

Analytical Modeling of a Piezoelectric Bimorph Beam

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Abstract Smart structures based on piezoelectric materials are now finding applications in a wide variety of environmental conditions. Such materials are capable of converting mechanical energy into electrical energy. Indeed, when subjected to mechanical stress become electrically charged at their surface and vice versa. In the current paper, the research is focused on a simple analytical model based on Euler–Bernoulli beam theory with the following assumptions was proposed: (a) the piezoelectric layer thickness in comparison to the length of the beam is very thin and (b) the electrical field between the upper surface and lower surface of the piezoelectric layer is uniform. We have applied this model to study its static responses and predict the ambient deformations into usable electrical energy from a cantilever piezoelectric beam. The proposed model was numerically investigated for validity. Analytical data showed that the proposed model simulations are in good agreement with the FE results. The results of the modeling are very promising.

Keywords: smart structure, piezoelectric material, modeling

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1. Introduction

Piezoelectric materials, such as Lead Zirconate Titanate (PZT), have the ability to convert mechanical forces into an electric field in response to the application of mechanical stresses or vice versa [1,2,3]. This effect is due to the spontaneous separation of charge within certain crystal structures, thereby producing an electric dipole. For a piezoelectric material, it is known that the output voltage of the material is a function of stress. Typically, stress is achieved through the displacement or bending of the piezoelectric beam. These novel properties of the materials have found applications in sensor and actuator technologies, and recently, in the new field of energy harvesting [4,5,6,7].

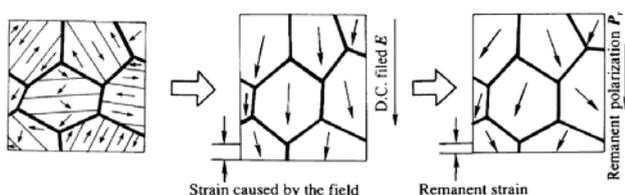


Figure 1. Schematic illustration of the poling process

The origin of the piezoelectric effect is related to an asymmetry in the unit cell and the resultant generation of electric dipoles due to the mechanical distortion. These materials do not possess any piezoelectric properties owing to the random orientations of the ferroelectric domains in the ceramics before poling. During poling, an electric field is applied on the ferroelectric ceramic sample to force the domains to be oriented or poled. After poling,

the electric field is removed and a remnant polarization and remnant strain are maintained in the sample, and the sample exhibits piezoelectricity. A simple illustration of the poling process is shown in Figure 1.

In recent years, there have been a considerable number of publications using various models for the electromechanical behavior of piezoelectric energy harvester beams, as can be seen in Anton and Sodano [8]. Eggborn [9] developed the analytical models to predict the energy harvesting from a cantilever beam and a plate using Bernoulli-beam theory and made a comparison with the experimental result. Ajitsaria et al. [10] developed modeling and analysis of a bimorph piezoelectric cantilever beam for voltage generation using on the analytical approach based on Euler–Bernoulli beam theory and Timoshenko beam equations, which is then compared with two previously described models in the literature: the electrical equivalent circuit and energy method. Lead Zirconate titanate (PZT) is the most used piezoelectric material because of its high electromechanical coupling characteristics in single crystals [11,12]. Sodano et al. [13] developed an analytical model of a beam with attached PZT elements to provide an accurate estimate of the power generated by the piezoelectric effect.

Erturk and Inman have recently published a series of papers on energy harvesting using the cantilever model and their provide a broad coverage of several important modeling aspects that were validated with experimental data [14,15,16,17,18].

Works presented in this report concerns the problems the characteristic phenomena of piezoelectricity. In the current paper, the research is focused on the different deformations effect by voltage generation of the piezoelectric beam. The relation between the voltage imposed and the curvature is derived which is used to

explain the effect of placement on voltage generation is investigated. The predictive models are validated by being compared to numerical data.

