

# Mathematical Modeling of Wheeled Mobile Robots

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**Abstract** This article is about designing and constructing a four-wheel chassis. Current changes to the terms of advanced mechatronic systems (currently in robotic systems) and current intensity of demands on them in practice require new insights into the processes and techniques of designing and verification of the project designs. Analyzed is a used concept of chassis motion control of mechatronic systems on the principle of differential wheel control for the task of active tracking of planned chassis path. Created was a simulation model of chassis mobile system in terms of kinematics, which will be usable in concept of chassis locomotion control on diverse terrain.

**Keywords:** mathematical model, wheeled mobile robot

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## 1. Introduction

Current changes to the terms of advanced mechatronic systems (currently in robotic systems) and current intensity of demands on them in practice require new insights into the processes and techniques of designing and verification of the project designs.

A complex understanding of technical subject and computer-aided simulation is one of the main characteristic features of the mechatronic approach to the design of modern products. There are integrated in the simulation model the mechanical, electrical and control subsystems and in this way can be obtained information about mutual interactions among these individual subsystems.

In the case of simulation model of a mobile robot equipped with the sectional undercarriage it is useful to apply methods of kinematic analysis and synthesis of bonded mechanical systems. A functional model of the wheeled robot, which is created afterwards, has a better manoeuvrability during movement across obstacles in a terrain, thanks to a hinge, which is installed in the middle of the undercarriage.

Application of methods and techniques for modeling and simulation for the design of robotic systems has its professional and technical requirements, but also professional and social topicality.

## 2. Control Concept

The designed chassis uses for its motion differential method of wheel control. Chassis motion should be stable and fluent to avoid slipping between wheels and terrain and to avoid mechanical shock resulting from rapid changes in chassis motion. However during chassis motion unavoidable trajectory deviations between current position and requested trajectory do occur because of path tracking control imperfection using wheels velocity and fault variables from environment (terrain roughness, friction forces changes between wheels and terrain and so forth) [1,2,3].

Trajectory deviations should be corrected online using requested linear and rotational chassis motion velocities by path tracking control [5,6].

In work [1] layout of kinematic control is designed, comprising of three levels of links (dynamic, kinematic and planning). This approach is similar to those, which are applied in robotic manipulators.

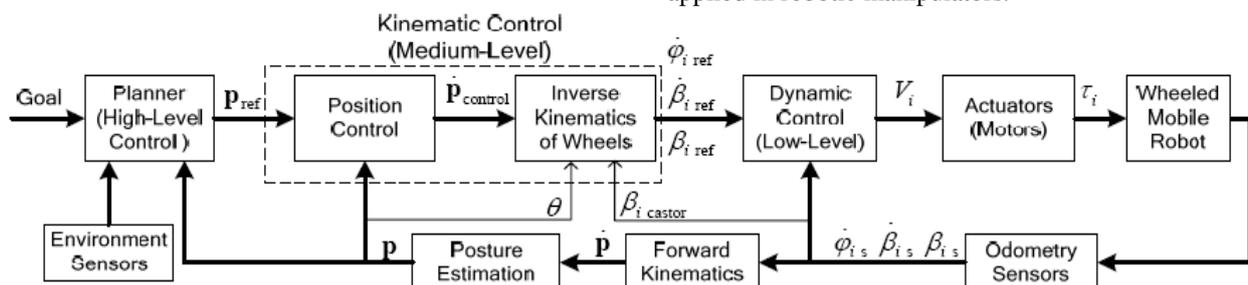


Figure 1. Control scheme of the wheeled mobile robot [1]

### 3. Kinematic Model

Creation of kinematic model was based on mobile robot GTR 2010, which was designed and constructed in department of applied mechanics, and also on experience from previous model GTR2006.

Chassis with four wheels is controlled differentially. Each one of the wheels is independently driven by separate speed servomechanism.

Mobile robot always moves in circular trajectory with diameter  $R$  (on trajectory projection of which on plane  $XY$  is circle). Direct motion occurs when trajectory radius approaches infinity. This radius changes in time depending on desired motion direction [7].

Axes of all wheels have to intersect in one point and that point is an immediate center of rotation [4].

