

Numerical Solution of the Total Probability Theorem in a Three Diensional Earthquake Source Domain for Developing Seismic Hazard Map and Hazard Spectrum

Lalu Makrup^{1,*}, Masyhur Irsyam², I Wayan Sengara², Hendriyawan³

¹Department of Civil Engineering, Islamic University of Indonesia, Yogyakarta

²Department of Civil Engineering, Institute of Technology Bandung

³Department of Ocean Engineering, Institute of Technology Bandung

*Corresponding author: lalu_makruf@yahoo.com

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Abstract Probabilistic seismic hazard analysis is a method of assessing seismic threat on a region in the earth surface. Total probability theorem was utilized here has solved on earthquake source domain and employed it in seismic hazard analysis. Solution of the theorem is performed in a computer program and has been used to develop the Hazard Map and the Uniform Hazard Spectrum (UHS). The map is peak ground acceleration (PGA) of Sumatra Indonesia with hazard level 10% probability of exceedance in 50 years and the UHS is from a site in Surabaya Indonesia with hazard level 10% probability of exceedance in 50 years. The two result of the program are verified by the two previous study results. The result of the program is very similar to two other studies.

Keywords: development, probability, seismic hazard, seismic source, map

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1. Introduction

Previously, methodology of probabilistic seismic hazard calculation was based on an assumption that seismic source as a point. Calculation was executed through a common statistical procedure that is a combination of regression technique and Gumbel standard probability distribution. Cornell [6] introduced new seismic source, it was area source zone. The source zone has been used as a basis to calculate seismic hazard. The area source is defined based on epicenter distribution of earthquakes that were occurred in a region (Figure 1).

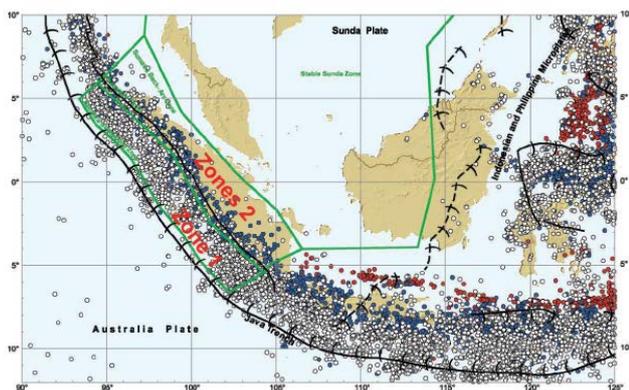


Figure 1. Seismic area source zone [17], used by permission

At this time, the earthquake source zone that was used as a basis to calculate seismic hazard are faults. The faults comprise subduction and transform fault source zone. As area source zones, these sources were defined also based on the epicenter distribution in their region. For earthquakes that were occurred on a region where the fault cannot be identified, their source is considered to area or background source [17].

Seismic hazard study has been performed [12] to develop Indonesian seismic hazard map. Irsyam *et al.*, [12] employed USGS analysis seismic hazard computer program in that study. The map is proposed for revising of Indonesian seismic building code. Generally, the study used the fault source zone. By employing USGS analysis seismic hazard program Petersen *et al.*, [18] developed seismic hazard map for Southeast Asia utilized the fault source zones. The Petersen study is funded as a USGS project.

In the present study I developed computer program to solve total probability theorem with earthquake source is fault. The two results of the program are verified by the two previous result study.

2. Total Probability Theorem

Probabilistic concept has allowed uncertainties in the site, location, and rate of recurrence of earthquake and in the variation of the ground motion characteristic with

earthquake size and location to be explicitly considered in the evaluation of seismic hazards. Probabilistic Seismic Hazard Analysis (PSHA) provides a framework in which these uncertainties can be identified, quantified, and combined in rational manner to provide more complete picture of seismic hazard. For a given earthquake occurrence, the probability of a ground motion parameter A will exceed a particular value a can be computed using total probability theorem [11], that is,

$$P_A(a) = \int_M \int_R P(A > a | m, r) f_M(m) f_R(r) dr dm \quad (1)$$

where $P(A > a | m, r)$ is a probability distribution of a particular value a will be exceeded a ground motion parameter A (the distribution was a log normal), $f_M(m)$ is a probability distribution of earthquake magnitude that commonly used is an exponential distribution which developed firstly by Gutenberg-Richter [9] $f_R(r)$ is a relative probability distribution of distance.

Equation (1) very difficult to solve analytically, even it almost cannot be solved analytically. Therefore the equation should be solved numerically. Commonly, each problem that has to be evaluated numerically need solution domain. The domain of this problem is an earthquake source (i.e. fault plain), see Figure 2.