2. Theory and System Modeling

Piezoelectricity involves the interaction between electrical and mechanical behaviors of the material. Meanwhile, the static linear relations between the electrical and mechanical variables can approximate this interaction. In strain-charge form, these relations can be given by the following equations:

$$\varepsilon = s^E \sigma + dE_3 \quad (1)$$

$$D = d\sigma + \varepsilon^\sigma E \quad (2)$$

Where s^E , ε^σ and d are the elastic compliance coefficient, electric field related electrostriction coefficient and linear permittivity respectively and E_3 are the electric field intensity [19].

In equations (1) and (2) above, the electrical quantities (E , D) have vector nature, while the mechanical quantities (σ , s) have tensor nature of six components. In piezoelectric materials, the constants d , e and ε depend on the directions of electric field, displacement, as well as stress and strain. The piezoelectric strain constant (d) can be presented as d_{ij} which can be interpreted as charge created in the i^{th} direction under a stress applied in the j^{th} direction.

The linear constitutive equations for a piezoelectric material [20] have been employed in terms of the piezoelectric coefficient e_{31} and the electric field applied across the thickness of the layer E_3 .

3. Cantilever Beam Analysis

The following section describes the development of the PZT models and the analytical estimations of local deformation. The schematic view of the beam undertaken in this study is shown in Figure 2. If the thickness of the both host substrate layers equal to $200\mu\text{m}$, respectively h_{p1} , h_{p2} are far greater than the thickness of the electrode patch (500nm), and the PZT beam length (L) equal 10mm is approximately the length of the electrode layer. The PZT beam width (w) equal 1mm is approximately thirty times the width of the electrode layer ($w_0=30\mu\text{m}$, neglected in the analytical model proposed). A thin electrode layer was attached to the PZT layer using two-part epoxy glue in a typical bimorph configuration (see Figure 3). The PZT PCI-151 was used to manufacture the cantilever beam device. The dimensions and properties of the materials are shown in Table 1.

Based on piezoelectric cantilever, the most important piezoelectric strain coefficients when designing harvesting devices are d_{31} and d_{33} (Roundy et al., 2003). In the 3-1 mode, imposed strain in the 1-direction is perpendicular to the electric field in the piezoelectric material (i.e. in the 3-direction). In the 3-3 mode, the strain and electric field are parallel to each other (in the 3-direction).

The electrical-mechanical coupling of the 3-3 mode (or z-z mode) is higher than the 3-1 mode (or z-x mode) and

the 3-3 mode piezoelectric cantilever will allow for a much higher open circuit voltage compared to a similarly sized 3-1 cantilever. Another key advantage of the 3-1 mode of operation over the 3-3 system is that the 3-1 system is much more compliant, hence larger strains can be produced with smaller input forces. The piezoelectric constitutive equation (2) can be rewritten as:

$$D_3 = d_{31}\sigma_1 + \varepsilon_{33}^\sigma E_3. \quad (3)$$

Table 1. The Dimensions and Properties of Electrode and Pzt Beam

	Parameters	Value
PZT Beam	length	10 mm
	width	1 mm
	thickness	200 μm
	Young modulus	66 GPa
	piezoelectric constant d_{33}	$-210.10 \cdot 10^{-12}$ m/V
Electrode	length	10 mm
	width	30 μm
	thickness	500 nm
	Young modulus	200 GPa

