

A Multivariable Adaptive Control Design with Applications to Air-heat Tunnel Using Delta Models

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Abstract The article describes the design of adaptive controller for autonomous and non-autonomous control of nonlinear laboratory model hot-air tunnel using delta models. Synthesis of the controller is based on a matrix approach and polynomial theory. Autonomous control is solved using compensators. The controller was verified by simulation and the real-time experiment on nonlinear laboratory model hot-air tunnel. The recursive least squares method in delta domain is used in identification part of the proposed controller.

Keywords: adaptive control, delta model, real time control, multivariable control

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1. Introduction

Most of the control circuits are implemented as one-dimensional circuits. For a large number of objects it is necessary to control several variables relating to one system simultaneously. There are a number of possible approaches to design multivariable control systems. These approaches are based on different mathematical apparatus and hence the different forms of mathematical description of dynamic systems. This problem can be solved using the method of synthesis based on matrix approach and polynomial theory. This method is based on the description of multivariable systems using matrix fractions. Synthesis is easily algorithmizable for a digital computer. All the linear control tasks can be converted to an equation of the same type, only the coefficients of the equation depends on the task condition.

To avoid loop interactions, multivariable systems can be decoupled into separate loops known as single input, single output (SISO) systems [7,12]. Decoupling may be done using several different techniques. In our case the decoupling is realized by means of compensator placed ahead of the system.

2. Delta Models

The Z-transfer function is used to describe discrete-time dynamic system. When the sampling period decreases the z-transfer functions have some disadvantages [1]. The disadvantage of the discrete models can be avoided by introducing a more suitable discrete model [5,6,10]. It is possible to introduce new discrete operator [10]. This operator has following properties:

- leads to a model that provides a simple linear constraints on models with the shift-operator

- converges to the continuous derivatives with sampling period goes to zero
 - converges so that the inverse operator is causal
- Define operator and associated complex variable to fulfilled following condition

$$\lim_{T_0 \rightarrow 0} \gamma = s \quad (1)$$

Where T_0 denote sampling period, γ stands for complex variable of delta transformation and s is complex variable of Laplace transformation. Delta model is generally defined as

$$\gamma = \frac{z-1}{\alpha T_0 z + (1-\alpha)T_0} \quad 0 \leq \alpha \leq 1 \quad (2)$$

By substituting α in equation (2) we obtained an infinite number of new δ -models. In practice, the best known and most widely used are

Forward δ -model ($\alpha = 0$)

$$\gamma = \frac{z-1}{T_0} \quad (3)$$

Backward δ -model ($\alpha = 1$)

$$\gamma = \frac{z-1}{zT_0} \quad (4)$$

Tustin δ -model ($\alpha = 0.5$)

$$\gamma = \frac{2}{T_0} \frac{z-1}{z+1} \quad (5)$$

In following parts only the forward δ -model is taken into consideration. The δ -models will be used in process modeling for adaptive control based on the self-tuning

controller (STC). The STC consists of two integral parts – recursive identification and controller synthesis. The controller synthesis is based on parameters of controlled system obtained from recursive identification. For this reason it is necessary to apply suitable recursive identification algorithm to this model. The parameters of the δ - model are estimated using recursive least squares method (RLSM) with directional forgetting [1,3,9].

Regression model (ARX) is useful to apply this method of identification. This model can be express in its compact form

$$y(k) = \theta^T(k)\phi(k-1) + n(k) \tag{6}$$

where $\theta^T(k)$ is the vector of parameter and $\phi^T(k-1)$ is data vector ($y(k)$ is the process output variable, $u(k)$ is the controller output variable and $n(k)$ is the non-measurable random component).

The description of the model and relations for feedback control of the model with two inputs and two outputs are derived in the following sections. The polynomials of the second degree are supposed in the description using the matrix fraction.

3. Description of Two-input Two-output System

The system with two inputs and two outputs is the simplest and also the most common multivariable circuit. The internal structure of the system is described by single input single output transfer functions. These transfer functions uniquely identifies relationships between variables.

The internal structure of the system is depicted in Figure 1.