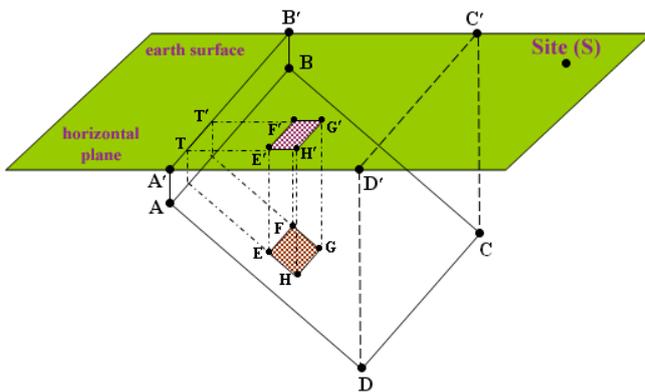


Figure 2. Fault plain in a three dimensional representation (drown based on McGuire, [15])

where:

A'B'AB = the plain where no energy release in this portion

ABCD = dipping fault plain

A'B'C'D' = projection of fault plain on earth surface

EFGH = rupture area

EF = rupture length and EH = rupture width

E'F'G'H' = projection of rupture area on horizontal plain

A'B' = fault trace, TT' = rupture portion on fault trace

EE' = FF' = top rupture depth

GG' = HH' = bottom rupture depth

2.1. Magnitude Probability Distribution

Magnitude distribution $f_M(m)$ is developed originated with Gutenberg-Richter's law [9], equation (2).

$$\log(\lambda_m) = a - bm \text{ or } \lambda_m = e^{\alpha - \beta m} \quad (2)$$

where $\alpha \approx 2.303a$, $\beta \approx 2.303b$, λ_m is number of event per year, a and b are regression constant that can be obtained by statistical procedure.

A truncated-below magnitude m_0 can be introduced to the preceding formulation (equation 2) to exclude small magnitudes that can be ignored in engineering analysis. In mostly hazard analysis, m_0 ranges between 3 and 5 [8]. So, from equation (2) can be derived probability density function as:

$$f_M(m) = \beta e^{-\beta(m-m_0)} \quad (3)$$

Equation (3) allows probabilities calculation even for very large magnitude (i.e. unrealistic magnitude). To overcome this problem, an upper bound magnitude m_u is introduced. It is defined as the largest earthquake likely to occur along an active source [16]. Cornell and Vanmarcke [7] propose a modification to the original Gutenberg-Richter curve, accounting for m_u as well as what has already accounted for m_0 . The actual value of m_u should be determined by geological investigation of the region that will provide information about maximum fault rupture and therefore the maximum energy and magnitude that can be produced. The complete probability density function $f_M(m)$ for the magnitude range are expressed by:

$$f_M(m) = \frac{\beta e^{-\beta(m-m_0)}}{1 - e^{-\beta(m_u-m_0)}}, \quad m_0 < m < m_u \quad (4)$$

The equation (4) is known as a truncated exponential distribution function (Figure 3).

Geologic and seismologic studies on a number of faults have shown that sources tend to repeat large earthquake that closes to their magnitude maximum, call characteristic earthquake. This was explained by observation that fault segment move by the same distance in each earthquake (constant fault slip rate).

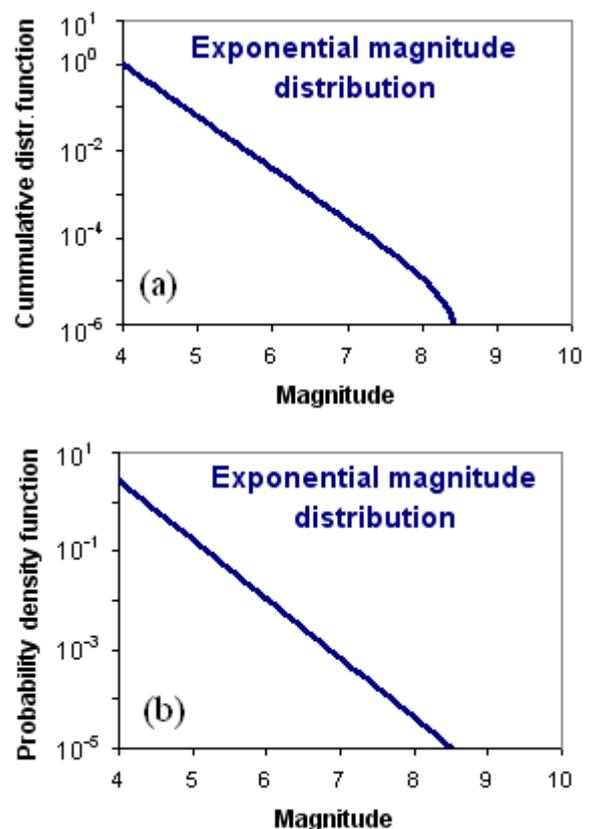


Figure 3. Exponential magnitude distribution (a) cumulative distribution function and (b) probability density function (drown based on equation 4)

