

# Flow of a Maxwell Fluid in a Porous Orthogonal Rheometer under the Effect of a Magnetic Field

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**Abstract** The steady flow of a Maxwell fluid in a porous orthogonal rheometer with the application of a magnetic field is studied. It is shown that there exists an exact solution for the problem. The effects of the parameters controlling the flow are analyzed by plotting graphs. It is observed that the effects of the Deborah number, the suction/injection velocity parameter, and the Reynolds number in the absence of a magnetic field are similar to those in the presence of a magnetic field. It is displayed that the Hartmann number that is based on the applied magnetic field reveals the tendency to slow down the flow.

**Keywords:** *magnetohydrodynamics, Maxwell fluid, orthogonal rheometer, porous disk, exact solution*

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## 1. Introduction

Magnetohydrodynamics (MHD) is the study of electrically conducting fluids in the presence of magnetic and electric fields and examines the phenomena associated with electro-fluid-mechanical energy conversion. Its applications in engineering are MHD generators, pumps, bearings, flow meters, plasma jets, fusion machines for power and space power generators. The MHD flow of non-Newtonian fluids has been a subject of great interest due to its widely spread applications in the design of cooling systems with liquid metals, purification of crude oil and polymer technology.

In nature, there are many fluids whose behavior cannot be described by the classical Navier-Stokes theory. The inadequacy of the theory of Newtonian fluids in predicting the behavior of some fluids, especially those with high molecular weight, has led to the development of non-Newtonian fluid mechanics. The constitutive equations of non-Newtonian fluids such as polymer solutions, greases, melts, muds, emulsions, paints, jams, soaps, shampoos and certain oils are of higher order and much more complicated than the Navier-Stokes equations. The mechanical behavior of non-Newtonian fluids cannot be described by a single constitutive equation. Therefore, many constitutive equations have been suggested in the literature. One of the most popular models is the Maxwell model. The importance of the Maxwell model lies in the fact that it can show the relaxation effects. The properties of polymeric fluids can be explored by the Maxwell model for small relaxation time. However, in some more concentrated polymeric fluids, the Maxwell model is also useful for large relaxation time. Maxwell fluids include

glycerin, toluene, crude oil, flour dough and dilute polymeric solutions.

Maxwell and Chartoff [1] devised an instrument called the orthogonal rheometer consisting of two parallel disks rotating about non-coincident axes with the same angular velocity. They pointed out that it is possible to determine the complex dynamic viscosity of a viscoelastic fluid by using it. Abbott and Walters [2] were the first to obtain an exact solution of a Newtonian fluid in the orthogonal rheometer. They also carried out a perturbation analysis in the case of a viscoelastic fluid. Rajagopal [3] showed that the velocity field corresponding to the motion in an orthogonal rheometer is a motion with constant principal relative stretch history. He further demonstrated that the adherence boundary condition is sufficient for obtaining a determinate problem since the flow of any homogeneous incompressible simple fluid which undergoes a flow characterized by this velocity field results in a second-order partial differential equation. The reader may also consult [4,5,6] for the studies that deal with the flow in an orthogonal rheometer.

The flow in the orthogonal rheometer in the presence of a magnetic field has been studied by a number of researchers. Mohanty [7] obtained an exact solution of the MHD flow in the case of a Newtonian fluid. Rao and Rao [8] investigated the flow in this geometry for a second grade fluid under the effect of a magnetic field. Kasiviswanathan and Rao [9] investigated the unsteady MHD flow of a Newtonian fluid in the same geometry when the disks are subjected to non-torsional oscillations. Kasiviswanathan and Gandhi [10] studied the flow of a micropolar fluid in the presence of a magnetic field. Ersoy [11] and Siddiqui et al. [12] investigated the MHD flow for an Oldroyd-B fluid and a Burger's fluid, respectively. Guria et al. [13] examined both the hydromagnetic flow

and the heat transfer phenomenon when the disks are porous. Recently, Das et al. [14] studied unsteady hydromagnetic flow due to concentric rotation of eccentric disks.