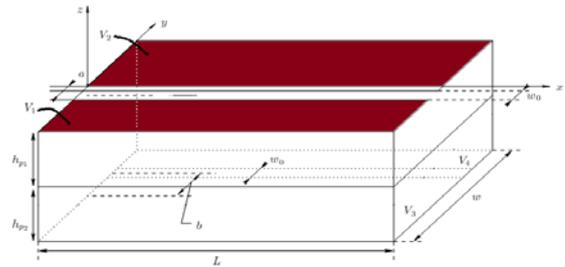


Figure 2. Schema a piezoelectric cantilever beam and electrode patches

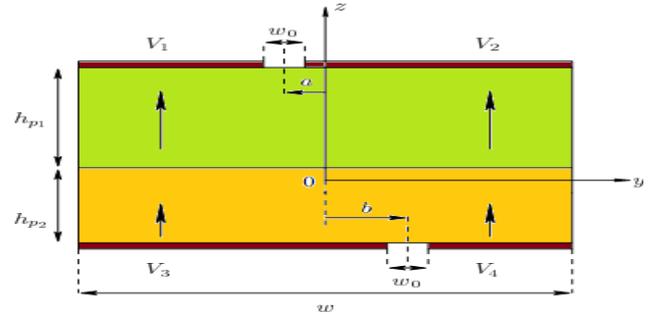


Figure 3. The attachment of electrode on the PZT beam

4. Analytical Results and Discussions

The modeling of a cantilever beam by means of the substitution potential V_1 , V_2 , V_3 , V_4 , by the two command variables of the beam axes V_y et V_z of such so that:

$$\begin{aligned} V_1 &= V_z + V_y, & V_2 &= V_z - V_y \\ V_3 &= V_z - V_y, & V_4 &= V_z + V_y. \end{aligned}$$

The moment equilibrium equation along y is expressed by:

$$\iint \sigma_1(x, y, z) dA = 0 \quad (4)$$

The relative deformation of the beam due to the elasticity is given by:

$$\varepsilon_1 = \frac{z}{\rho_z} \quad (5)$$

Furthermore, the slope at an abscissa x of the beam is in a first approximation:

$$\frac{dz}{dx} = \frac{x}{\rho_y} \quad (6)$$

The resolution of equation (4) by using equation (6) we obtain the following deflection z, δ_z :

$$\delta_z = \frac{3}{8} \frac{d_{311} L^2 h_{p1} \left\{ (w - w_0 + 2a) V_1 + (w - w_0 - 2a) V_2 \right\}}{s_{11}^{p1} w B_1} + \frac{3}{8} \frac{d_{312} L^2 h_{p2} \left\{ (w - w_0 + 2b) V_3 + (w - w_0 - 2b) V_4 \right\}}{s_{11}^{p2} w B_1}$$

$$\text{With } B_1 = \left(\frac{h_{p2}^3}{s_{11}^{p2}} + \frac{h_{p1}^3}{s_{11}^{p1}} \right)$$

Taking our case the two piezoelectric plates are the same, therefore if the centering error of the electrodes is the same on both faces of the bimorph, or $a = b$ or $a = -b$.

$$\delta_z = \frac{3}{4} \frac{d_{31} L^2}{w h_p^2} \left[(w - w_0) V_z - 2b V_y \right]$$

Moreover, it is possible to draw on the same graph of the deflection curve of the bimorph beam obtained for different values of a and b errors. The superposition of these curves to quantify the value of the deflection at the end of the cantilever. It was found that this deflection is directly proportional to the value of the electric field V_y by setting the voltage $V_z = 100V$ (cf. Figure 4).

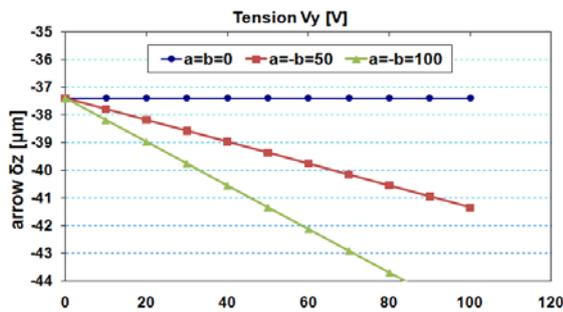


Figure 4. Deflections δ_z according to V_y for several centering errors as $a = -b$ and $V_z = 100V$

In this case we observe that for $a = b = 0$ there is a stabilization of the deflection for any value of the tension V_y against to $V_y = 100V$ the results obtained without taking account the error values a and b (cf. Figure 5).

However, the values of a and b depend primarily on the technical means of realization implemented for the development of these actuators. From a technological point of view, it is difficult to strictly satisfy the condition $a = b = 0$, or even $a = b$.

Let us now try fixing the two tensions and traced the deflections based on centering errors electrodes a and b on the coupling in the case of $a = -b$.