The proposed four-wheel undercarriage will be steered differentially. Each of the wheels is driven individually by means of the servomechanism. This is a proposed design solution of the wheeled undercarriage [5]. In the middle-point of the undercarriage is situated a hinge, which enables tilting of both parts of the mobile robot and in this way it increases stability of the robot in terrain.



Figure 2. Mobile robot with sectional undercarriage

Individual transformation matrixes among the neighbouring local coordinate systems will be multiplied symbolically using the software Mathcad in order to obtain the global transformation matrix, which describes

the geometrical position and orientation of the centre of gravity in the global coordinate system, as well as there are obtained in this way the transformation matrixes describing the position and orientation of the individual undercarriage wheels.

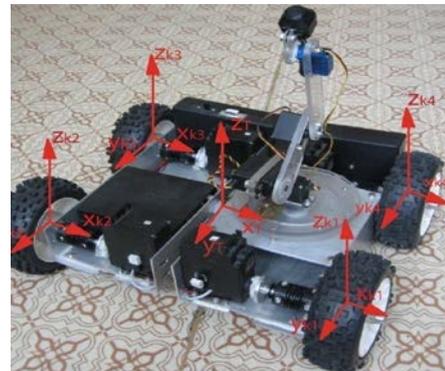


Figure 3. Local coordinate systems

The coordinate  $q1$  represents a transformation of geometrical position of the gravity centre. This transformation corresponds to the rotation around the axis  $z$ . The coordinate  $q2$  represents a transformation corresponding with translation toward the axis  $x$ .

According to the principle of the differential steering it is evident that if there is actual a requirement of a linear movement, so it is necessary that the turning radius is approximating to infinity. In this case this radius is given by the coordinate  $q1$ , which has to approximate to infinity. There was chosen also the third coordinate –  $q3$ , which represents transformation of a translation along the  $y$ -axis. This coordinate ensures the linear movement of the undercarriage, as well as it simplifies the control of movement and calculations.

Based on constructed transformation matrixes it is possible to construct simulation models for defining center of gravity geometric position of chassis and wheels in Matlab/Simulink environment.