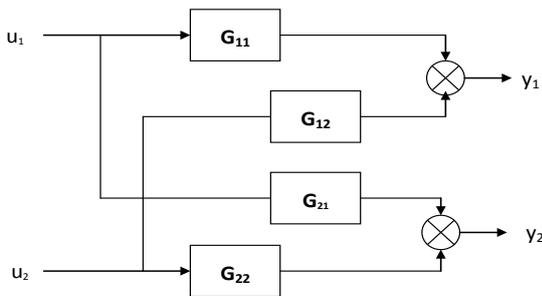


Figure 1. A two input –two output system – the "P" structure

The transfer matrix of the system is

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{7}$$

The matrix \mathbf{G} is a rational matrix (its elements are rational functions). Every rational matrix can be expressed using a polynomial matrix in the form of the left or right matrix fraction. It is possible to assume that the system is described by the matrix fraction

$$\mathbf{G}(\gamma) = \mathbf{A}^{-1}(\gamma)\mathbf{B}(\gamma) = \mathbf{B}_1^{-1}(\gamma)\mathbf{A}_1(\gamma) \tag{8}$$

Polynomial matrices \mathbf{A} , \mathbf{B} are left indivisible decomposition of matrix $\mathbf{G}(\gamma)$, polynomial matrices \mathbf{A}_1 , \mathbf{B}_1 are right indivisible decomposition of matrix $\mathbf{G}(\gamma)$.

The matrices of discrete model take following form

$$\mathbf{A}(\gamma) = \begin{bmatrix} \gamma^2 + \alpha_1\gamma + \alpha_2 & \alpha_3\gamma + \alpha_4 \\ \alpha_5\gamma + \alpha_6 & \gamma^2 + \alpha_7\gamma + \alpha_8 \end{bmatrix} \tag{9}$$

$$\mathbf{B}(\gamma) = \begin{bmatrix} \beta_1\gamma + \beta_2 & \beta_3\gamma + \beta_4 \\ \beta_5\gamma + \beta_6 & \beta_7\gamma + \beta_8 \end{bmatrix} \tag{10}$$

and the differential equations of the model are

$$\begin{aligned} y_{1\delta}(k) &= -\alpha_1 y_{1\delta}(k-1) - \alpha_2 y_{1\delta}(k-2) \\ &\quad - \alpha_3 y_{2\delta}(k-1) - \alpha_4 y_{2\delta}(k-2) \\ &\quad + \beta_1 u_{1\delta}(k-1) + \beta_2 u_{1\delta}(k-2) \\ &\quad + \beta_3 u_{2\delta}(k-1) + \beta_4 u_{2\delta}(k-2) \\ y_{2\delta}(k) &= -\alpha_5 y_{1\delta}(k-1) - \alpha_6 y_{1\delta}(k-2) \\ &\quad - \alpha_7 y_{2\delta}(k-1) - \alpha_8 y_{2\delta}(k-2) \\ &\quad + \beta_5 u_{1\delta}(k-1) + \beta_6 u_{1\delta}(k-2) \\ &\quad + \beta_7 u_{2\delta}(k-1) + \beta_8 u_{2\delta}(k-2) \end{aligned} \tag{11}$$

3.1. Recursive Identification

In the case of the above described system with two inputs and two outputs it is necessary to identify a total of sixteen unknown parameters of ARX model described by equation (11). The unknown parameters of the δ - model are estimated using recursive least squares method (RLSM) with directional forgetting.

The parameter vector takes the form:

$$\theta_{\delta}^T(k) = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & \beta_5 & \beta_6 & \beta_7 & \beta_8 \end{bmatrix} \tag{12}$$

The data vector is

$$\begin{aligned} \phi_{\delta}^T(k) &= [-y_{1\delta}(k-1), -y_{1\delta}(k-2), \\ &\quad -y_{2\delta}(k-1), -y_{2\delta}(k-2), \\ &\quad u_{1\delta}(k-1), u_{1\delta}(k-2), \\ &\quad u_{2\delta}(k-1), u_{2\delta}(k-2)] \end{aligned} \tag{13}$$

where

$$\begin{aligned} y_{1\delta}(k-1) &= \frac{y_1(k-1) - y_1(k-2)}{T_0} \\ y_{2\delta}(k-1) &= \frac{y_2(k-1) - y_2(k-2)}{T_0} \\ y_{1\delta}(k-2) &= y_1(k-2), y_{2\delta}(k-2) = y_2(k-2) \\ u_{1\delta}(k-1) &= \frac{u_1(k-1) - u_1(k-2)}{T_0} \\ u_{2\delta}(k-1) &= \frac{u_2(k-1) - u_2(k-2)}{T_0} \\ u_{1\delta}(k-2) &= u_1(k-2), u_{2\delta}(k-2) = u_2(k-2) \end{aligned} \tag{14}$$

The detailed description of recursive identification algorithm for TITO system is designed in [8].

