

Analysis of Reliability of Modal Parameters Estimation Using High-speed Digital Image Correlation Method

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Abstract This paper deals with description of modification and using high-speed digital image correlation method in experimental modal analysis. In its theoretical part the basic principle of digital image correlation and description of modification of correlation system Q-450 Dantec Dynamics for the purposes of SIMO (Single Input / Multiple Output) assumption of modal parameters is introduced. The practical part – analysis of reliability of program Modan3D, developed for modal parameters estimation – was realized on a simple steel specimen in laboratory conditions. The obtained outputs are present in a form of graphs and tables.

Keywords: digital image correlation, experimental modal analysis, reliability, Modan3D

Cite This Article: Martin Hagara, Róbert Huňady, Pavol Lengvarský, and Peter Pavelka, “Analysis of Reliability of Modal Parameters Estimation Using High-speed Digital Image Correlation Method.” *American Journal of Mechanical Engineering*, vol. 3, no. 6 (2015): 190-194. doi: 10.12691/ajme-3-6-7.

1. Introduction

Currently the work of engineers and constructors is focused on designing of reliable structures with comfortable noiseless operation. For a reason that newer materials are increasingly lighter it is necessary to ensure their safety. These requirements suggest that it is even more necessary to pay attention to the dynamic characteristic of the structures.

Dynamic characterization of structures is of significant importance in a wide variety of industries including aerospace, traffic [1], petroleum industry [2], civil structures, appliances, however also in sport [3]. In general, for the investigation of the dynamic behavior of structures [4] a method serving for estimation of modal parameters can be used. Such method, by which natural frequencies, mode shapes and modal damping are estimated, is called modal analysis [5]. There are two forms of modal analysis – experimental modal analysis (EMA) and operational modal analysis (OMA). While the first one uses for excitation of the structures an additional hardware as impact hammer or shakers, the second one investigates the structures in situ, i.e. during operation.

The responses of the structures are commonly captured using traditional mechanical transducers of acceleration. Several past years ago the non-contact optical devices, based e.g. on laser-Doppler principle, have been used for measuring of structure surface velocity. To the group of optical techniques belongs also a method of digital image correlation (DIC).

DIC is a modern non-contact method serving for 2D or 3D reconstruction of investigated object displacement and deformation fields. The possibility of 3D measurement is dependent on the use of minimally two cameras with CCD

(Charge Coupled Devices) or CMOS (Complementary metal–oxide–semiconductor) sensor for sampling of the object during its loading.



Figure 1. Low-speed CCD cameras of correlation system Q-400 Dantec Dynamics



Figure 2. High-speed CMOS cameras of correlation system Q-450 Dantec Dynamics

According to maximal sampling frequency of camera sensor digital image correlation systems can be divided

into two groups – low-speed and high-speed systems. While the low-speed cameras (Figure 1) are able to capture maximally several frames per second, the high-speed ones (Figure 2) can take several thousand frames per second.

During correlation digital images are not correlated (compared) as whole, but among small picture elements called facets (Figure 3). Commonly delivered correlation systems allow determine displacement fields in directions x , y and z as well as strain fields ϵ_x , ϵ_y and γ_{xy} for each nodal point of virtual grid created on investigated object surface. The spacing between grid nodal points should be equal to 3/4 of facet size.

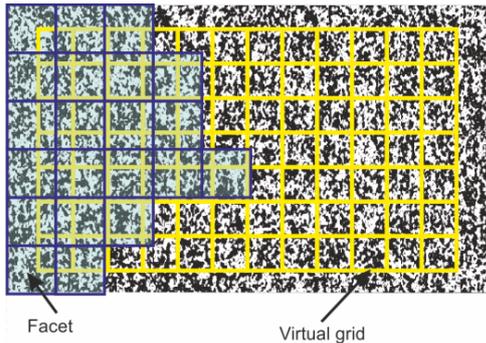


Figure 3. Investigated object surface divided into facets and virtual grid

There are many possibilities, how to utilize digital image correlation technique. The low-speed correlation systems are commonly used in stress/strain analysis [6], or by prediction of locations of strain concentrators [7]. The high-speed systems serve as tools for motion analysis [8], vibration analysis [9], crash tests as well as drop tests and can be used also in failure analysis.

For a reason that high-speed digital image correlation systems allow measurements at relatively high sampling frequencies and the results are obtained in every nodal point, it is very convenient to use correlation systems for the purpose of experimental modal analysis.