The MHD flow due to the pull of disks in an orthogonal rheometer has also been one of the subjects which has attracted attention. After Ersoy's pioneering work [15], Asghar et al. [16] and Siddiqui et al. [17] investigated the influence of Hall current and heat transfer on the steady flow of an Oldroyd-B fluid and a Burgers' fluid, respectively. Hayat et al. [18] and Hayat et al. [19] studied the Hall current and heat transfer effects on the steady flow in the case of a generalized Burgers' fluid and of a Sisko fluid, respectively. Ersoy [20] obtained the velocity field and examined the distribution of force applied by the fluid in the same geometry for a second order/grade fluid.

We refer the readers to some papers by Hsiao [21,22,23,24] for flows of non-Newtonian fluids in the presence of a magnetic field.

The aim of this paper is to extend the study considered in [25] to the magnetohydrodynamic flow. The results obtained for the velocity field are presented graphically in terms of the Deborah number, the suction/injection velocity parameter, the Reynolds number, and the Hartmann number. It is shown that the presence of magnetic field results in the deceleration of flow.

## 2. Basic Equations and Solution

Let us consider a Maxwell fluid between two porous disks rotating with the same angular velocity  $\Omega$  about two parallel and distinct axes perpendicular to the disks. The top and bottom disks are located at  $z = h$  and  $z = -h$ , respectively. The distance between the axes of rotation placed in the plane  $x = 0$  is  $2\ell$  (Figure 1). A uniform magnetic induction of strength  $B_0$  is applied to the insulated disks in the  $z$ -direction. The induced magnetic field is neglected under the assumption of a small magnetic Reynolds number.

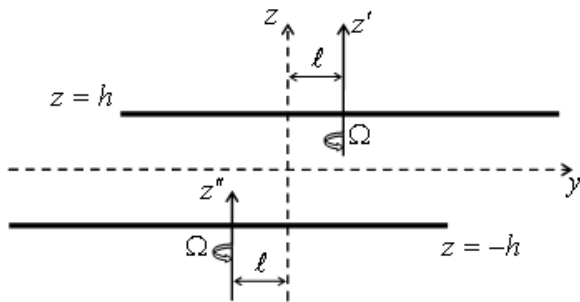


Figure 1. Flow geometry

The Cauchy stress  $\mathbf{T}$  for a Maxwell fluid is related to the fluid motion in the form

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S} \quad (1)$$

$$\mathbf{S} + \lambda(\dot{\mathbf{S}} - \mathbf{L}\mathbf{S} - \mathbf{S}\mathbf{L}^T) = \mu\mathbf{A}_1 \quad (2)$$

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \mathbf{L} = \nabla\mathbf{v} \quad (3)$$

where  $p$  denotes the pressure,  $\mathbf{I}$  the unit tensor,  $\mathbf{S}$  the extra stress tensor,  $\lambda$  the relaxation time,  $\mathbf{L}$  the velocity gradient,  $\mathbf{L}^T$  the transpose of  $\mathbf{L}$ ,  $\mu$  the dynamic viscosity,  $\mathbf{A}_1$  the first Rivlin-Ericksen tensor, and  $\mathbf{v}$  the velocity vector. The dot represents the material time derivative. When  $\lambda = 0$ , the model (2) reduces to the classical linearly viscous model.

The governing equations are

$$\rho\dot{\mathbf{v}} = \nabla \cdot \mathbf{T} + \mathbf{J} \times \mathbf{B} \quad (4)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (6)$$

$$\nabla \times \mathbf{B} = \mu_m \mathbf{J} \quad (7)$$

$$\nabla \times \mathbf{E} = \mathbf{0} \quad (8)$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (9)$$

where  $\rho$  is the density,  $\mathbf{J}$  the current density,  $\mathbf{B}$  the magnetic induction,  $\mu_m$  the magnetic permeability,  $\mathbf{E}$  the electric field, and  $\sigma$  is the electrical conductivity of the fluid.

