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Design of Compact Compliant Devices – Mathematical Models vs. Experiments

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Abstract The wide range of applications requires using compact compliant structures that exhibit specific functional features different from traditional mechanisms. Therefore the design process, beside known design methods, should include procedures for mathematical modeling and performance simulation of final product. The best way is recommended to build the physical / hybrid model and to compare results from both models. This paper describes the design procedure of a micro-positioning device (gripper) based on compliant mechanisms. The complex performance test of the hybrid (mathematical – physical) model of the device are made on the test bed have been developed for this purpose. The laboratory test equipment includes linear actuator with force sensing and webcam for sensing positions / deflections. It enables experimentations in order to verify quasi real functional characteristics of the designed device with expected features from theoretical / mathematical models. Results from experimental measurements and their comparison with data from mathematical models are shown.

Keywords: *compact compliant devices, micro-gripper, modeling and simulation, image processing*

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1. Introduction

Compliant devices seem as powerful tools in wide range of technical applications. They offer advantages such as increased performance (high precision, low weight), lower cost (simplified manufacture based on equal methods as are used in integrated circuit manufacturing) and ability to miniaturize (this same physical principles are used in macro, micro and nano-scale devices) [1]. Incidentally, such devices have strong functionality connected with application usage. Sometime compliant structure could be use as positioning mechanism but this same structure could be used as a sensor. The difference between such applications is in using actuator in case of positioning device, or analyzing deformations in case of sensor application [2]. On the other side, the design of such devices is more complicated comparing to traditional robotic devices [3]. Design methods are relatively new and are similar to design methods of classical kinematical structures where particular revolute or prismatic joints are replaced by elastic connections. Such connection exhibits cross deflection effects that deteriorate final accuracy of the mechanisms and its performance, as whole. The other types of compliant devices are devices with distributed compliance [1,6,7].

Both designs methods (for traditional and compliant devices) are currently based on mathematical modeling and simulation with careful analysis of results. However in case of compact compliant devices the main building elements are flexible joints and flexural characteristics of these elements usually differ from theirs mathematical models. The cumulated positional errors can reach up to 5 – 10% [4,5]. This is one reason why it is necessary to build hybrid models (combination of physical models/prototypes and mathematical models) and to verify expected and quasi real performance characteristics of a final product.

2. Mathematical Models of Compliant Device

As suitable example of compact compliant device will be used design of our micro-gripper [8,9]. Such device has minimal number of flexural joints which minimize errors of end position [4,5]. The initial design conditions were: maximal dimensions 10x10mm and maximal dimensions of grasped object 0.2mm. After careful analysis and shape optimization the final design proposed principal dimensions of the gripper body as follows: width 5.0mm, height 6.2mm, distance of fingers 0.2mm, thickness of flexure joint 0.05mm and the thickness of the flexure plate 0.5mm. The elastic material is aluminum alloy.

Considering that designed device is relatively small and available manufacturing technologies, building the physical sample / model in scale 20:1 was chosen. In this case there are two technologies that can be used: precise machining or 3D printing. Standard 3D printers support only specific types of materials like PLA or ABS, but these materials exhibit small flexibility. As a suitable material can be processed by 3D printers it seems the polyamide. Naturally, when other material is used, in calculation of flexural characteristics different values of material Young's elasticity modulus must be taken into account.

The physical sample, made for experiments, is produced by 3D printing technology by using polyamide (Young's modulus is 1.65GPa, Poisson ratio 0.34, density 930kg/m^3 and Yield strength is 48MPa). Principal dimensions are: width of gripper - 80mm, high of gripper - 154.2mm, thickness of the flat elastic material - 3mm, the thickness of all elastic joints made in form of two-side circular notches is 1mm. The form of the physical model is shown in the Figure 1.

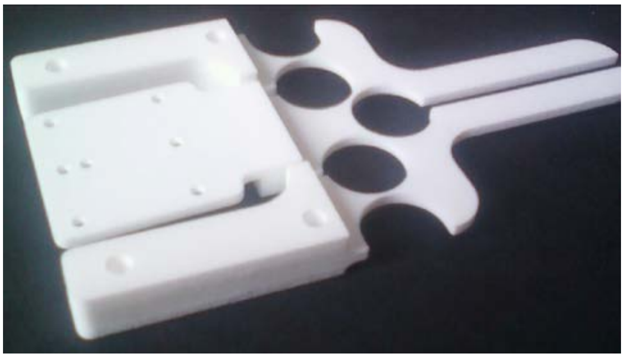

Figure 1. The form of compliant micro-gripper physical model

Considering that the aim of the paper is comparing results from mathematical models and simulations with results from experimental measurements. All models are created with dimensions of physical sample.