2. Modan3D

Modan3D is a tool, created in Matlab, allowing import and processing of data obtained by Istra4D, what is a software delivered with digital image correlation systems Dantec Dynamics. Istra4D allows export data in a form of AVI, STL as well as HDF5 AVI format. While exporting in AVI format is used for creating of videos, data exported in STL format serves as input for CAX software, HDF5 data format (Figure 4) with hierarchical structure allows relatively easy processing.

As the frequency response function (FRF), which describes the dynamic behavior of structure in frequency spectrum, is given by relation between output and input signal, it was necessary to modify high-speed correlation system Q-450 Dantec Dynamics for acquisition of force signal from impact hammer. For that reason two additional devices were added to measuring string – CCLD (Constant Current Line Drive) amplifier and AC/DC converter NI USB-4431 with four input and one output channel. The block scheme of connection between high-speed correlation system and additional devices can be seen in Figure 5.

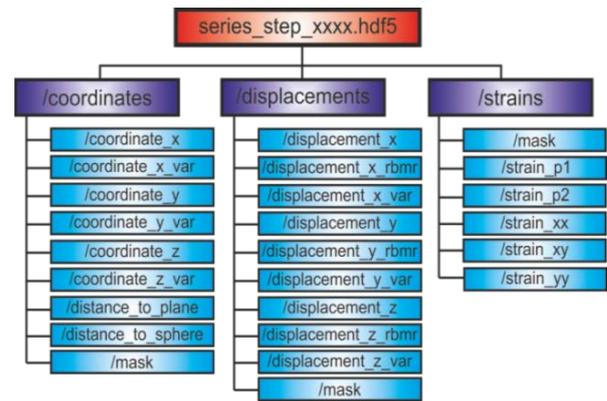


Figure 4. Hierarchical structure of HDF5 file exported from Istra4D

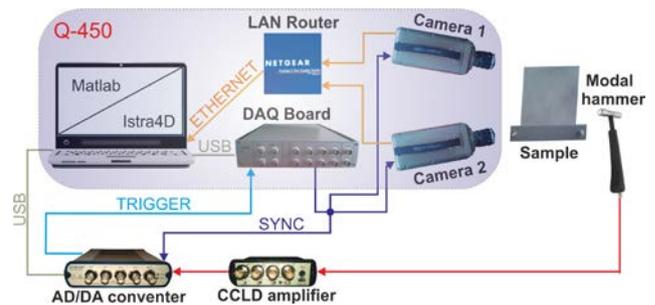


Figure 5. Block scheme of modified correlation system Q-450 Dantec Dynamics for the purposes of experimental modal analysis

Program Modan3D allows read the data exported from Istra4D in a form of displacements in particular directions x , y and z as well as the time dependence of force signal obtained from impact hammer. Mentioned data are then transformed into frequency spectrum using Fast Fourier Transform (FFT).

Frequency response matrices in each point of object surface are obtained using following formula:

$$\begin{aligned} H_x &= \frac{\text{Dis}X_{\text{fft}}}{F_{\text{fft}}}, \\ H_y &= \frac{\text{Dis}Y_{\text{fft}}}{F_{\text{fft}}}, \\ H_z &= \frac{\text{Dis}Z_{\text{fft}}}{F_{\text{fft}}}. \end{aligned} \quad (1)$$

where $\text{Dis}X_{\text{fft}}$, $\text{Dis}Y_{\text{fft}}$ and $\text{Dis}Z_{\text{fft}}$ are matrices of object displacements in frequency spectrum and F_{fft} is a force input in frequency spectrum.

Currently Modan3D utilizes for assumption of natural frequencies two functions – Normal Mode Indicator Function (NMIF) and Complex Mode Indicator Function (CMIF) [10]. The values of NMIF for each spectral line of the frequency spectrum are obtained using:

$$\text{NMIF}(f) = \frac{\sum_p |H_{Rp}(f)|^2}{\sum_p |H_p(f)|^2}, \quad (2)$$

where summation is performed on p measured frequency response functions.

The second function, CMIF, is obtained using singular value decomposition from:

$$\mathbf{H}(\mathbf{f}) = \mathbf{U}(\mathbf{f}) \cdot \mathbf{\Sigma}(\mathbf{f}) \cdot \mathbf{V}(\mathbf{f}), \quad (3)$$

where \mathbf{U} symbolizes the left singular matrix corresponding to modal vectors matrix, \mathbf{V} denotes the right singular matrix corresponding to modal vectors participation matrix and $\mathbf{\Sigma}$ represents a diagonal singular values matrix. Particular singular values are then expressed as:

$$\text{CMIF}_k(f) = \Sigma_k(f). \quad (4)$$

Ideal courses of NMIF and CMIF functions can be seen in Figure 6.

