the solar cell and conversion efficiency, which is associated with an increase in individual power losses, the total power loss $\left(F_{sum}\right)$ in a square cell for maximum power output is 9.27 %. The dominant power loss of 5.39 % is the fractional power loss due to shadowing from busbars and fingers (sunlight blockage). Thus, the maximum power output is given by the product of the power density, $0.0191\ \dfrac{W}{cm^2}$ and the area of the solar cell, 25cm^2 as 0.4770 W, for a square cell.

In the case of a rectangular cell, the maximum power output is found to be 0.4770 W with the conversion efficiency of 19.08 %, nearing 93 % of the maximum conversion efficiency (20.54 %). The difference in total power loss between maximum conversion efficiency and maximum power output is 1.93 %, an indication that the total power loss in maximum power output is higher as a result of differing sizes of the solar cell. The main fractional power loss (5.56%) is caused by shadowing from fingers and busbars because the number of fingers and busbars has increased from 12 to 41 and from 3 to 7, respectively. In addition, the widths of the finger and busbar associated with the power losses have increased from 20.03 μm to 37.64 μm and from 100.21 μm to

188.21 μm, respectively. With consideration of cell structure, cell size, grid contact design, and conversion efficiency, the power output of 0.4770 W can be found as the product of density, 0.0191 $\frac{W}{cm^2}$, and the area of the solar cell, 25 cm^2 for a rectangular cell. This indicates that the relationship between the conversion efficiency and power output plays a role in the optimization of solar cell performance.

The single-objective optimization problems for maximum conversion efficiency provide a solar cell design having a maximum conversion efficiency of 20.28 % and 20.54 % for square and rectangular cells, respectively. In this section, the maximum power output of a solar cell is investigated through sensitivity analysis with respect to applying different constraint values on the maximum conversion efficiency (formulated as the ratio η_c / η_{max_c}) in the range of 70 % to 100 %.

The difference between the lowest and highest power output of a solar cell is mainly caused by changes in power density, collected amount of sunlight, and total fractional power loss. An increase in the thicknesses of a solar cell emitter and base has the influence of improving the conversion efficiency (η_c). The widths of fingers and busbars contribute to conversion efficiency and power density since they prevent collection of the proper amount of sunlight. Beyond a value of 90 % for minimum permissible value of the maximum conversion efficiency, the cell areas have steeply decreased from 25.00 cm^2 to 0.66 cm^2 for a square cell and from 25.00 cm^2 to 1.23 cm^2 for a rectangular cell. This indicates that an increase in the conversion efficiency combined with an increase in the power output occur due to changes in the geometric design and the number of fingers and busbars; these are also associated with total power loss and power density.

Table 6 shows the variations of conversion efficiency, total power loss, power density, cell area, and power output with respect to different minimum permissible values of constraint on conversion efficiency in finding the maximum power output. The relationship between the conversion efficiency and power output has been observed through variations in power density and cell area associated with geometric design parameters, which indicated that the amount of power output has increased even though the power density has decreased by the reduction of conversion efficiency. Thus, the area of a solar cell contributes to an increase in the total power output in both types of cells. This indicates that the behaviors of conversion efficiency and power output are approximately opposite; hence, a compromise solution is to be found in a practical solar cell design.

Table 6. Variations of conversion efficiency, power density, cell area, and power output with respect to maximum conversion efficiency ratio

Results							
$\frac{\eta_c}{\eta_{max\_c}}$		Conversion efficiency (%)	Total power loss (%)	Power density ($\frac{W}{cm^2}$)	Cell area (cm^2)	Power output (W)
70%	Square	19.08	9.27	0.0191	25.00	0.4770
	Rect.	19.08	9.28	0.0191	25.00	0.4770
75%	Square	19.08	9.27	0.0191	25.00	0.4770
	Rect.	19.08	9.28	0.0191	25.00	0.4770
80%	Square	19.08	9.27	0.0191	25.00	0.4770
	Rect.	19.08	9.28	0.0191	25.00	0.4770
85%	Square	19.08	9.27	0.0191	25.00	0.4770
	Rect.	19.08	9.28	0.0191	25.00	0.4770
90%	Square	19.08	9.27	0.0191	25.00	0.4770
	Rect.	19.08	9.27	0.0191	25.00	0.4770
95%	Square	19.24	8.53	0.0192	15.51	0.2984
	Rect.	19.51	10.15	0.0195	15.16	0.2958
100%	Square	20.28	11.06	0.1217	0.66	0.0798
	Rect.	**20.54**	**9.93**	**0.1232**	**1.23**	**0.1514**