2.1. MATLAB/SimMechanics

MATLAB and its toolbox SimMechanics is a suitable and a strong tool for constructions powerful mathematical models of kinematics and dynamics of our device. Considering that the flexure hinge is the main building element of the compliant structure the carefully built model is required. The lack of chosen simulation environment has no options for calculation of problems focused on elasticity and rigidity of the elastic bodies. As the compact structures belong to this problem group too, the representation is made by description of mass-spring-damper system. The mathematical model of compliant joint goes out from splitting the joint on small parallel elastic elements and assumption that the joint deflections are within limit of linear elastic deformation i.e. within the validity of the Hooke's law.

The principal block diagram of the joint model represents Figure 2. Solution of a given problem lies in the segmentation of the beam (joint) on n elastic segments. Each of these segments consists of elementary three components: the rigid coupling - connector, the deformable elements - body and the core - core of the elasticity. Mutual connection of two rigid components (in one segment) is made through the stiffness matrix. So, for the planar case two lateral elastic deflections (as sliding motions) in directions x and y and one rotation around the z axis should be calculated. One side of the segment – body is strongly fixed to the imaginary frame and the force, calculated from the total load, affects on the second side of the segment. The transformations of forces and displacements are made through the calculation block Weld, which is imaginarily connected to the end of model. This segmentation enables to calculate the direct interaction between the solid parts of the mechanism and flexible segments / joints [10].

The input parameters for flexure hinge model are dimensions, mass of joint and Young's modulus of used material. From these parameters are calculated all require matrixes for mass-spring-damper system. Usually, properties for inertial moment of are not specified in SimMechanic environment when the blocks from the Joint group are used.

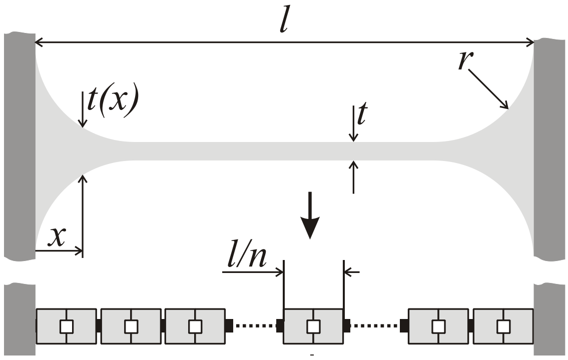

Figure 2. Basic block diagram of flexible joint model

For the mass the one element (of model of flexure joint) the following relation can be used

$$m_i = \rho w \int_{\frac{l}{n}(i-1)}^{\frac{l}{n}i} t(x)\,dx; \quad i = 1,2,\ldots n \tag{1}$$

where n is the number of elements, t(x) is the function describing the shape of the notch. The mass matrix for whole flexible joint is

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{pmatrix} \tag{2}$$

where m is the total weight of the flexible joint and I_z is the inertia moment around the z axis [11].

For solving planar problem the elements of the flexibility matrix (compliance) are evident from equation (3). It represents the relations between acting load and elastic deflection [12]

$$\begin{pmatrix} u_x \\ u_y \\ \theta_z \end{pmatrix} = \begin{pmatrix} C_{xFx} & 0 & 0 \\ 0 & C_{yFy} & C_{yMz} \\ 0 & C_{\theta zFy} & C_{\theta zMz} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} \tag{3}$$

Considering that material properties related to damping characteristics are usually not available the damping matrix is calculated as dependence of mass and compliance/stiffness matrix [13].

$$B = Me^{-\left(M^{-1}K\right)^2/2} \sinh\left(K^{-1}M \ln\left(M^{-1}K\right)^{2/3}\right) + K\cos^2\left(K^{-1}M\right)\sqrt[4]{K^{-1}M}\,\tan^{-1}\frac{\sqrt{M^{-1}K}}{\pi} \tag{4}$$