4. Designing of Feedback MIMO System

Transfer function of the controller takes the form matrix fraction.

The stability of the closed loop is given by solution of following diophantine equation

$$\mathbf{F}\mathbf{P}_1 + \mathbf{Q}_1 = \mathbf{M} \quad (26)$$

The structure of polynomial matrices of controller

$$\mathbf{P}_1(\gamma) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (27)$$

$$\mathbf{Q}_1(\gamma) = \begin{bmatrix} q_{1\delta} & 0 \\ 0 & q_{2\delta} \end{bmatrix} \quad (28)$$

and matrix \mathbf{M} was chosen to be

$$\mathbf{M}(\gamma) = \begin{bmatrix} \gamma + m_1 & 0 \\ 0 & \gamma + m_2 \end{bmatrix} \quad (29)$$

The controller parameters are given by solution of diophantine equation (29) using the uncertain coefficients method. The control law is described by matrix equation

$$\mathbf{F}\mathbf{U} = \mathbf{B}^{-1}\mathbf{A}\mathbf{Q}_1\mathbf{P}_1^{-1}\mathbf{E} \quad (30)$$

Compensator \mathbf{C}_2 is adjugated to matrix \mathbf{B} . When \mathbf{C}_2 was included in the design of the closed loop the model was simplified by considering matrix \mathbf{A} as diagonal. The multiplication of matrix \mathbf{B} and adjugated matrix \mathbf{B} results in diagonal matrix \mathbf{H} . The determinants of matrix \mathbf{B} represent the diagonal elements. When matrix \mathbf{A} is non-diagonal, its inverted form must be placed ahead of the system in order to obtain diagonal matrix \mathbf{H} , otherwise it may increase the order of the controller and sophistication of the closed loop system. Although designed for a diagonal matrix, compensator \mathbf{C}_2 also improves the control process for non – diagonal matrix \mathbf{A} in the controlled system. This is demonstrated in the simulation results.

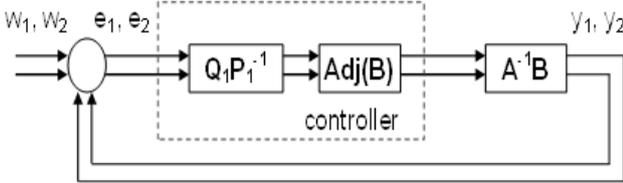


Figure 5. Closed loop systems with compensator \mathbf{C}_2

Equation for system output takes the form

$$\mathbf{Y} = \mathbf{P}_1(\mathbf{A}\mathbf{F}\mathbf{P}_1 + \mathbf{B}_v\mathbf{Q}_1)\mathbf{B}_v\mathbf{Q}_1\mathbf{P}_1^{-1}\mathbf{W} \quad (31)$$

The matrix \mathbf{B}_v is

$$\mathbf{B}_v = \mathbf{B}\mathbf{adj}(\mathbf{B}) = \begin{bmatrix} \det(\mathbf{B}) & 0 \\ 0 & \det(\mathbf{B}) \end{bmatrix} \quad (32)$$

The stability of the closed loop is given by solution of following diophantine equation

$$\mathbf{A}\mathbf{F}\mathbf{P}_1 + \mathbf{B}_v\mathbf{Q}_1 = \mathbf{M} \quad (33)$$

The structure of the matrix \mathbf{P}_1 and \mathbf{Q}_1 is chosen so that the number of algebraic equations after multiplication of matrix diophantine equation corresponds to the number of unknown parameters. The structure of controller polynomial matrices takes the form

$$\mathbf{P}_1(\gamma) = \begin{bmatrix} \gamma^2 + p_{1\delta}\gamma + p_{2\delta} & 0 \\ 0 & \gamma^2 + p_{3\delta}\gamma + p_{4\delta} \end{bmatrix} \quad (34)$$

$$\mathbf{Q}_1(\gamma) = \begin{bmatrix} q_{1\delta}\gamma^2 + q_{2\delta}\gamma + q_{3\delta} & 0 \\ 0 & q_{4\delta}\gamma^2 + q_{5\delta}\gamma + q_{6\delta} \end{bmatrix} \quad (35)$$