Modan3D can calculate damping ratios, whereby it utilizes the half-power method [11]. More information about program Modan3D, created at authors department, can be found in publication [12].

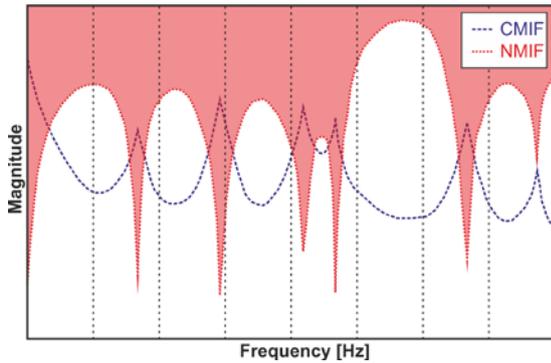


Figure 6. Ideal courses of CMIF and NMIF functions

3. Experimental Modal Analysis of Rectangular Plate Using Modan3D

The estimation of modal parameters in a form of natural frequencies, modal shapes and damping ratios was performed on a steel sheet of thickness 0.8 mm, which shape and dimensions are depicted in Figure 7. The boundary conditions for the specimen were fixation along its narrower edge with free other three edges.

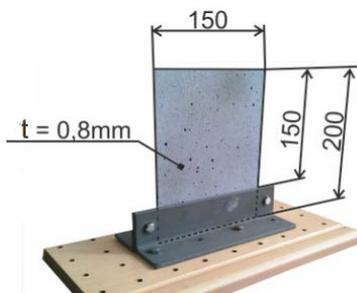


Figure 7. Dimensions and boundary conditions of the specimen

For the excitation an impact hammer Brüel & Kjær 8206. The impact hammer is supplied with three interchangeable impact tips of aluminum, plastic and rubber. The choice of impact tip determines the impulse shape (amplitude and duration) and the bandwidth of the excitation [13]. The sampling frequency of high-speed cameras was set to 2000 fps, what means that a frequency spectrum from 0 Hz to 1000 Hz was investigated. According to courses depicted in Figure 8 and Figure 9, characterizing the impulse shape and force spectrum of the

impact, respectively, the impact hammer with plastic tip for the excitation of the specimen was chosen.

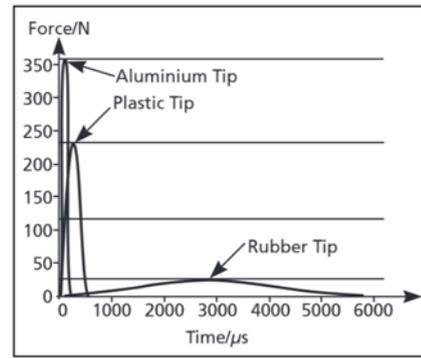


Figure 8. Time responses of impulse shapes for three available hammer tips [13]

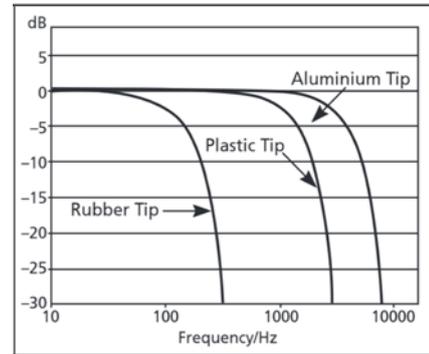


Figure 9. Force spectrum of an impact on an aluminum plate [13]

For the analysis of reliability of Modan3D it was necessary to perform the same measurement many times at various conditions such as alternating magnitude of force impacts, locations of impacts etc. The best solution should be that several different people perform the same measurement. As there was a problem to ensure sufficient amount of qualified people, we decided to repeat the measurement ten times (Figure 10).



Figure 10. Excitation of the specimen using impact hammer

The amounts of impact forces measured during experiment varied from minimal value of 12.4 N up to maximal value of 19.8 N are mentioned in Table 1.

The results of each measurement obtained in a form of CMIF and NMIF functions were compared together. As can be seen from Figure 11 and Figure 12, the courses of mentioned courses correspond well together.