It seems reasonable to assume the following form for the velocity field

$$u = -\Omega y + f(z), v = \Omega x + g(z), w = w_0 \quad (10)$$

where  $u$ ,  $v$ , and  $w$  are the  $x$ ,  $y$ , and  $z$ -components of the velocity vector, respectively, and  $w$  is a constant as a result of (5). The direction of  $w$  is upward, so  $w_0$  is taken to be positive.

The appropriate boundary conditions for the velocity field are

$$u = -\Omega(y - \ell), v = \Omega x, w = w_0 \text{ at } z = h \quad (11)$$

$$u = -\Omega(y + \ell), v = \Omega x, w = w_0 \text{ at } z = -h \quad (12)$$

$$u = -\Omega y, v = \Omega x, w = w_0 \text{ at } z = 0 \quad (13)$$

The boundary conditions for  $f(z)$  and  $g(z)$  from Eqs. (10)-(13) are

$$f(\pm h) = \pm\Omega\ell, g(\pm h) = 0 \quad (14)$$

$$f(0) = 0, g(0) = 0$$

We shall suppose that the extra stress tensor  $\mathbf{S}$  depends only on  $z$  (see [25]). Using Eqs. (2), (3) and (10), we obtain the following equations:

$$S_{xx} + \lambda(w_0 S'_{xx} + 2\Omega S_{xy} - 2f' S_{xz}) = 0 \quad (15)$$

$$S_{xy} + \lambda(w_0 S'_{xy} + \Omega(S_{yy} - S_{xx}) - f' S_{yz} - g' S_{xz}) = 0 \quad (16)$$

$$S_{xz} + \lambda(w_0 S'_{xz} + \Omega S_{yz} - f' S_{zz}) = \mu f' \quad (17)$$

$$S_{yy} + \lambda(w_0 S'_{yy} - 2\Omega S_{xy} - 2g' S_{yz}) = 0 \quad (18)$$

$$S_{yz} + \lambda(w_0 S'_{yz} - \Omega S_{xz} - g' S_{zz}) = \mu g' \quad (19)$$

$$S_{zz} + \lambda w_0 S'_{zz} = 0 \quad (20)$$

where a prime denotes differentiation with respect to  $z$ . The solution of (20) gives

$$S_{zz} = C \exp(-z/(\lambda w_0)) \quad (21)$$

where  $C$  is a constant. We shall investigate the possibility of a solution to the steady problem in which  $C = 0$  [25,26,27,28].

We obtain the following equation by using Eqs. (17) and (19)

$$\lambda w_0 G' + (1 - i\lambda\Omega)G = \mu F' \quad (22)$$

where  $i = \sqrt{-1}$ ,  $G(z) = S_{xz} + iS_{yz}$  and  $F(z) = f + ig$ .

Using Eqs. (4), (6) and (10), we get

$$\frac{\partial p}{\partial x} = \rho\Omega(\Omega x + g) - \rho w_0 f' + S'_{xz} + B_0 J_y \quad (23)$$

$$\frac{\partial p}{\partial y} = \rho\Omega(\Omega y - f) - \rho w_0 g' + S'_{yz} - B_0 J_x \quad (24)$$

$$\frac{\partial p}{\partial z} = 0 \quad (25)$$

Using Eq. (9), we have

$$J_x = \sigma(E_x + \nu B_0), J_y = \sigma(E_y - u B_0), J_z = \sigma E_z \quad (26)$$

Since the disks are insulated, we get  $J_z = 0$  and  $E_z = 0$ . From Eq. (8), we have  $\partial E_x / \partial z = 0$  and  $\partial E_y / \partial z = 0$ . We obtain the following equations by cross-differentiating Eqs. (23)-(25)

$$\rho(\Omega g - w_0 f') + S'_{xz} - \sigma B_0^2 f = \text{constant} \quad (27)$$

$$\rho(\Omega f + w_0 g') - S'_{yz} + \sigma B_0^2 g = \text{constant} \quad (28)$$