Polynomial matrix \mathbf{M} takes the form selected with regard to the structure of other matrices in diophantine equation

$$\mathbf{M}(\gamma) = \begin{bmatrix} \gamma^5 + m_1\gamma^4 + m_2\gamma^3 + & 0 \\ +m_3\gamma^2 + m_4\gamma + m_5 & \\ 0 & \gamma^5 + m_6\gamma^4 + m_7\gamma^3 + \\ & +m_8\gamma^2 + m_9\gamma + m_{10} \end{bmatrix} \quad (36)$$

Solving the diophantine equation defines a set of algebraic equations. These equations are subsequently used to obtain the unknown controller parameters.

The control law is given by the block diagram

$$\mathbf{F}\mathbf{U} = \mathbf{adj}(\mathbf{B})\mathbf{Q}_1\mathbf{P}_1^{-1}\mathbf{E} \quad (37)$$

6. Simulation Examples

A program and diagrams to simulate and verify the algorithms was created in the program system MATLAB - SIMULINK. Verification by simulation was carried out on a range of systems with varying dynamics. The control of the model below is given here as our example.

$$\mathbf{A} = \begin{bmatrix} s^2 + 2s + 0,7 & 0,2s + 0,4 \\ -0,5s - 0,1 & s^2 + 2s + 0,7 \end{bmatrix} \quad (38)$$

$$\mathbf{B} = \begin{bmatrix} 0,5s + 0,2 & 0,1s + 0,3 \\ 0,5s + 0,1 & 0,3s + 0,4 \end{bmatrix} \quad (39)$$

The right side control matrices are denoted as follows: without compensator – \mathbf{M}_1 , with compensator \mathbf{C}_1 – \mathbf{M}_2 , and with compensator \mathbf{C}_2 – \mathbf{M}_3 .

$$\mathbf{M}_1(\gamma) = \begin{bmatrix} \gamma^4 + 2\gamma^3 + 1,5\gamma^2 + & 0 \\ +0,5\gamma + 0,0625 & \\ 0 & \gamma^4 + 2\gamma^3 + 1,5\gamma^2 + \\ & +0,5\gamma + 0,0625 \end{bmatrix} \quad (40)$$

$$\mathbf{M}_2(\gamma) = \begin{bmatrix} \gamma + 0,5 & 0 \\ 0 & \gamma + 0,5 \end{bmatrix} \quad (41)$$

$$\mathbf{M}_3(\gamma) = \begin{bmatrix} \gamma^5 + 2,5\gamma^4 + & \\ +2,5\gamma^3 + 1,25\gamma^2 + & 0 \\ +3,125\gamma + 0,0313 & \\ 0 & \delta^5 + 2,5\gamma^4 + \\ & +2,5\gamma^3 + 1,25\gamma^2 + \\ & +3,125\gamma + 0,0313 \end{bmatrix} \quad (42)$$

The same initial conditions for system identification were used for all the types of adaptive control we tested. The initial parameter estimates were chosen to be

$$\boldsymbol{\theta}_\delta^T = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 \\ 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 \end{bmatrix} \quad (43)$$

The simulation results are shown in Figure 6-8.

It is possible to draw several conclusions from the simulation results of the experiments on linear static systems. The basic requirement to ensure permanent zero control error was satisfied in all cases. The criteria on which we judge the quality of the control process are the overshoot on the controlled values and the speed with which zero control error is achieved. According to these criteria the controller incorporating compensator C_1 performed the best. However, this controller appears to be unsuited to adaptive control due to the size of the overshoot and the large numbers of process and controller outputs. The controller which uses compensator C_2 seems to work best in adaptive control. The addition of compensators in series ahead of the system caused that change in one of control variables change only the corresponding process variable in all cases. Compensators actually provide autonomous control loop. With regards to decoupling, it is clear that controllers with compensators greatly reduce interaction.

7. Laboratory Experiment

The verification of the proposed algorithms for autonomous and non-autonomous adaptive multivariable control on the real object under laboratory conditions has been realized using experimental laboratory model – air-heat tunnel. It is a suitable tool for the laboratory experimental verification of control algorithms and controller parameter settings.