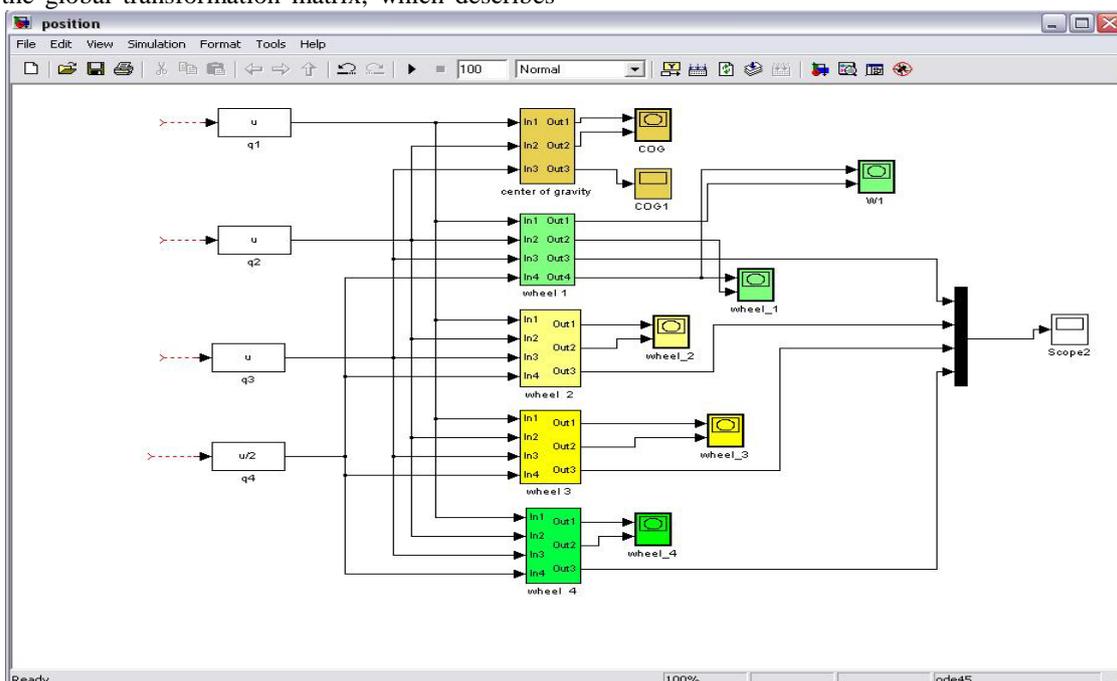


Figure 4. Simulation model of position of center of gravity and wheels

Simulation model of position of geometric center of gravity of chassis and individual wheels comprises of several subsystems that have common input. Every

subsystem represents position of wheel, or position of center of gravity to beginning O (Instantaneous Center of Rotation).

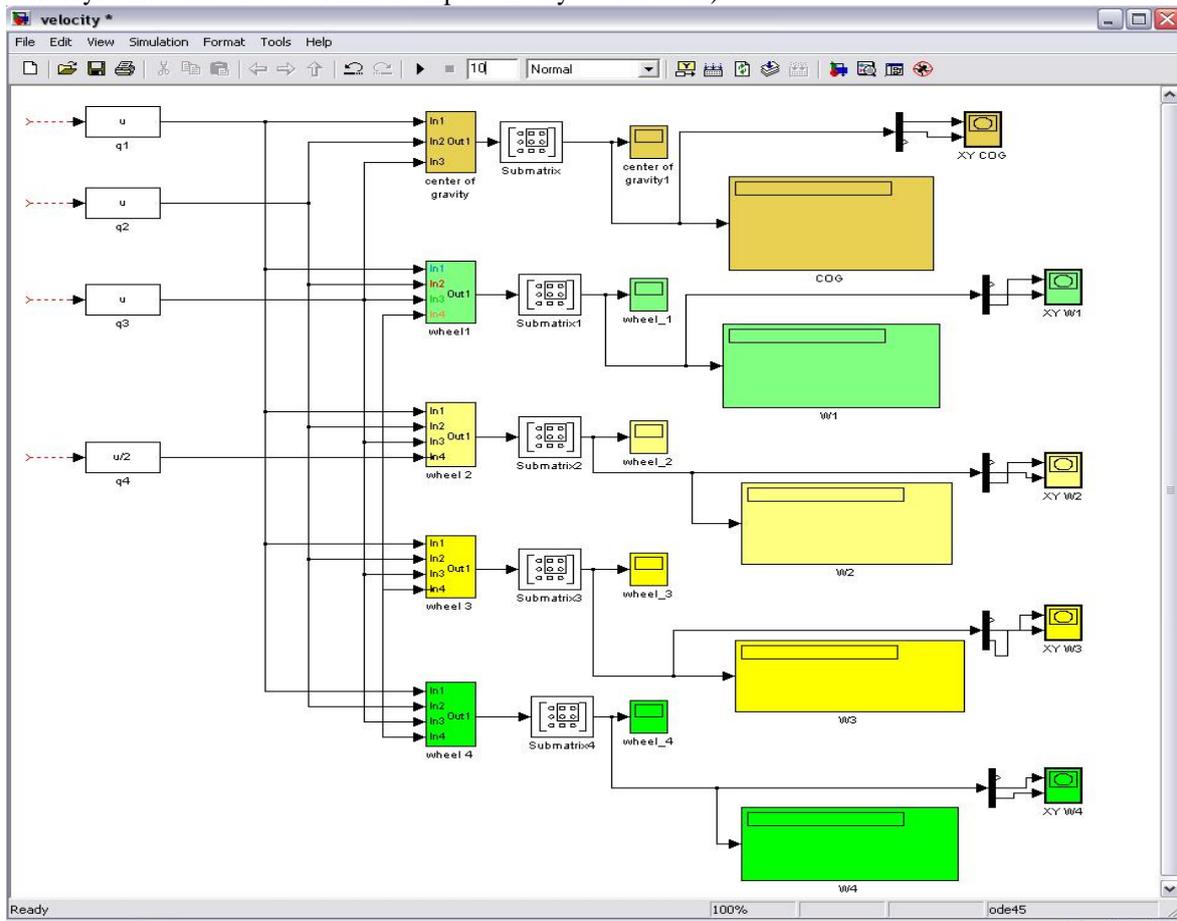


Figure 5. Simulation model of velocity of chassis, center of gravity and wheels

It is possible to use this simulation model to calculate estimated trajectory of center of gravity and wheels of chassis. It is also possible to examine behavior of model in any given combination of chassis dimensions. In connection to this there is also an option of chassis geometry optimization. Furthermore created model provides room for examining influence of terrain roughness on behavior of chassis of mechatronic system.

