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Matehmatical Study of Thermosolutal Convection in Heterogeneous Vscoelastic Fluid in the Presence of Porous Medium

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Abstract The present paper a thermo-solutal convection in Walters B' heterogeneous visco-elastic fluid through Brinkman permeable effect is investigated. The investigation of thermosolutal convection is proposed by its complexities of double diffusive dissemination and importance of geophysics and nuclear physics. The results are investigation of the oscillatory modes exists under different conditions and non-oscillatory modes are unstable.

Keywords: *Thermosolutal Convection, Heterogeneous Walters B' Viscoelastic Fluid, Porous Medium, Linear Stability Theory, dispersion*

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1. Introduction

The theoretical and experimental results on thermal convection in a fluid layer, in the absence and presence of rotation and magnetic field have been given by Chandrasekhar [1]. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [2]. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz, [3]) and some Antarctic lakes (Shirtcliffe, [4]). The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat.

The scientific importance of the field has increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust (Lister, [5]). The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnel, [6]).

The stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below has been investigated by Vest and Arpaci [7]. The nature of instability and some factors may have different effects, on viscoelastic fluids as compared to the Newtonian fluids. Bhatia and Steiner [8] have profounded the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. Sharma and Sharma [9] have investigated the thermal instability of a rotating Maxwell fluid through porous medium and found that, for stationary convection, the rotation has stabilizing effect whereas the permeability of the medium has both stabilizing as well as destabilizing effect, depending on the magnitude of rotation. In another study, Sharma [10] has studied the stability of a layer of an electrically conducting Oldroyd fluid [11] in the presence of a magnetic field and has found that the magnetic field has a stabilizing influence. There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such class of viscoelastic fluids is Walters B′ fluid [12] having relevance and importance in geophysical fluid dynamics, chemical technology, and petroleum industry. Walters′ [13] reported that the mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters B′ viscoelastic fluid. Walters B′ viscoelastic fluid forms the basis for the manufacture of many such important and useful products. Chakraborty and Sengupta [14] have investigated the flow of unsteady viscoelastic (Walters B′ liquid) conducting fluid through two porous concentric non-conducting infinite circular cylinders rotating with different angular velocities in the presence of uniform axial magnetic field. Sharma and Kumar [15] found the stability of the plane interface separating two viscoelastic (Walters B′) superposed fluids of uniform densities. In another study, Sharma and Kumar [16] studied Rayleigh-Taylor instability of superposed conducting Walters B′ viscoelastic fluids in hydromagnetics. Kumar [17] has considered the thermal instability of a layer of Walters B′ viscoelastic fluid acted on by a uniform rotation and found that for stationary convection, rotation has a stabilizing effect. Kumar et al.

[18] have considered the stability of plane interface separating the Walters B′ viscoelastic superposed fluids of uniform densities in the presence of suspended particles. Kaur, et al. [19]. Analysis of heat transfer in hydrodynamic rotating flow of viscous fluid through a non homogenious porous medium with constant heat source/sink, Kaur et al. [20] investigated finite difference technique for Unsteady MHD Periodic Flow of Viscous Fluid through a planer channel. Kumar et al. [21] investigated Perturbation technique of MHD free convective flow through infinite vertical porous plate with constant heat flux. Kumar et al. [22] developed a mathematical and simulation of lid driven cavity flow at different aspect ratios using single relaxation time lattice Boltzmann technique, Kumar et al. [23], investigated finite difference technique for reliable MHD steady flow through channels permeable boundaries.

2. Mathematical Model

In the present communication of the paper we are consider an infinite horizontal layer of incompressible and heterogeneous Walters B' viscoelastic fluid of thickness 'd ', in porous medium of porosity ε and medium permeability k_1, bounded by the planes $z=0$ and $z=d$.

The effective density is given by.

$$\rho = \rho_0[f(z)+\alpha(T_0-T)-\alpha'(S_0-S)] \qquad (1)$$

where α and α' are the thermal and solute expansion coefficients.

The relevant Brinkman-Oberbeck-Boussinesq equations areas given below:

$$\rho_0\frac{\partial D}{\partial t} - grad\ \ p + \rho\overline{g} + (\mu - \mu'\frac{\partial}{\partial t})[\Delta^2\overline{q} - \frac{1}{k_1}\overline{q}] \qquad (2)$$

$$div\,\overline{q} = 0 \qquad (3)$$

$$\frac{\partial\rho}{\partial t} + (\overline{q}.\nabla)\rho = 0 \qquad (4)$$

$$\frac{\partial T}{\partial t} + (\overline{q}.\nabla)T = K\nabla^2 T \qquad (5)$$

$$\frac{\partial S}{\partial t} + (\overline{q}.)\nabla S = K'\nabla^2 S \qquad (6)$$

where $\overline{q}$, μ, μ', ρ and p are the velocity, coefficient of viscosity, viscoelastcity, density and pressure of the fluid respectively, T the temperature, S the solute concentration, $\overline{g}(0,0,-g)$ is the acceleration due to gravity, K and K' are the thermal and solute diffusivities and k_1 is the intrinsic permeability of the medium .