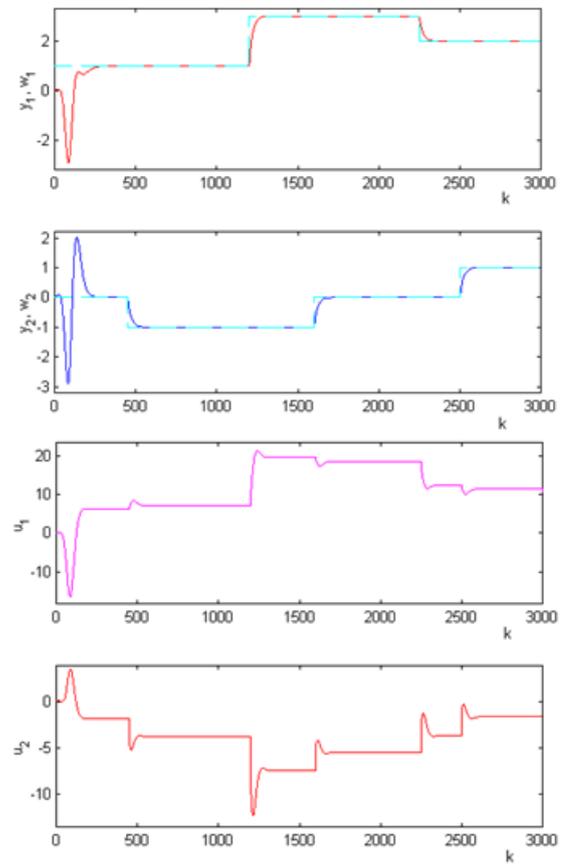


Figure 7. Simulation results: adaptive control without compensator C_1

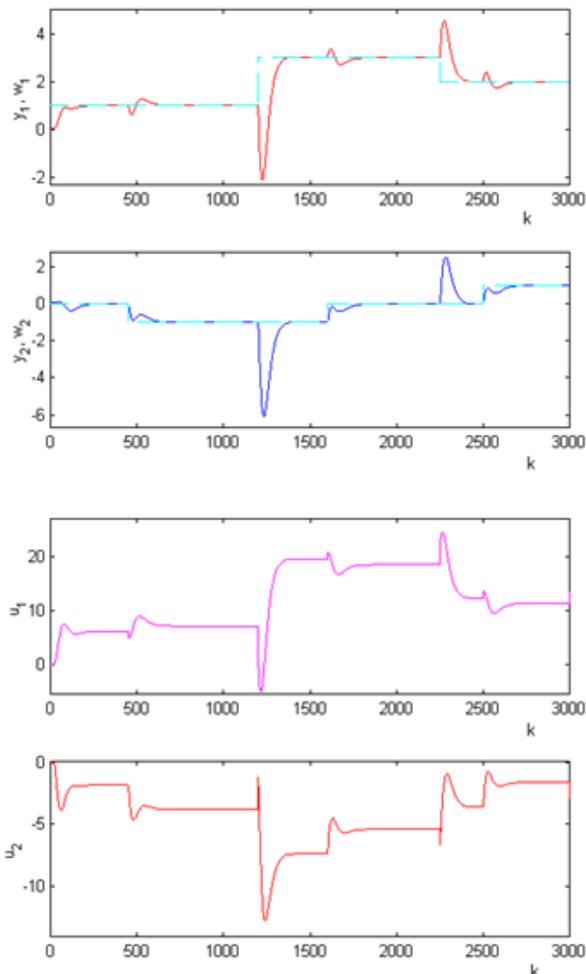


Figure 6. Simulation results: adaptive control without compensator

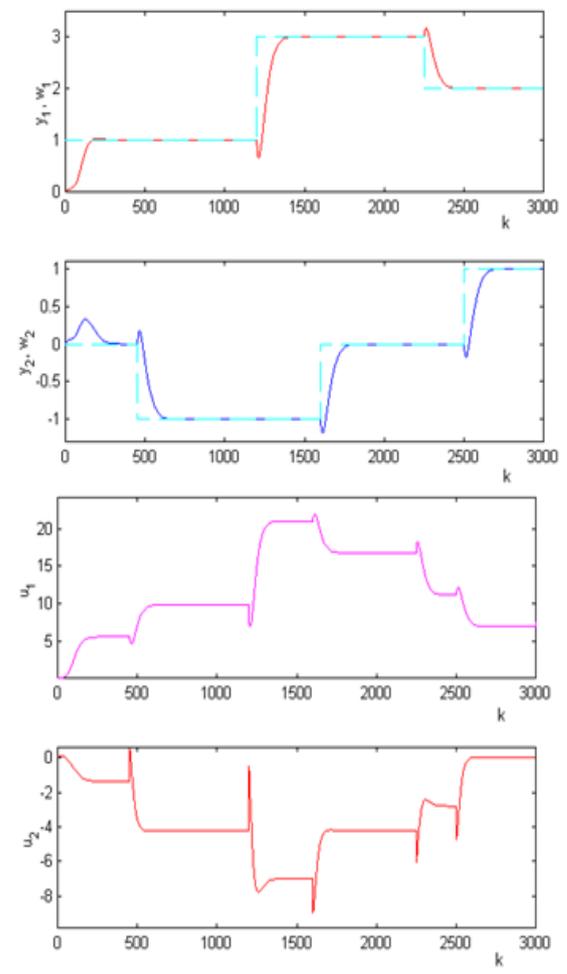


Figure 8. Simulation results: adaptive control without compensator C_2

The model is composed of the heating coils, primary and secondary ventilator and a thermal resistor covered by tunnel. The heating coils are powered by controllable source of voltage and serves as the source of heat energy while the purpose of ventilators is to ensure and measure the flow of air inside the tunnel. Connecting the real model - hot-air tunnel is made using a technology card Advantech PCL 812, which is connected to the motherboard.