Table 1. The amounts of impact forces used for excitation of the specimen

Measurement no.	Force value
1.	12.6 N
2.	13.1 N
3.	12.4 N
4.	12.8 N
5.	13.3 N
6.	13.4 N
7.	16.9 N
8.	18.2 N
9.	17.5 N
10.	19.8 N

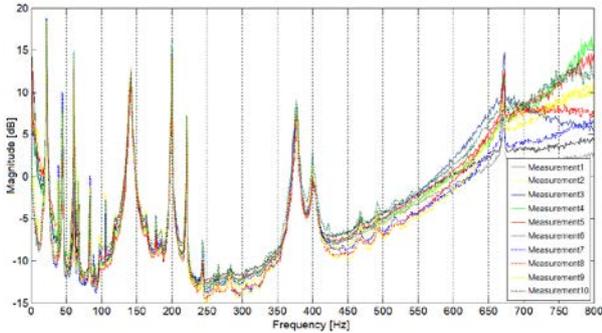


Figure 11. CMIF functions obtained from ten measurements by Modan3D

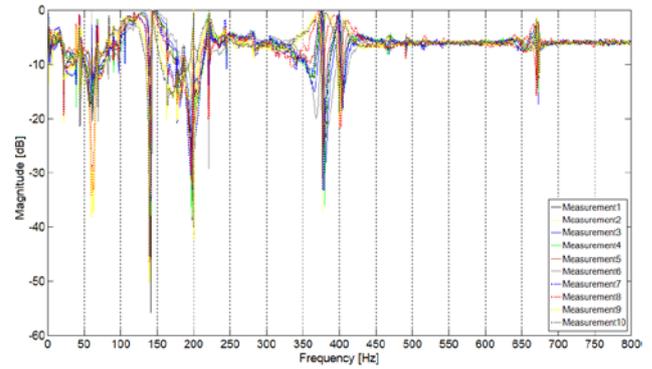


Figure 12. NMF functions obtained from ten measurements by Modan3D

The reliability of the modal parameters estimation using program Modan3D was tested by observation of standard deviations, dissipations and average relative deviations of natural frequencies as well as damping ratios amounts of particular modes obtained from mentioned measurements. The obtained data were processed and are present in [Table 2](#) and [Table 3](#).

Table 2. Specimen natural frequencies obtained by Modan3D

	1st mode	2nd mode	3rd mode	4th mode	5th mode	6th mode	7th mode	8th mode
M1	22.1583 Hz	61.0976 Hz	142.175 Hz	200.200 Hz	221.384 Hz	378.751 Hz	400.601 Hz	672.606 Hz
M2	22.1513 Hz	61.0611 Hz	142.158 Hz	200.183 Hz	221.358 Hz	378.654 Hz	400.541 Hz	672.479 Hz
M3	22.1752 Hz	61.0809 Hz	142.181 Hz	200.197 Hz	221.408 Hz	378.699 Hz	400.480 Hz	672.488 Hz
M4	22.1675 Hz	61.0611 Hz	142.251 Hz	200.175 Hz	221.325 Hz	378.859 Hz	400.448 Hz	672.547 Hz
M5	22.1888 Hz	61.0809 Hz	142.322 Hz	200.200 Hz	221.538 Hz	378.926 Hz	400.608 Hz	672.746 Hz
M6	22.1888 Hz	61.0539 Hz	142.363 Hz	200.200 Hz	221.495 Hz	378.822 Hz	400.514 Hz	672.600 Hz
M7	22.1675 Hz	61.1216 Hz	141.900 Hz	200.183 Hz	221.532 Hz	378.212 Hz	400.297 Hz	672.169 Hz
M8	22.1675 Hz	61.1289 Hz	141.905 Hz	200.175 Hz	221.580 Hz	378.455 Hz	400.404 Hz	672.542 Hz
M9	22.1544 Hz	61.0962 Hz	141.779 Hz	200.160 Hz	221.517 Hz	378.185 Hz	400.291 Hz	672.785 Hz
M10	22.1808 Hz	61.1029 Hz	141.917 Hz	200.160 Hz	221.577 Hz	378.484 Hz	400.285 Hz	672.029 Hz

AA	22.1700 Hz	61.0885 Hz	142.095 Hz	200.183 Hz	221.471 Hz	378.605 Hz	400.447 Hz	672.499 Hz
SD	0.01331	0.02561	0.20312	0.01583	0.09403	0.26132	0.12419	0.23550
D	0.00018	0.00066	0.04126	0.00025	0.00884	0.06829	0.01542	0.05546
ARD	0.0483%	0.0343%	0.1238%	0.0064%	0.0370%	0.0572%	0.0255%	0.0247%