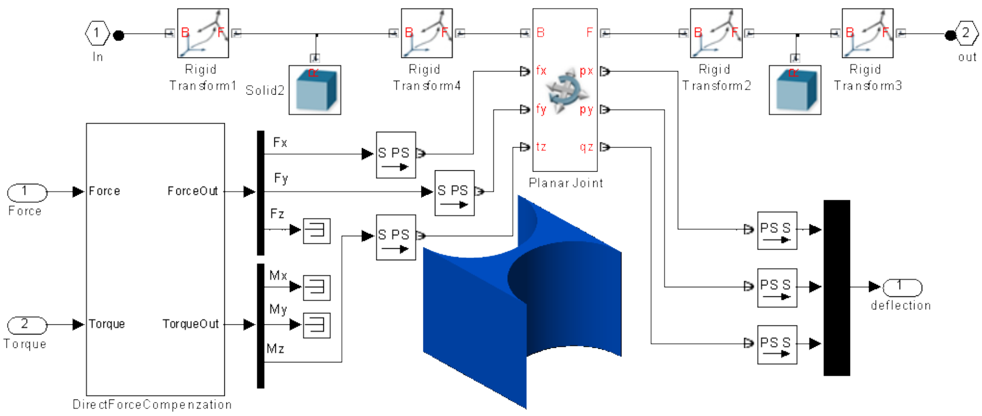

Figure 3. MATLAB/SimMechanics block scheme of planar flexure joint

2.2. Stiffness Model

The mathematical model build in MATLAB/SimMechanics can not be applied for a complete stiffness analysis of our compliant device. It considers only stiffness of the flexural hinges in their working directions. The stiffness model of whole compliant device needs calculations to specify other actuator parameters. The principal condition is that input force/energy exerted by actuator should deflect the mechanisms and produce desired force at output point/surface. For our case it is force between fingers.

The compliant gripper consists of two symmetric mechanisms as illustrates Figure 4. Therefore stiffness model is build for one half of mechanism and final stiffness of gripper includes both parts.

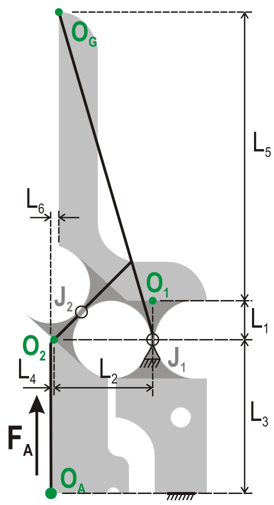

Figure 4. Geometry of symmetric part of compliant micro-gripper

The dependence between acting load and resulting deformations for a flexure joint describes (3). The compliance matrix is expressed in local frame located at end of flexure part of joint. Joint is in horizontal position. Then, it is necessary to transform this compliance matrix to references that correspond to joint J_1 and point O_1. Compliance matrix with respect to point O_1 is expressed as

$$C_{01} = T_{01} C_{J1} T_{01}^T \tag{5}$$

where C_{J1} is common compliance matrix of flexure joint, T_{01} is transformation matrix from ground frame to frame located in point O_1. It can be expressed [14,15].

$$T_{01} = \begin{pmatrix} R_{01} & R_{01}P_{01} \\ 0 & R_{01} \end{pmatrix} \tag{6}$$

where R_{01} denotes the rotation matrix of the frame O_0 (located in ground) with respect to coordinate system in O_1, P_{01} is displacement matrix between O_0 and O_1. The rotation matrix R_{01} is

$$R_{01} = \begin{pmatrix} \cos\alpha_{01} & -\sin\alpha_{01} & 0 \\ \sin\alpha_{01} & \cos\alpha_{01} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

where α_{01} is rotation angle around z axis between frame O_0 and O_1. The displacement matrix is

$$P_{01} = \begin{pmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{pmatrix} \tag{8}$$

where p_i are particular displacement.