Here in writing equations (2)-(6), porosity ε ($0<\varepsilon<1$ and $\varepsilon \to 1$ corresponds to non-porous medium) corrections have not been included for avoiding the involvement of too many constants.

The initial state stability is given below:

$$\begin{aligned} &q \to 0 + \vec{\delta}\vec{q}, T \to T_0 - \beta z, S = S_0 - \beta' z \\ &\rho = \rho_0\left[f(z) + \alpha\beta z - \alpha'\beta' z\right], p = p_0 - \int_0^z g\rho dz. \end{aligned} \qquad (7)$$

Suppose the system be slightly disturbed and as a result of this small perturbation, the different physical quantities as given below:

$$\begin{aligned} &q = 0, T \to T + \theta \quad S \to S + \gamma, p \to p + \delta p \\ &and \\ &\rho \to \rho_0\begin{bmatrix} f(z) + \alpha(T_0 - T - \theta) \\ -\alpha'(S_0 - S - \gamma) \end{bmatrix}, +\delta p \end{aligned} \qquad (8)$$

The linearized perturbations equations are as given below:

$$\rho_0\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - (\mu - \mu'\frac{\partial}{\partial t})\left[\frac{1}{k_1}u - \nabla^2 u\right] \qquad (9)$$

$$\rho_0\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - (\mu - \mu'\frac{\partial}{\partial t})\left[\frac{1}{k_1}v - \nabla^2 v\right] \qquad (10)$$

$$\begin{aligned} &\rho_0\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - g(\delta\rho - \alpha\rho_0\theta + \alpha\rho_0\gamma') \\ &-(\mu - \mu'\frac{\partial}{\partial t}\left[\frac{1}{k_1}w - \nabla^2 w\right] \end{aligned} \qquad (11)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (12)$$

$$\frac{\partial}{\partial x}\delta\rho + \rho_0 w\frac{df}{dz} = 0 \qquad (13)$$

$$\frac{\partial\theta}{\partial t} - \beta w = K\nabla^2\theta \qquad (14)$$

$$\frac{\partial\gamma}{\partial t} - \beta' w = \nabla^2\gamma \qquad (15)$$

Where

$$\delta\overline{q} = (u, v, w). \qquad (16)$$

3. Mathematical Analysis

We seek solutions of the equations (9)-(15) whose dependence on space-time coordinates are of the form [Chandrasekhar (1)].

$$\begin{aligned} &[u, v, w, \theta, \gamma, \delta p, \delta\rho] = \\ &U(z), V(z), W(z), \Theta(z), \Gamma(z), L(z), Y(z)] \\ &\exp[ik_x x + ik_y y + nt] \end{aligned} \qquad (17)$$

where k_x and k_y are the horizontal wave numbers and n is the frequency of the harmonic disturbances. Also

$$k = \sqrt{(k_x{}^2 + k_y{}^2} \qquad (18)$$

gives the wave number of the perturbation propagation.

Using expression (17), equations (9)-(15), on simplification, we get

$$-k^2 L = \rho_0\left[\left(n + \frac{\nu - \nu' n}{k_1}\right) - (\nu - \nu' n)(D^2 - k^2)\right]DW \qquad (19)$$

$$\rho_0\left[\left(n+\frac{\nu-\nu' n}{k_1}\right)W\right]=-DL+g\rho_0$$
$$\left[\frac{1}{n}(\frac{df}{dz})W+\alpha\Theta-\alpha'\Gamma\right] \quad (20)$$
$$+\rho_0(\nu-\nu' n)(D^2-k^2)W$$

$$n\Theta-\beta W-K(D^2-k^2)\Theta \quad (21)$$

$$n\Gamma-\beta' W-K'(D^2-k^2)\Gamma \quad (22)$$

where $v(=\frac{\mu}{\rho_0})$ and $v'(=\frac{\mu'}{\rho_0})$ are respectively the kinematic viscosity and kinematic viscoelasticity.