### 4. Dynamic Model

The aim of the system analysis from mechatronic point of view is to draw a dynamic model of system for control.

DC motor as a dynamic system is described by a system of differential equations that can be derived from the simplified diagram.

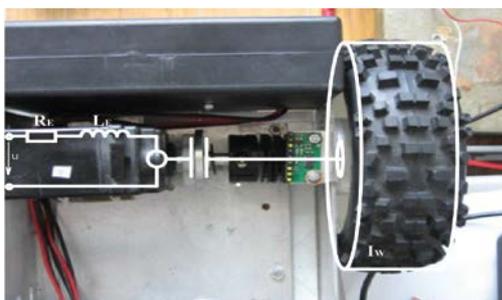


Figure 6. Simplified system diagram

Subsequently it is possible to draw the motion equation:

$$I_E + I_C + I_W = M_E - M_L \tag{1}$$

Where  $M_E$  is moment torque,  $I_E$ ,  $I_C$  and  $I_W$  is the sum of the moments of inertia of engine, clutch and wheel, where as following applies:

$$\omega_E = \omega_C = \omega_W \tag{2}$$

Losses occur when electrical energy is converted to mechanical energy (in the motor), or mechanical energy is converted to electrical energy. For the machine to be efficient, these losses must be kept to a minimum. Some losses are electrical, others are mechanical. Electrical losses are classified as copper losses and iron losses; mechanical losses occur in overcoming the friction of various parts of the machine. These losses represents the moment  $M_L$ .

Dynamic model of a DC motor with separate constant excitation can be expressed as follows:

- Equation describing the electrical balance in the armature circuit

$$u(t) = R_E i(t) + L_E \frac{di(t)}{dt} + u_i(t) \tag{3}$$

Equation (3) describes the sum of the voltages of individual members of the simplified electrical diagram, where  $R_E \cdot i(t)$  is the voltage on winding resistance of engine,  $L_E \frac{di(t)}{dt}$  is the voltage on engine inductance and

$u_i(t)$  is the induced voltage, for which the following applies:

$$u_i(t) = k_E \omega_E(t) \tag{4}$$

Where  $k_E$  is electric constant,  $\omega_E(t)$  is angular velocity acting on the engine shaft.

- Equation of mechanical balance of moment on shaft:

$$M_E - M_L = M_D \tag{5}$$

$$M_E = k_E i(t) \tag{6}$$

$$k_E i(t) - M_L = I \frac{d\omega(t)}{dt} \tag{7}$$

Where  $I$  is moment of inertia of engine,  $M_L$  is moment, by which engine applies load (passive resistances).

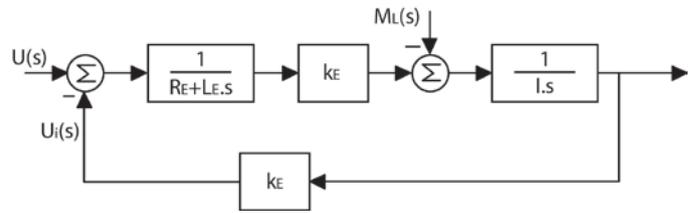
Using Laplace transformation it is possible to derive from equations (1-7) the transfer function of DC motor:

$$U(s) = R_E I(s) + sL_E I(s) + K_E \Omega_E(s) \tag{8}$$

Operator form of equation (8) of balance of moments is expressed by:

$$k_E I(s) - M_L = s\Omega(s) \tag{9}$$

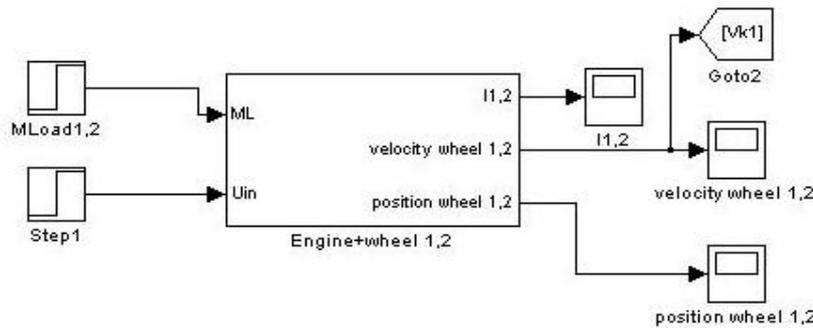
Block diagram of engine and wheels 1 and 2. **Figure 7** shows Dynamic model of DC motor with constant excitation.



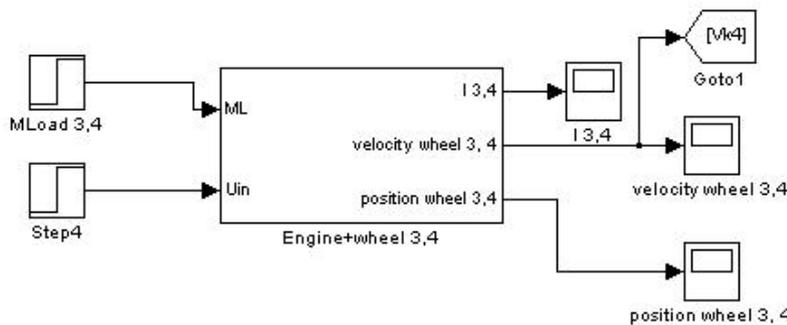
**Figure 7.** Dynamic model of DC motor with constant excitation

**Figure 8** shows Dynamic model of DC motor with constant excitation.

**Figure 8** and **Figure 9** shows a model describing the dynamics of the engine and wheel of chassis. Since the chassis is controlled differentially, wheel velocities on right and left side must be the same. Therefore, for purpose of simplification, the right block represents engines and wheels 1 and 2, left block engines and wheels 3 and 4.



**Figure 8.** Block diagram of engine and wheels 1 and 2



**Figure 9.** Block diagram of engine and wheels 3 and 4

The concept of control structure using kinematic and dynamic model at the same time provides the benefit of economical utilizing of energy sources. In terms of practical realization it is being considered to implement this structure into functional model, where this structure would in real time ensure chassis locomotion control of mobile mechatronic system. With this application the mobile mechatronic system gains a certain amount of intelligence.

Hence it is possible to use the model for calculating estimated or requested trajectory of chassis center of gravity and wheels. At the same time it is possible to examine model behavior in any given combination of chassis dimensions and relating to this there is an option of chassis geometry optimization in terms of obstacles negotiability on diverse terrain.

Thanks to the simulation software products it is relatively easy to fulfil requirements without a necessity to design a real physical model, or there is also another

advantage that the real equipment cannot be damaged during testing. After verification of the simulation model is possible realisation of the real functional mechanism.

The created simulation model can be applied for calculation of assumed or intended path of the undercarriage centre of gravity and path of wheels. It offers a possibility to analyse impacts of terrain unevenness on behaviour of the undercarriage mechatronic system.

Dynamic model is after the identification of unknown parameters and relations applicable for control structure of chassis motion for the task of active trajectory tracking.

## 5. Conclusion

Thanks to the simulation software products it is relatively easy to fulfil requirements without a necessity to design a real physical model, or there is also another advantage that the real equipment cannot be damaged during testing. After verification of the simulation model is possible realisation of the real functional mechanism.

The concept of control structure using kinematic and dynamic model at the same time provides the benefit of economical utilizing of energy sources. In terms of practical realization it is being considered to implement this structure into functional model, where this structure would in real time ensure chassis locomotion control of mobile mechatronic system. With this application the mobile mechatronic system gains a certain amount of intelligence.

The created simulation model can be applied for calculation of assumed or intended path of the undercarriage centre of gravity and path of wheels.

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