4. Conclusion

The optical and ohmic losses are the major causes that reduce the efficiency of a solar cell in the practical conversion of solar energy to electricity. To maximize the conversion efficiency and power output from sunlight to electricity, the parameters of the solar cell structure and the mechanism of collecting solar energy are determined using mathematical programming approaches. Solar cell modeling has revealed that, in optimizing solar cell performance, a correlation exists between the cell structure and geometric design parameters for maximizing conversion efficiency of the solar cell. The solar cell size, power density, and total power loss influence the maximizing power output. Heavy doping levels reduce the open-circuit voltage and short-circuit current of a solar cell due to the emitter layer with resistive properties, while close spacing between fingers results in high power losses. This is the most important reason why optimal geometric design factors should be found for a solar cell for maximum conversion efficiency. The design of solar cells under specified constraints on design parameters, finds the optimal intensity values of *C* suns, the values of the solar cell structure, such as the thicknesses of emitter and base and grid contact design parameters when the conversion efficiency is maximized. Even when the areas of square and rectangular solar cells are restricted to lie in any specific ranges, such as, 0.5 × 0.5 cm^2 to 5 × 5 cm^2 and 0.5 to 5 × 0.5 to 5 cm^2, respectively, the values of the optimal geometric design parameter can be determined through optimization. It is found that, in some cases, higher total power losses do not always correspond to maximum conversion efficiency on account of concentrated sunlight. Therefore, the correlation of cell structure parameters, grid contact design, cell size, conversion efficiency, and power output under intensity of sunlight should be deliberated by using optimum design procedure for the solar cell.

Nomenclature

C = Intensity of sunlight (an integer)
D_b = Spacing between the busbars (cm)
D_f = Spacing between the fingers (cm)
D_p = Minority electron diffusion coefficient (cm^2/s)
D_n = Minority hole diffusion coefficient (cm^2/s)
F_b =Fractional power loss of the resistivity of the busbars (%)
F_c = Fractional power loss of contact resistance (%)
F_f = Fractional power loss of the resistivity of the fingers (%)
F_{sum} = Total fractional power losses (%)
F_s = Fractional power loss of shadowing (%)
F_{sr} = Fractional power loss of sheet resistance (%)
H_c = Height of cell (cm)
J_B = Current density of base (mA/ cm^2)
J_E = Current density of emitter (mA/ cm^2)
J_L = Light-generated current density (mA/ cm^2)
J_m = Maximum operating current density (mA/ cm^2)
J_{SCR} = Current density of space-charge region (mA/ cm^2)
J_S = Saturation current density (mA/ cm^2)
J_{sc} = Short-circuit density (mA/ cm^2)
k = Boltzmann constant (8.617×10^{-5}eV/K)
L_p = Minority electron diffusion length (μm)
L_n = Minority hole diffusion length (μm)
L_T = Current transfer length (μm)
L_c = Length of cell (cm)
N_a = Acceptor concentration (cm^3)
N_d = Donor concentration (cm^3)
n_i = Intrinsic carrier concentration (cm^3)
n_{ph} = Photon flux ($cm^{-2}\ s^{-1}$)
P_m = Maximum operating power density (W/ cm^2)
P_{in} = Input incident power density (W/ m^2)
P_o = Power output (W)
q = Electron charge (1.602×10^{-19} *colulomb*)
R = Reflection coefficient of the anti-reflective coating
R_{sh} =Sheet resistance (Ω/ cm^2)
S_p = Recombination velocity of the front surface (cm/s)
S_n = Recombination velocity of the back surface (cm/s)
t_{scr} = Width of the space charge region (μm)
T = Temperature (K)
T_e = Thickness of the emitter region (μm)
T_b = Thickness of the base region (μm)
V_{oc} = Open-circuit voltage (mV)
V_m = Maximum operating voltage (mV)
W_c = Width of cell (cm)
α = Absorption coefficient (cm^{-1})
η_c = Conversion efficiency of a solar cell (%)
η_{max_c} = Maximum conversion efficiency of a solar cell (%)
τ_p = Minority carrier lifetime in the emitter region (μs)
τ_n = Minority carrier lifetime in the base region (μs)
δE_g = Shrinkage of the energy gap (eV)
ρ_c = Contact resistance ($\Omega\cdot cm^2$)
ρ_m = Metal resistivity (Ω·cm)
λ = Wavelength (μm)