Elimination of L from equations (19) and (20) we get

$$n\left[\left(n+\frac{\nu-\nu' n}{k_1}\right)-(\nu-\nu' n)(D^2-k^2)\right](D^2-k^2)W$$
$$+gk^2(\frac{df}{dz})W+g\alpha nk^2\Gamma=0. \quad (23)$$

The equations (21)-(23) in non-dimensional form as given below:

$$[(D^2-k^2-\sigma p_1]\Theta=-\frac{\beta' d^2}{K}W \quad (24)$$

$$[\tau(D^2-a^2-\sigma p_1]\Gamma=-\frac{\beta' d^2}{K'}W \quad (25)$$

$$\sigma v(D^2-a^2)[1-A\sigma(D^2-a^2)-(\sigma+B-B' A\sigma)]W$$
$$-\frac{a^2gd^4}{v}(\frac{df}{dz})W-g\alpha\sigma a^2d^2\Theta+g\alpha'\sigma a^2d^2\Gamma=0. \quad (26)$$

Now put $\hat{D}=dD$, $\hat{a}=kd$, $\hat{\sigma}=\frac{nd^2}{v}$ and thereafter dropping the caps for convenience. Also we have put

$$p_1=\frac{v}{K},\tau=\frac{K'}{K},A=\frac{v'}{d^2},B=\frac{d^2}{k_1},R=\frac{g\alpha\beta d^4}{Kv}$$
$$R'=\frac{g\alpha'\beta' d^4}{K'v},R_2=\frac{gd^4(\frac{df}{dz})}{Kv} \quad (27)$$

Equation (26) with the help of equations (24) and (25) is written as

$$\sigma p_1(D^2-a^2)[D^2-a^2-\sigma p_1][\tau(D^2-a^2)-\sigma p_1]$$
$$[(1-A\sigma)(D^2-a^2)-(\sigma+B-B' A\sigma]W$$
$$-a^2R_2[D^2-a^2-\sigma p_1][\tau(D^2-a^2)-\sigma p_1]W \quad (28)$$
$$+a^2\sigma p_1R[\tau(D^2-a^2)-\sigma p_1]W-a^2\sigma p_1R'$$
$$[\tau(D^2-a^2)-\sigma p_1]W=0$$

The equations (24)-(26) and (28) are to be solved using boundary conditions.

The appropriate boundary conditions for this case are

$$W=D^2W=0,\Gamma=0 \ at\ z=0\ and\ z=1 \quad (29)$$

4. Results and Discussion

(a) Stationary Convection

When the instability sets in as stationary convection, the marginal state have been characterized by $\sigma=0$ Hence the substitution of $\sigma=0$ in equations (21)-(23) we get

$$\left.\begin{aligned}&(D^2-a^2)\Theta=-\left(\frac{\beta d^2}{K}\right)W\\&\tau(D^2-a^2)\Gamma=-\left(\frac{\beta' d^2}{K'}\right)\Gamma\\&(\frac{df}{dz})W=0\end{aligned}\right\} \quad (30)$$

Now integrating equation (30) and using the boundary conditions (29), we see that $W=0,\Theta=0,\Gamma=0$ etc. are the only possible solutions which led to contradiction to the hypothesis that initial state solutions are perturbed.

(b) Oscillatory Convection

The equation (28),[using Chandrasekhar [1]], we get

$$a^2R-\frac{a^2R'[\pi^2+a^2+\sigma p_1]}{[\tau(\pi^2+a^2)+\sigma p_1}+\frac{a^2R_2[\pi^2+a^2+\sigma p_1]}{\sigma p_1}$$
$$=[\pi^2+a^2(\pi^2+a^2+\sigma p_1)][(1-A\sigma)(\pi^2+a^2) \quad (31)$$
$$+(\sigma+B-B' A\sigma)]$$

Using non dimensional quantity are given below:

$$R_3=\frac{R_2}{\pi^4},R_1=\frac{R}{\pi^4},$$
$$R_4=\frac{R'}{\pi^4},x=\frac{a^2}{\pi^2} \quad (32)$$
$$\sigma_2=\frac{\sigma_2}{\pi^2}\ and\ B_1=\frac{B}{\pi^2}.$$

Putting (32) in equation (31), we get

$$xR_1-\frac{xR_4[1+x+i\sigma_2p_1]}{[\tau(1+x)+i\sigma_2p_1]}+\frac{xR_3[1+x+i\sigma_2p_1]}{i\sigma p_1}$$
$$=[1+x][1+x+i\sigma_2p_1]\begin{bmatrix}(1-iA\pi^2\sigma_2)(1+x)\\+(i\sigma_2+B_1+B_2i\pi^2\sigma^2)\end{bmatrix}. \quad (33)$$

Separating equation (33) in real and imaginary parts, we get

$$R_1=\frac{1}{x}\begin{bmatrix}(1+x)^2\left\{1+x+B1+\sigma^2{}_2p_1A\pi^2\right\}-\\(1+x)\sigma^2{}_2p_1\left\{1+B_2\pi^2\right\}-xR_3\\+\frac{xR_4\left\{\tau(1=x)^2+\sigma^2{}_2p_1\right\}}{\{\tau^2(1=x)^2+\sigma^2{}_2p_1\}}\end{bmatrix} \quad (34)$$

$$A_0\sigma_2{}^4+A_1\sigma_2{}^2+A_2=0 \quad (35)$$

$$\left.\begin{aligned} A_0 &= p^3{}_1(1+x)\left\{\begin{matrix} -\pi^2(1+x)^2 \\ +(1+x)(1+B_2\pi^2+p_1)+p_1B_1 \end{matrix}\right\} \\ A_1 &= \tau^2 p_1(1+x)^3\left\{\begin{matrix} -\pi^2(1+x)^2+(1+x) \\ +B_2\pi^2(1+x)+p_1(1+x) \\ +B_1p_1(1+x) \end{matrix}\right\} \\ &+xp^2{}_1(1+x)\{R_4(\tau-1)+R_3\} \\ A_2 &= xR_3(1+x)^3\tau^2 \end{aligned}\right\} \quad (36)$$