The exponential model described in the previous paragraph is based on historical data solely and underestimate the rates of large earthquake as compare to geologic information. Youngs and Coppersmith [23] suggest an alternative recurrence law to account for the seismicity and rate of large event. Their model is called the characteristic earthquake recurrence law. By this approach, the cumulative distribution function flattens close to the maximum magnitude (Figure 4a). The probability density function that result from this model is a combination of truncated exponential Gutenberg-Richter model at small magnitude and uniformed distribution close to the maximum magnitude (Figure 4b).

$$f_M(m) = \frac{\beta e^{-\beta(m-m_0)}}{1 - e^{-\beta(m_u-m_0)}} , m_0 < m < m_u - 1/2 \quad (5)$$

$$f_M(m) = \frac{\beta e^{-\beta(m_u-3/2-m_0)}}{1 - e^{-\beta(m_u-m_0)}} , m_u - 1/2 < m < m_u \quad (6)$$

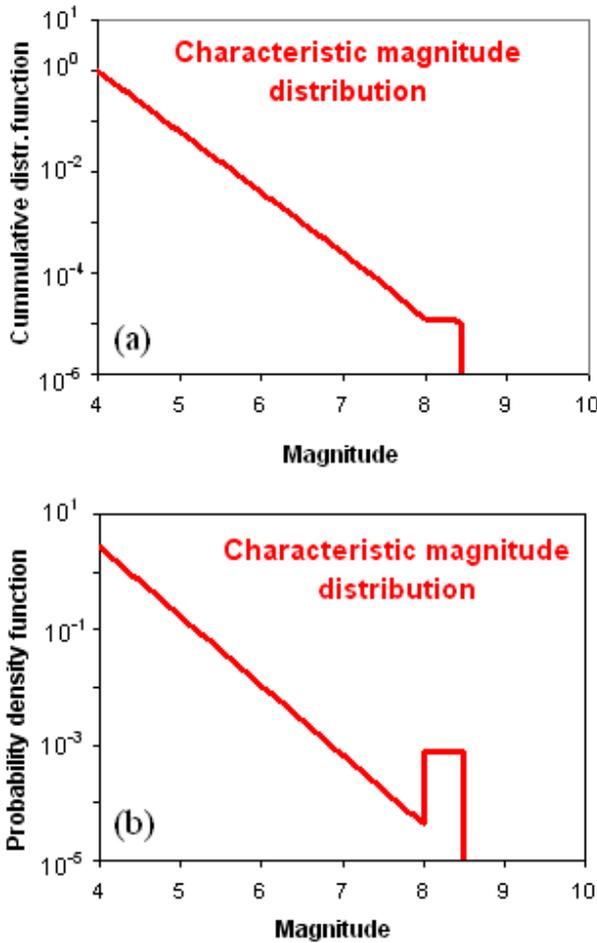


Figure 4. Characteristic magnitude distribution (a) cumulative distribution function and (b) probability density function [20]

The recurrence laws both Gutenberg-Richter [9] and Youngs and Coppersmith [23] are used in the program to describe the aleatory (or random) uncertainties in magnitude distribution.

2.2. Total Probability Theorem Solution and Distance Probability Distribution

A rupture and its magnitude can be occurred on different time and in everywhere on a fault plain. Thus

probably occurrence of the rupture can be drawn as most of overlapped-rupture-area on whole of the fault plain (see Figure 5a). All equations related to the total probability theorem should be evaluated on fault plain (i.e. on each rupture area). Relative probability of a rupture that can be occurred on a fault will be the same as relative probability $f_R(r)$ of earthquake rupture-to-site distance.

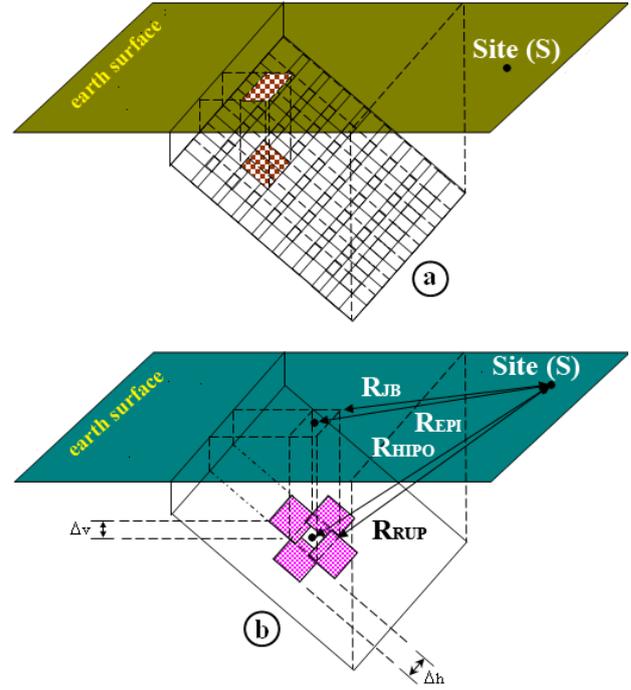


Figure 5. Likely occurrence of rupture on a fault plain (a) and definition of distance (b)