When we use Eqs. (27)-(28), it is obtained

$$G' - \rho w_0 F' - (\sigma B_0^2 + i\rho\Omega)F = \text{constant} \quad (29)$$

Combining Eqs. (22) and (29), we have

$$\begin{aligned} (\mu - \rho\lambda w_0^2)F'' - w_0(\rho + \lambda\sigma B_0^2)F' \\ - (1 - i\lambda\Omega)(\sigma B_0^2 + i\rho\Omega)F = E \end{aligned} \quad (30)$$

where  $E$  is a constant that can be determined from the boundary conditions. Let us use the following non-dimensional quantities:

$$\Gamma = \frac{F}{\Omega\ell}, \zeta = \frac{z}{h}, D = \lambda\Omega, \alpha = \frac{w_0}{\sqrt{\Omega\mu/\rho}}, \quad (31)$$

$$R = \frac{\rho\Omega h^2}{\mu}, M = \sqrt{\frac{\sigma}{\mu}} B_0 h$$

where  $D$  is the Deborah number,  $\alpha$  the suction/injection parameter,  $R$  the Reynolds number, and  $M$  is the Hartmann number. The solution of Eq. (30) with the conditions  $\Gamma(\pm 1) = \pm 1$  and  $\Gamma(0) = 0$  is

$$\Gamma = \frac{Z_1 \exp(A\zeta) + Z_2 \exp(B\zeta) + Z_3}{Z_4} \quad (32)$$

where

$$\begin{aligned} Z_1 &= 1 - \cosh(B), Z_2 = \cosh(A) - 1, \\ Z_3 &= \cosh(B) - \cosh(A), \\ Z_4 &= \sinh(A) - \sinh(B) - \sinh(A - B), \\ A &= \frac{a + \sqrt{a^2 + bc + ibd}}{e}, \\ B &= \frac{a - \sqrt{a^2 + bc + ibd}}{e}, \\ a &= \alpha(R + DM^2), b = 4R(1 - D\alpha^2), \\ c &= M^2 + DR, d = R - DM^2, e = 2\sqrt{R}(1 - D\alpha^2) \end{aligned} \quad (33)$$

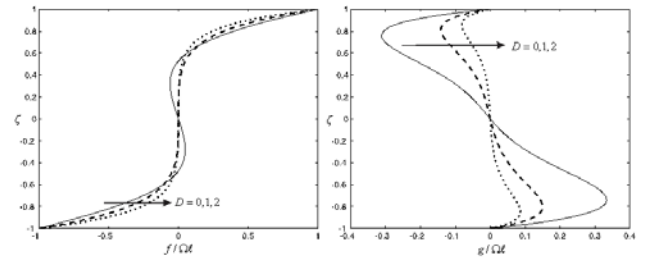


Figure 2. Variation of  $f/\Omega\ell$  and  $g/\Omega\ell$  with  $\zeta$  ( $R = 20$ ,  $\alpha = 0.1$ ,  $M = 0.5$ ,  $D = 0,1,2$ )

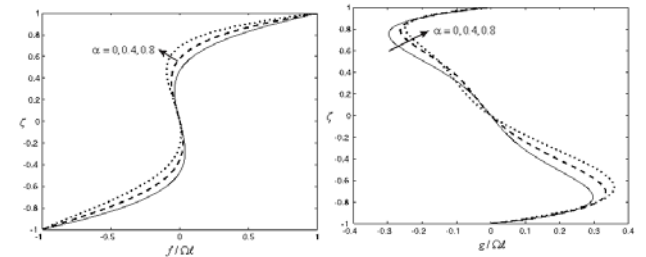


Figure 3. Variation of  $f/\Omega\ell$  and  $g/\Omega\ell$  with  $\zeta$  ( $R = 20$ ,  $D = 0.1$ ,  $M = 0.5$ ,  $\alpha = 0,0.4,0.8$ )