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Appendix

Appendix A: Determination of the maximum operating power density (P_m) and total fraction power loss (F_{sum})

Calculations of the short-circuit current density (J_{sc}) and open circuit voltage (V_{oc}) for the maximum operating power (P_m) and the total fractional power losses (F_{sum}) from ohmic resistance losses are the essential factors to optimize a solar cell based on the geometric design parameters.

- **Calculation of short-circuit current density (J_{sc}) and open circuit voltage (V_{oc})**

The total current density (J_L) can be expressed as

$$J_L = J_E + J_{SCR} + J_B \quad \text{(A-1)}$$

Where the individual value of J_E, J_{SCR}, and J_B can be refereed by the publication of Jain, Heasell, and Roulston (1986) [9] and Singal (1980) [10].

Total current density (J_L) can be calculated as

$$J_E = qn_{ph}(1-R)\left[\frac{\alpha L_p}{L_p^2\alpha^2 - 1}\right]\times\left[\frac{\left(\left(\frac{S_pL_p}{D_p}\right)+\alpha L_p\right) - e^{-\alpha T_e}\left(\left(\frac{S_pL_p}{D_p}\right)\cosh\left(\frac{T_e}{L_p}\right)+\sinh\left(\frac{T_e}{L_p}\right)\right)}{\left(\frac{S_pL_p}{D_p}\right)\sinh\left(\frac{T_e}{L_p}\right)+\cosh\left(\frac{T_e}{L_p}\right)} - \alpha L_p e^{-\alpha T_e}\right] \quad \text{(A-2)}$$

$$J_B = qn_{ph}(1-R)\left(\frac{L_n\alpha}{L_n^2\alpha^2-1}e^{(-T_e+t_{scr})\alpha}\right)\times\left[L_n\alpha - \frac{\left(\frac{S_nL_n}{D_n}\right)\left[\cosh\left(\frac{T_b}{L_n}\right)-e^{-\alpha T_b}\right]+\sinh\left(\frac{T_b}{L_n}\right)+L_n\alpha e^{-\alpha T_b}}{\left(\frac{S_nL_n}{D_n}\right)\sinh\left(\frac{T_b}{L_n}\right)+\cosh\left(\frac{T_b}{L_n}\right)}\right] \quad \text{(A-3)}$$

$$J_{SCR} = qn_{ph}(1-R)e^{-T_e\alpha}(1-e^{-t_{scr}\alpha}) \quad \text{(A-4)}$$

$$t_{scr} = \sqrt{\frac{2K_s\varepsilon_0V_{bi}}{q}\frac{(N_a+N_d)}{N_aN_d}} \quad \text{(A-5)}$$

The photon flux density $n_{ph}(\lambda)$ and Si absorption coefficient were described by Liou and Wong (1992) [11] under AM1.5 global normal sun condition can be approximated with two linear curves as

$$n_{ph}(\lambda) = C(19.7\lambda - 4.7)\times 10^{15} \quad for\ 0.24 \le \lambda \le 0.47\mu\text{m} \quad \text{(A-6)}$$

$$n_{ph}(\lambda) = C(-2.5\lambda + 5.7)\times 10^{15} \quad for\ 0.48 \le \lambda \le 1.1\mu\text{m} \quad \text{(A-7)}$$