The rotation angles and particular displacements for transformation matrixes and compliance matrixes are expressed as follows

From ground frame to point O_1

$$\alpha_{01} = \frac{\pi}{2}; \quad p_{01} = [0, L_3 + L_1, 0]^T \tag{9}$$

From frame in point O_1 to frame in point O_2

$$\alpha_{12} = \frac{135.15\pi}{180}; \quad p_{12} = [-L_2, -L_1, 0]^T \tag{10}$$

Separately compliance matrix of flexible joint J_2 is calculated

$$\alpha_{J2} = \frac{227.7}{180}\pi \tag{11}$$

The compliance matrix expressed to frame in point O_2 is

$$C_{12} = T_{12} C_{01} T_{12}^T + C_{J2} \tag{12}$$

Rem.: Both joints are serially arranged with respect to O_2. From frame located in point O_2 to frame in frame O_A (place where actuator is located)

$$\alpha_{2A} = \frac{135\pi}{180}; \quad p_{2A} = [-L_4, -L_3, 0]^T \tag{13}$$

$$C_{2A} = T_{2A} C_{2A} T_{2A}^T \quad (14)$$

The stiffness of whole mechanism located in point O_A is expressed as

$$K_{2A} = 2C_{2A}^{-1} \quad (15)$$

Rem.: A connection of two symmetric parts is expressed as parallel connection. (It is sum of all stiffness located to specified point)

2.3. FEM Analysis

The FEM analysis was used as strong tool for stress analysis in elastic parts of compliant mechanisms. As software tool the Comsol Multiphysics is used. The purpose of analysis was verification if a stress in elastic parts is within limits of Hooke ´s law.

The input parameters for model are: material of whole device is polyamide PA2200 with parameters: Young's modulus 1.65GPa, Poisson ratio 0.34, density 930kg/m^3 and Yield strength is 48MPa. The dimensions of model mechanism without actuator mounting part are: width 80mm, height of gripper 100mm, thickness of flat material 3mm, min. thickness of flexure joint 1mm, distance of fingers 4mm. The Figure 5 shows the form and fixation frame with placement of the actuator.

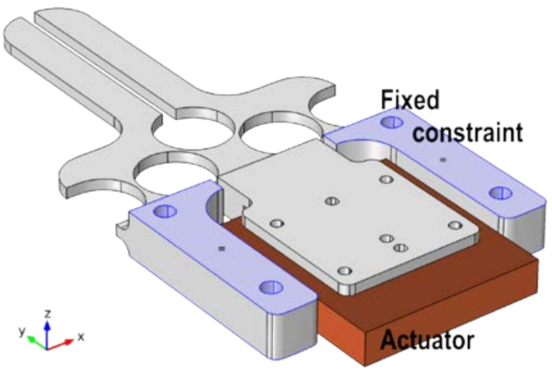

Figure 5. The gripper mechanism with the place of actuator

The Figure 6 illustrates distribution of stress in the elastic joints of the mechanism. As can be seen the maximum stress is much lower then stress allowed for PA2200. This fact enables that physical sample can be produced by 3D printing technology. The minimum printed thickness of walls is stated on 1mm what corresponds to the thickness of flexure joints.

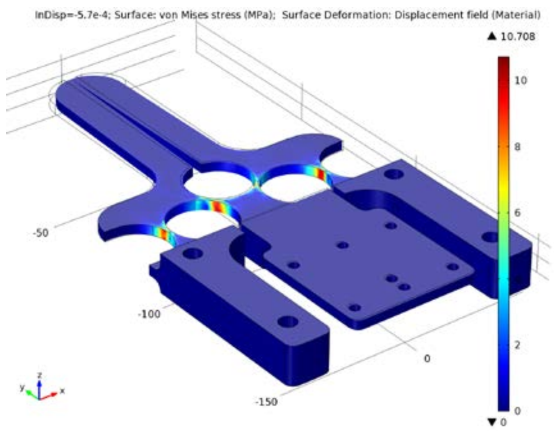

Figure 6. Distribution of stress in compliant micro-gripper

3. Test Bed for Performance Tests of Compliant devices

Performance characteristics of a device can be obtained if a hybrid approach and corresponding models will be applied. Considering the specific group of compliant robotic devices such approach is really needed. For this reason, the experimental platform for performance tests has been developed in our laboratory. More general purpose of this test bed is verification of performance characteristics of various devices frequently used in robotics (e.g. precise positioning mechanisms, grippers and mechanical or force amplifiers). It enables to apply HIL (Hardware-in-the-Loop) and SIL (Software-in-the-Loop) approaches, as suitable procedures in designing complex mechatronic devices.