Table 3. Specimen damping ratios obtained by Modan3D

	1st mode	2nd mode	3rd mode	4th mode	5th mode	6th mode	7th mode	8th mode
M1	0.022582	0.005685	0.011125	0.002386	0.002613	0.005843	0.0107	0.002483
M2	0.022396	0.004884	0.011118	0.002379	0.00257	0.006031	0.010175	0.002326
M3	0.02305	0.005473	0.011035	0.002444	0.002729	0.006087	0.009433	0.002079
M4	0.023415	0.004857	0.010909	0.002391	0.002533	0.005869	0.010255	0.002631
M5	0.024593	0.005452	0.010785	0.002398	0.002932	0.005708	0.01208	0.002389
M6	0.023635	0.004907	0.011157	0.002308	0.002816	0.005804	0.011058	0.002188
M7	0.021931	0.005376	0.012713	0.002597	0.002893	0.006248	0.010738	0.0029081
M8	0.022573	0.005397	0.012762	0.002563	0.00296	0.006478	0.009919	0.0025453
M9	0.021628	0.0048916	0.012937	0.002636	0.002824	0.006809	0.00955	0.002209
M10	0.022875	0.0049489	0.012505	0.002569	0.002897	0.006507	0.008278	0.0028672

AA	0.0228678	0.00518715	0.0117046	0.0024671	0.0027767	0.0061384	0.0102186	0.00246256
SD	0.000863	0.000317	0.000895	0.000113	0.000157	0.000363	0.001032	0.000281
D	0.00000074	0.00000010	0.00000080	0.00000001	0.00000002	0.00000013	0.00000107	0.00000008
ARD	2.8241%	5.5801%	7.0034%	4.0258%	4.7668%	4.8495%	7.3161%	9.1108%

The abbreviations M1-M10, mentioned in [Table 2.](#) and [Table 3.](#), denote particular measurements, AA – average amount of the obtained results, SD – standard deviations, D – dissipations and ARD – average relative deviations.

Mode shapes corresponding to mentioned natural frequencies can be found in Figure 13.

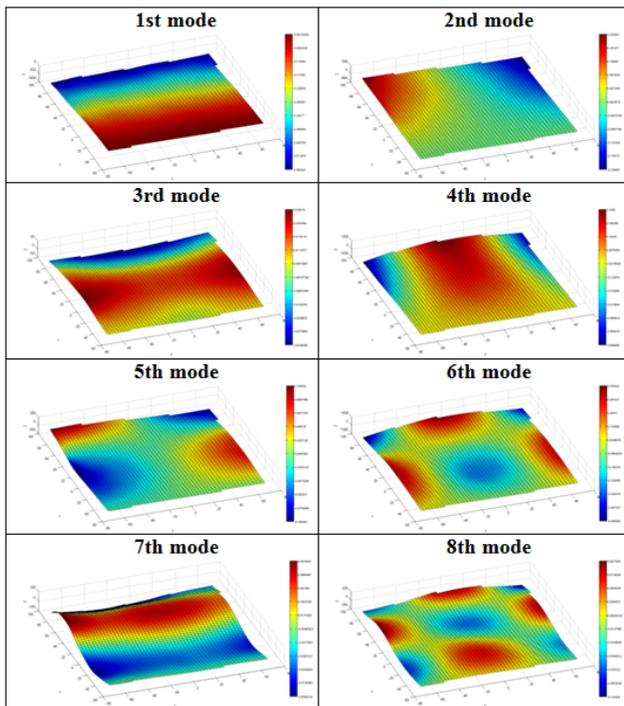


Figure 13. Mode shapes of the investigated specimen obtained by Modan3D

4. Conclusions

Digital image correlation is a modern technique allowing a wide range of applications. In this article its utilization for the purposes of SIMO experimental modal analysis is described. With regard to classical EMA methodology such performing of modal parameters estimation is very convenient. The main reasons are: it is not necessary to create model of investigated structure, characterize the degrees of freedom and excite the structure (or measure the response of the structure) in all degrees of freedom, what make the measurement process relatively easy and not time-consuming. The data processed from ten measurements, from which average relative deviations lower than 0.1% for natural frequencies estimation and 10% for very small amounts of damping ratios were calculated, approve that the modified high-speed correlation system Q-450 Dantec Dynamics with program Modan3D gives reliable results. The obtained

mode shapes of the specimen correspond to the mode shapes computed using finite element method.

Acknowledgement

This contribution is the result of the projects implementation VEGA 1/0937/12 “Development of nontraditional experimental methods for mechanical and mechatronical systems” and APVV-0091-11 “The use of experimental and numerical methods for the increase of competitiveness and innovation of mechanical and mechatronical systems”.

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