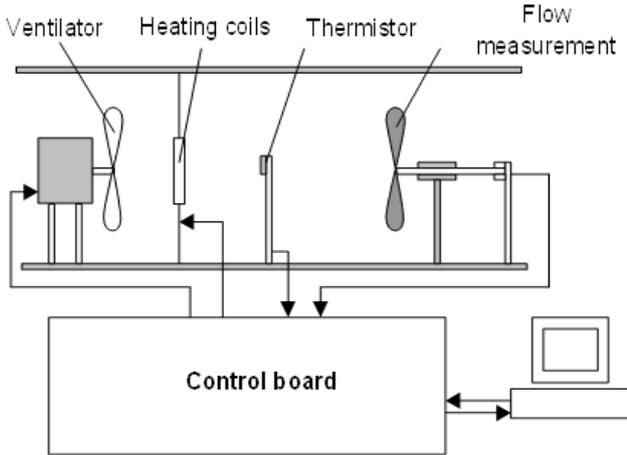


Figure 9. Scheme of hot-air tunnel

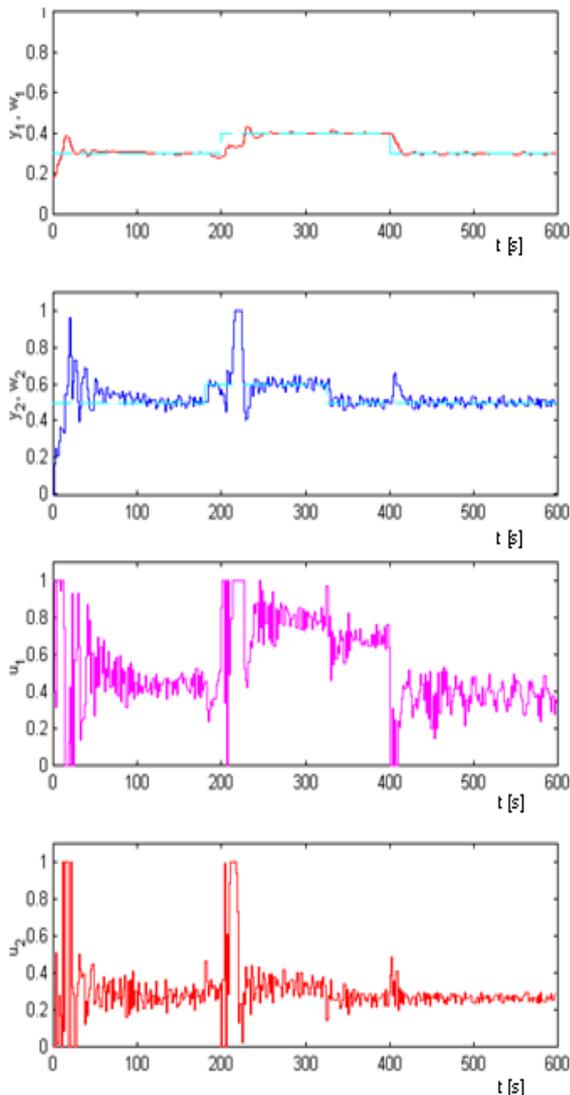


Figure 10. Adaptive control of a real model without compensator

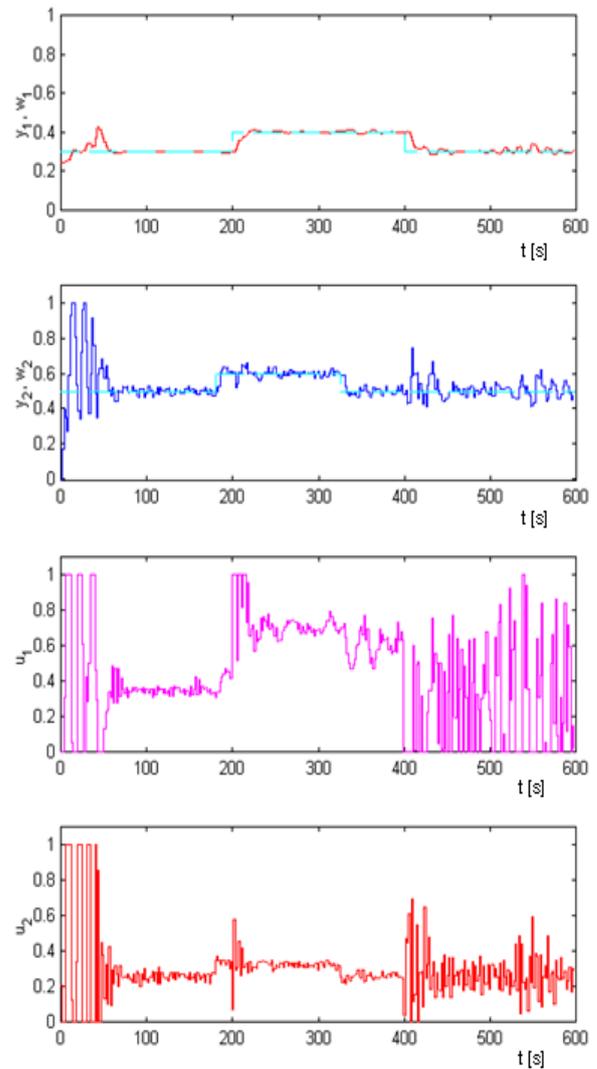


Figure 11. Adaptive control of a real model using compensator C_2

The controller output variables are the inputs to the ventilator and heating coils and the process output variables are temperature and airflow at the tunnel. There are interactions between the control loops.

The task was to apply the methods we designed for the adaptive control of a model representing a nonlinear system with variable parameters which is, therefore, hardly to control deterministically.

Adaptive control using recursive identification both with and without the use of compensators was performed.

As indicated in the simulation, compensator C_1 was shown to be unsuitable and control broke down. The other two methods provided satisfactory results. The time responses of the control for both cases are shown in Figure 10 and Figure 11. The figures demonstrate that control with a compensator reduces interaction. Process output variable y_1 is the temperature and process output variable y_2 is the airflow. The variables u_1 and u_2 are the controller outputs—inputs to the heating coils and ventilator.

8. Conclusion

The aim of this study was to use algebraic methods for synthesis of multivariable control systems for adaptive control using delta models. The used algorithms are based

on the pole placement method of the characteristic polynomial matrix. The adaptive control of a two-variable system based on polynomial theory and using delta models was designed. Decoupling problems were solved by the use of compensators. The designs were simulated and used to control a laboratory model. Experimental verification of proposed control algorithms were realized on laboratory model air-heat tunnel. The simulation results proved that these methods are suitable for the control of linear systems. The control tests on the laboratory model gave satisfactory results despite the fact that the nonlinear dynamics were described by a linear model. Due to the fact that the proposed controller is designed as an adaptive it can be used for control of non-linear and time-invariant systems.

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