Now from equation (35) and (36), the frequency of oscillations σ_2 in marginal state as given below:

$$\sigma^2{}_2 = \frac{-A_1\sqrt{A^2{}_1-4A_0A_2}}{2A_0} \quad (37)$$

and from (27) and (34), Rayleigh number R is given by

$$R=\pi^4R_1=\pi^4\left[\begin{matrix} \dfrac{1}{x}\left\{\begin{matrix} (1+x)^2\begin{pmatrix} 1+x+B_1 \\ +\sigma^2{}_2p_1A\pi^2 \end{pmatrix} \\ -(1+x)\sigma^2{}_2p_1(1+B_1\pi^2) \end{matrix}\right\} \\ -R_3+R_4\dfrac{\{\tau(1+x)^2+\sigma^2{}_2p_1{}^2\}}{\{\tau^2(1+x)^2+\sigma^2{}_2p_1{}^2\}} \end{matrix}\right]. \quad (38)$$

We are discussing the existence of over stable marginal states for different cases:

Case (i): When $R_3\rangle 0$, i.e. $\frac{df}{dz}\rangle 0$,

Since $R_3>0$ thus it implies $A_2>0$, therefore, if $A_0>0$ and $A_1>0$ i.e. $1>\pi^2A(1+x)$ and $\tau-1>0$ i.e. $1>\pi^2A(1+x)$ and $K'>K$, then there will be no real σ_2 resulting non-occurrence of overstable marginal state. But, if R_4 satisfies the inequality

$$R_4\rangle\frac{1}{(1-\tau)}\left[\frac{\tau^2(1+x)^2}{xp_1}\left\{\begin{matrix} [1-\pi^2A(1+x)] \\ +B_2\pi^2+p_1(1+B_1) \end{matrix}\right\}+R_3\right], \quad (39)$$

besides $K'<K$, $A>\frac{1}{\pi^2(1+x)}$ and $A_1^2-4A_0A_2>0$, then the marginal state may exist even when $R_3>0$.

Case (ii): When $R_3<0$, i.e. $\frac{df}{dz}<0$

When $R_3<0$, the marginal state always exist whatever be the values of other parameters provided $A_1^2-4A_0A_2>0$ and then σ_2 is given by equation (37).

(c) Nature of Non-Oscillatory Modes

For $R_3>0$ i.e. $R_2>0$ and $K'>K$, the only modes that may exist are non-oscillatory modes for which $\sigma_2=0$ and $\sigma=\sigma_1$ (σ_1 is real). Hence substitution of $\sigma=\sigma_1$ and $W=W_0\sin\pi z$ in equation (28) gives

$$D_0\sigma^4{}_1+D_1\sigma^3{}_1+D_2\sigma^2{}_1D_3\sigma_1+D_4=0, \quad (40)$$

Where

$$\left.\begin{aligned} D_0 &= p_1^3(\pi^2+a^2)\left[-A(\pi^2+a^2)+1-B'A\right] \\ D_1 &= p_1^2(\pi^2+a^2)\begin{bmatrix} -A(\pi^2+a^2) \\ +1-B'A \end{bmatrix}(1+\tau) \\ &+p_1^3(\pi^2+a^2)\left[(\pi^2+a^2)+B\right] \\ D_2 &= p_1(\pi^2+a^2)\begin{bmatrix} \tau\{-A(\pi^2+a^2)+1-B'A\} \\ +p_1(1+\pi) \end{bmatrix} \\ &+p_1^2(\pi^2+a^2)^2B(1+\pi) \\ &-a^2p_1^2(R_2+R-R') \\ D_3 &= p_1\tau(\pi^2+a^2)^3\{(\pi^2+a^2)+B\} \\ &-a^2p_1(\pi^2+a^2)\{R_2\tau+R\tau-R'\} \\ &-a^2R_2\tau p_1 \\ D_4 &= -a^2(\pi^2+a^2)R_2\tau^2 \end{aligned}\right\} \quad (41)$$

5. Conclusion

In this paper we are consider the thermosolutal convection in a layer of heterogeneous Walters B' viscoelastic fluid heated and solute from below through porous medium is investigated.

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