Also, the absorption coefficient (α) of silicon material can be divided into 4 sections to obtain the data based on particular wavelength range, and is given by

$$\alpha(\lambda) = \begin{cases} 0 & for\ \lambda \ge 1.1\mu m \\ 10^{-6.7\lambda+8.4}cm^{-1} & for\ 0.8 \le \lambda \le 1.1\mu m \\ 10^{-3.3\lambda+5.6}cm^{-1} & for\ 0.5 \le \lambda \le 0.8\mu m \\ 10^{-6.7\lambda+8.4}cm^{-1} & for\ \lambda \le 0.5\mu m \end{cases} \quad \text{(A-8)}$$

The short-circuit current density (J_{sc}) is due to the generation and collection of light-generated carriers and can be expressed as

$$\begin{aligned} J_{sc} &= J_L - J_S \\ &= J_L - J_{01}\left[\left(e^{\frac{qV_{oc}}{kT}}\right)-1\right] - J_{02}\left[\left(e^{\frac{qV_{oc}}{nkT}}\right)-1\right] \end{aligned} \quad \text{(A-9)}$$

where the reverse saturation current density (J_0) can be computed using the following equations, this is also known as the diode equations

$$J_{01} = qn_i^2\left(\frac{D_n}{N_aL_n}+\frac{D_p}{N_dL_p}\right) \quad \text{(A-10)}$$

$$J_{02} = \frac{qn_it_{scr}}{2(\tau_n\tau_p)^2}e^{\left(\frac{\delta E_g}{2kT}\right)} \quad \text{(A-11)}$$

Also, the open-circuit voltage (V_{oc}) can be found as

$$V_{oc} = \left(\frac{kT}{q}\right)\log\left(\frac{J_{sc}}{J_0}+1\right) \quad \text{(A-12)}$$

Thus, the maximum operating power at one sun (P_m) can be found as

$$P_m = J_m \times V_m \quad \text{(A-13)}$$

where the maximum current (J_m) and voltage (V_m) are given by

$$\begin{aligned} J_m &= J_L\left(1-\frac{1}{v+1-\log(v)}\right), \\ V_m &= V_{oc}(1-\frac{1}{v}\log(v+1-\log(v))), \\ and\ v &= \frac{nkT}{q}V_{oc} \end{aligned} \quad \text{(A-14)}$$

- **Calculation of total fractional power loss (F_{sum})**

The calculation of total fractional power loss (F_{sum}) was explained by Shabana, Saleh, and Soliman (1988) [12] and can be expressed as

$$F_{sum} = \sum_{i=1}^{n}\frac{P_{loss}}{P_{generation}} = \sum_{i=1}^{n}\frac{\sum_{i=1}^{n}F_{sum}J_mV_m}{\sum_{i=1}^{n}J_mV_m} \quad \text{(A-15)}$$

where n is the number of individual terms of fractional power loss. While calculating the total fractional power loss (F_{sum}), the weighted fractional losses for the individual terms are calculated, with the weights being the power contributions from the respective terms.

The total fractional power loss (F_{sum}) can also be expressed in terms of the individual fractional power losses as

$$F_{sum} = F_{sr} + F_f + F_b + F_s + F_c \quad \text{(A-16)}$$

The resistance of the sheet can be expressed in a differential form as

$$\mathrm{dR} = \left(\frac{\textit{Sheet resistance}}{\textit{Distance along finger}}\right) \quad \text{(A-17)}$$

Distance between two fingers.