Experimental platform consists of a linear actuator working on electromagnetic principle [16] with the stroke 10mm, maximal motion speed 400mm/s, resolution by optical sensor 30μm, positioning accuracy ±90μm, maximum force 5.5N. As feedback components could be used some sensors with analog output e.g. force sensitive resistor. The data from sensors are processed by microprocessor unit on Arduino platform. In our platform the common webcam is used for main positional feedback. For sensing displacements of end element on relatively small area the evaluation of displacement are made by processing follow-up images. For example: in case scanning the area 10x10mm by relatively cheap webcam with resolution 640x480, one pixel presents approximately 20μm. On the other side, it is not possible to apply this method for fast moving objects; only for initial deflection measurements. Others principal components of the test bed are: power source, galvanic isolation module or motor controller. The modular concept of our experimental platform enables to add or replace any component (sensors, linear actuators with their controller). Photo in Figure 7 is shown the actual state of the whole test bed.

The necessary condition for application in hybrid models is mutual connection to higher control system. All principal components should have the possibility for interconnection to the higher control system (MATLAB environment in our system). The connection is realized by universal serial bus (USB).

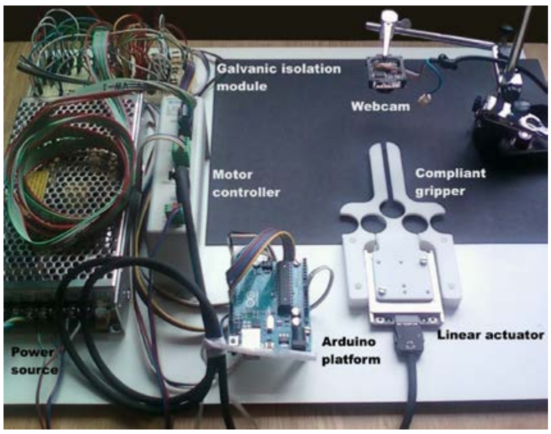

Figure 7. Experimental platform for performance tests of compliant devices

In the MATLAB environment the GUI application for "real-time" control of linear actuator was developed.

Actually it is in progress the development mutual communication between MATLAB and motor controller. It works in some modes for instance: the jog mode where only speed and direction is defined, the move mode where target position, speed and direction are specified and the normal mode, where all available Step Data blocks (15 blocks) are used. In this mode it is possible to define particular blocs in cycle time entry method (target position, positioning time, pushing/moving operation, load mass and movement mode absolute coordinate vs. relative coordinate with the current position) and speed entry method where positioning time is replaced by specification of speed, acceleration and deceleration. Any desired input motion for our physical sample could be defined by using these blocks.

In development of control algorithm based on microprocessor unit it is possible to arrange the main mutual communication thought Arduino platform, which can control motor controller by parallel digital inputs/outputs. All analog inputs on Arduino platform are still available for connection of some types of sensors.

There are two approaches that enable positional feedback from experimental platform. The first approach uses information for input displacement readings from motor controller, as output from linear encoder with resolution 30µm. The second approach is based on reading data from webcam and following image processing. Such approach is frequently used in industrial applications [17] and [18].

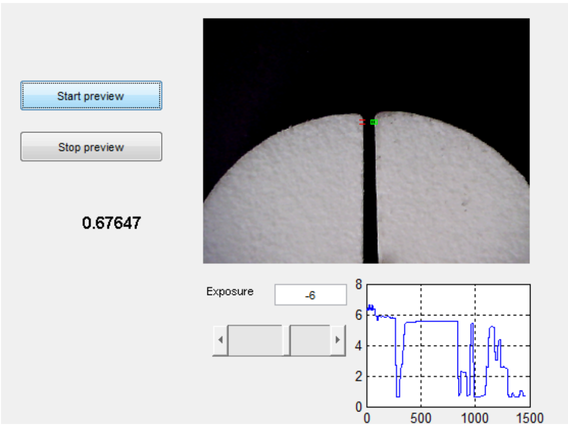

Figure 8. MATLAB GUI application for measure distance of gripper fingers based on image processing