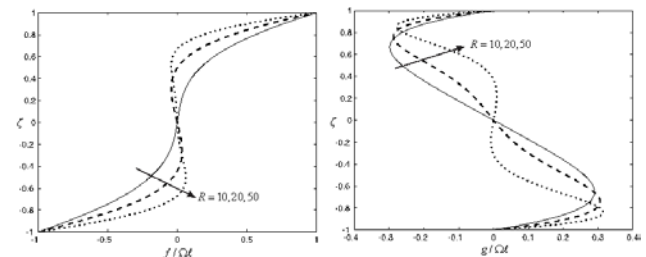


Figure 4. Variation of  $f/\Omega\ell$  and  $g/\Omega\ell$  with  $\zeta$  ( $D = 0.1$ ,  $\alpha = 0.1$ ,  $M = 0.5$ ,  $R = 10,20,50$ )

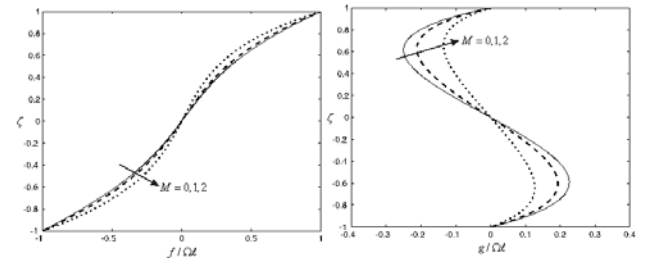


Figure 5. Variation of  $f/\Omega\ell$  and  $g/\Omega\ell$  with  $\zeta$  ( $D = 0.1$ ,  $\alpha = 0.2$ ,  $R = 5$ ,  $M = 0,1,2$ )

### 3. Results and Discussions

The problem that reflects the steady flow of a Maxwell fluid in a porous orthogonal rheometer has an exact solution of the velocity field even in the presence of a magnetic field. However, the components  $S_{xz}$  and  $S_{yz}$  of stress tensor, which are related to the  $x$  - and  $y$  - components of the force per unit area exerted by the fluid on the disks, cannot be obtained since the constitutive equations corresponding to the Maxwell fluid in the case of porous disks are of higher order than those corresponding to a classical viscous fluid including the same conditions.

The velocity field is entirely determined when the translational velocity components  $f(z)$  and  $g(z)$  are found. The effects of embedded parameters on the translational velocity components are examined through plots. It is obvious from Figure 2 that the curves in the main part of fluid belonging to the translational velocity components get closer to the  $z$  - axis when the Deborah number that is based on the relaxation time increases. An examination of Figure 3 shows that the curves start to approach the top disk but move away the bottom disk with the increase of the suction/injection velocity parameter that represents the upward axial velocity. It may be inferred from Figure 4 that the curves in the core region come near to the  $z$  - axis for large Reynolds numbers. The influence of the Hartmann number that is based on the applied magnetic field can be seen in Figure 5. An increase in the Hartmann number discloses that the curves tend to become flatter.

It should also be mentioned that the results obtained by Ersoy [25] can be obtained as the special case of the present analysis by taking the Hartmann number to be zero.

### 4. Conclusions

In this paper, the steady flow of a Maxwell fluid between two porous disks rotating about two distinct axes under the influence of a magnetic field is investigated. The conclusions which are drawn from this analysis can be listed as follows:

- The main part of fluid having the constant axial velocity tends to rotate about the  $z$  - axis for large values of the Deborah number.
- The thickness of the boundary layer adjacent to the top disk becomes thinner when the suction/injection velocity parameter increases, whereas an opposite behavior is observed for that near the bottom disk.
- The well-developed boundary layers exist near both the disks and are separated by a core region with increasing the Reynolds number. The main body of fluid both rotates about the  $z$ -axis and has a constant axial velocity.
- It is observed that the applied magnetic field reacts back on the flow as in the case of a Newtonian fluid. In other words, a Lorentz force which decelerates the flow is produced. As a result of this, the curves about which the fluid layers having the constant axial velocity  $w_0$  rotate are closer to the  $z$  - axis.

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