Thus the power loss due to sheet resistance can be calculated as

$$P_{loss\_sheet} = \int_0^{D/2} \frac{\mathrm{J}_m{}^2 \mathrm{L}_f{}^2 D^2 \mathrm{R}_{sh}}{\mathrm{L}_f} d_x = \frac{\mathrm{J}_m{}^2 \mathrm{L}_f \mathrm{R}_{sh}}{24} \quad \text{(A-18)}$$

The power generated can be expressed as

$$P_{generation} = \mathrm{J}_m \times V_m \left(\mathrm{L}_f \times \frac{D}{2}\right) \quad \text{(A-19)}$$

The fractional power loss of sheet resistance (F_{sr}) is given by

$$F_{sr} = \frac{\mathrm{J}_m \mathrm{R}_{sh} \mathrm{D}^2}{12\mathrm{V}_m} \quad \text{(A-20)}$$

Under normal circumstances, the contact resistivity (R_c) can be written, using the concept of transfer length (L_T), as

$$R_c = \frac{\sqrt{\mathrm{R}_{sh}\rho_c}}{\mathrm{L}_c} coth\left(\mathrm{L}_c \sqrt{\frac{\mathrm{R}_{sh}}{\rho_c}}\right) = \frac{2\mathrm{L}_T \mathrm{R}_{sh}}{\mathrm{L}_c} \cot\left(\frac{\mathrm{W}_f}{2\mathrm{L}_T}\right) \quad \text{(A-21)}$$

The specific contact resistance was described by Harrison and Reeves (1982) [13] and the power loss of contact resistivity can be found as

$$P_{losss\_contact} = I^2 R_C = \left(\mathrm{J}_m \times \frac{\mathrm{L}_c}{2} \times \frac{\mathrm{D}}{2}\right)^2 \left(\frac{2\mathrm{L}_T \mathrm{R}_{sh}}{\mathrm{L}_c} \cot\left(\frac{\mathrm{W}_f}{2\mathrm{L}_T}\right)\right) \quad \text{(A-22)}$$

Thus, the fractional contact loss (F_c) is given by

$$F_c = \frac{\mathrm{J}_m \mathrm{D}}{2\mathrm{V}_m} \mathrm{L}_T \mathrm{R}_{sh} \coth\left(\frac{\mathrm{W}_f}{2\mathrm{L}_T}\right) \quad \text{(A-23)}$$

The top of a solar cell has a series of arranged fingers intended to collect current. The corresponding resistive loss is given by

$$P_{generation} = \mathrm{J}_m \times V_m \left(\frac{\mathrm{L}_c}{2} D\right) \quad \text{(A-24)}$$

Because of symmetry, the equation is applied precisely at the midway along the length of finger to obtain

$$P_{loss\_finger} = I^2 R_f = \int_0^{\frac{\mathrm{L}_f}{2}} \left(\mathrm{J}_m \frac{D}{2} \mathrm{L}_f\right)^2 \left(\frac{\rho_m}{\mathrm{W}_f H_f}\right) d_x \quad \text{(A-25)}$$

Thus, the fractional power loss of a finger (F_f) can be expressed as

$$F_f = \frac{\mathrm{J}_m \rho_m \left(\mathrm{L}_f\right)^2 D}{48\mathrm{V}_m \mathrm{W}_f H_f} \quad \text{(A-26)}$$

The ratio of width to thickness of a contact should be within the limits of the recommended aspect ratio, which is 0.23~0.25. Also, the fractional power loss of busbars (F_b) is given by

$$\mathrm{F}_b = \frac{\mathrm{J}_m \rho_m B(\mathrm{L}_b)^2}{6\mathrm{V}_m H_b \mathrm{W}_b} \quad \text{(A-27)}$$

The fractional power loss of shadowing (F_s) depends on the size and number of grid lines (N_f and N_b) because it prevents light from entering a solar cell. The fractional power loss of shadowing (F_s) is given by

$$\mathrm{F}_s = \left(1 - \left(\frac{L_c^2 - \left(\begin{array}{l}\mathrm{N}_f \mathrm{W}_f \mathrm{L}_f + \mathrm{N}_b \mathrm{W}_b \mathrm{L}_b \\ -\mathrm{N}_f \mathrm{W}_f \mathrm{N}_b \mathrm{W}_b\end{array}\right)}{L_c^2}\right)\right) \quad \text{(A-28)}$$