In our case the webcam scans area around 20mm to 14mm. This means that one pixel represents the distance approximately 29µm. The reason for scanning so big area is, that one can open the fingers of gripper to distance up to 18mm (as mentioned, the flexibility of material enables to produce relative big deformations). The whole image processing algorithm is included into MATLAB preview function, which enables to work with frequency 25 frames per second. The algorithm calculates the distance between fingers on number of pixels. To get relatively precise results the image is transformed to gray color scale. Next noise is removed and then the image is transformed to black and white color scale. In this step the boundary of fingers are detected and are find vertical edges of fingers. The distance is measured between two points which are located 0.3mm lower as is detected maximal point of finger. This approach gets us the distance between fingers with minimized error caused by roundedness of fingers. On the Figure 8 is shown MATLAB GUI application of position feedback based on image processing. Between drowned points is measured distance. In right corner is graph with historical data of position where number of samples and position is shown. The number under buttons represents actual distance of gripper fingers.

4. Comparison of Results From Simulations and Experiments

Dependence between input displacement and distance of fingers of compliant gripper is analyzed. As was mentioned, models in MATLAB/SimMechanics and Comsol Multiphysics were prepared and parameters from physical sample (material properties, dimensions, etc.) were used as inputs of models.

Table 1. Comparison of results from simulations and experiments

Input displacement / [m]	Distance of fingers / [mm]		
	MATLAB SimMechanics	Comsol Multiphysics	Experiment
-0.000570	0.075000	-0.023242	0.441180
-0.000510	0.487700	0.062176	0.470590
-0.000420	1.107000	0.757087	1.088000
-0.000300	1.993000	1.683635	2.352900
-0.000180	2.760000	2.610182	3.117600
-0.000090	3.938000	3.305093	3.792500
0.000000	4.000000	4.000004	4.000000
0.000150	5.340000	5.158188	5.852900
0.000300	6.690000	6.316373	6.764600
0.000450	7.105000	7.474558	7.940000
0.000600	8.141000	8.632742	8.823500
0.000750	9.177000	9.790927	10.441200
0.000900	10.210000	10.949112	11.558800
0.001050	11.250000	12.107296	12.500000
0.001200	12.290000	13.265481	13.705900
0.001500	14.360000	15.581850	15.823500
0.001590	14.990000	16.276761	17.050000

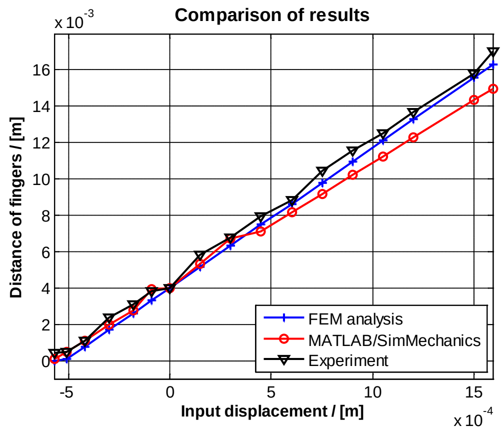

Figure 9. Comparison of results form simulations and experiments

At began of experiment; the positional feedback was calibrated, by measurement of object by known dimensions. The result of calibration is specifying of dependence between real distance and number of pixels. In our case the distance of 1mm is represented by 34pixels. Measurement of chosen dependence was based on data from linear encoder and for each step (approximately

0.09mm) was read distance of gripper fingers. This same approach was used on both models, where value of input displacement was defined. The results from models and experiments are shown on the Table 1 and Figure 9.

5. Conclusion

Some specifications connected by design of compact compliant devices are described. The complex design study requires include mathematical modeling and simulations, but in case of flexures such approach could not be sufficient. Therefore the best way is recommended to build the hybrid model and to compare results from both models.

Results of three different approaches (modeling, experiments) are gain and compared. The results from models prepared in MATLAB/SimMechanics, Comsol Multiphysics and from measurement on real device are shown on the Table 1 and Figure 9. Such results confirm our statement that in design process of compliant structures is the best way builds hybrid models. In other side presented differences between results form MATLAB and FEM analysis should be minimized by careful mathematical description and including out-of-plane deformations. Currently our models in MATLAB calculate only in-plane compliances connected by flexure hinge.

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