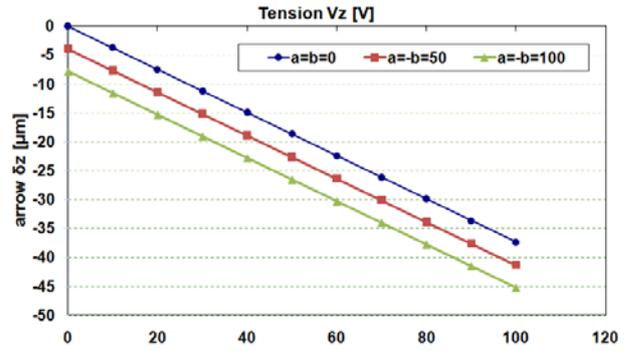


Figure 5. Deflections δ_z according to V_z for several centering errors as $a = -b$ and $V_y = 100V$

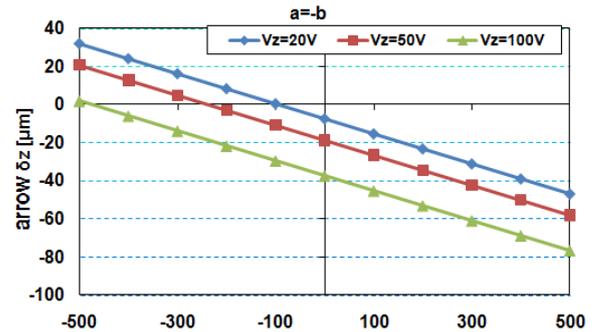


Figure 6. Deflections δ_z according to V_z for several centering errors as $a = -b$ and $V_y = 100V$

The Figure 6 presents the deflection as a function $a = -b$, for $V_y = 100V$ and for several values of V_z . In these conditions, we observed not only a decrease of the deflection when $a = b$ increases, but also a coupling V_z on δ_z .

5. FE Modeling Validation

As illustrated in Figure 7, the analytical solution in terms of the evolution of the deflection as a function of voltage conforms to the numerical simulation with the piezoelectric material cited below. The numerical curve being almost equivalent to the analytical.

We also assume that there is no centering error, or $a = b = 0$. We then obtain for a degree of freedom in z, the following equation:

$$\delta_z = \frac{3d_{31} L^2 (w - w_0)}{4 w h_p^2} V_z \quad (5)$$

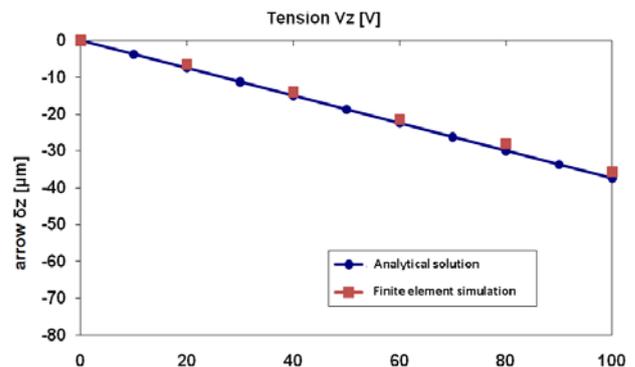


Figure 7. Free deflections δ_z of the piezoelectric beam depending on the control voltage V_z

6. Conclusion

Modeling the clamped-free piezoelectric beam small displacement was developed and presented in this article. The electromechanical coupling has been considered. The analytical results were valid for comparisons with numerical results found in the literature. The analytical results showing a good correlation and agree very well. It was also concluded that the actuator and the sensor will be better placed at underrun because it is the position or the actuator has the greatest impact and where the sensor gives the greatest signal. They are co-located as said Colles one below the other on either side of the beam.

This modeling is again applicable to structures of any shape and may include piezoelectric components to form a control or vibration. In addition, they allow us to know the magnitude of the force imposed equivalent to the applied field for a given displacement. These results are necessary for the design, optimization and development of systems for micromanipulation and assembly of micro objects.

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