Rupture-to-site distance can be defined as in Figure 5b where:

- R_{HIPO} = Hypocenter distance (i.e. assumed hypocenter point is central of the rupture)
- R_{JB} = Joiner-Boore distance
- R_{EPI} = Epicenter distance (i.e. projection of hypocenter point on the earth surface)
- R_{RUP} = Rupture distance

$$f_R(r) = \frac{a \text{ rupture area}}{\text{total rupture area}} \quad (7)$$

The distances that are defined above can be evaluated if length and width of rupture were known. There are many relations that can be used to calculate rupture length. Well and Coppersmith [21] have given the formulation for shallow crustal event for all types of slip (equation 8).

$$\log L = 0.69M_w - 3.22 \text{ with } \sigma_{\log L} = 0.22 \quad (8)$$

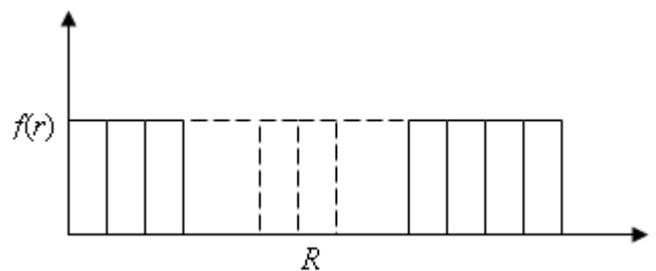


Figure 6. Probability distribution of distance

In PSHA the experts were commonly used the rupture width equal to rupture length [15]. Thus for the same magnitude relate to equation (8) the rupture length and width are obtained equal to L . Based on above paragraph, then all events that likely will be occurred in the future on a fault with a magnitude value give flatten probability distribution (Figure 6).

2.3. Probability of Seismic Parameter to be Exceeded

Probability of a ground motion parameter A will exceed a particular value a , $P(A>a|m,r)$, assuming log-normally distributed or logarithm of data are normally distributed (follow Gaussian distribution). According to probabilistic seismic hazard analysis, the standard normal deviate (z^*) of the ground motion parameter is:

$$z^* = \frac{\ln a - \ln(A)}{\sigma_{\ln(A)}} \quad (9)$$

where A is a ground motion parameter will exceed a particular value a . Probability of A exceed a $P(A>a|m,r) = p(z^*)$, see Figure 7, can be looked for in normal distribution table or it can be calculated using approximate formula [14]:

$$p(z^*) = w[\{(c_1 w - c_2)w + c_3\}w - c_4]^* w + c_5] \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad (10)$$

where

$$w = 1 + c_0 z \quad (11)$$

$c_0 = 0.2316419$	$c_3 = 11.781478$
$c_1 = 1.330274$	$c_4 = 0.3565638$
$c_2 = 1.821256$	$c_5 = 0.3193815$

3. Seismic Hazard Curve and Ground Motion Design

Frequency of a seismic event $\lambda(A>a)$ for n number of earthquake sources was accounted for by the function:

$$\lambda_A(a) = \sum_{i=1}^n v_i \int_M \int_R P(A>a | m, r) f_M(m) f_R(r) dr dm \quad (12)$$

where R is distance rupture-to-site, and M is magnitude.

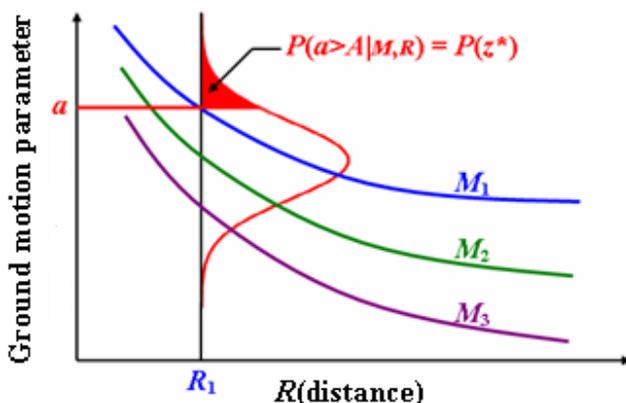


Figure 7. Probability diagram of $P(a>A|M,R)$

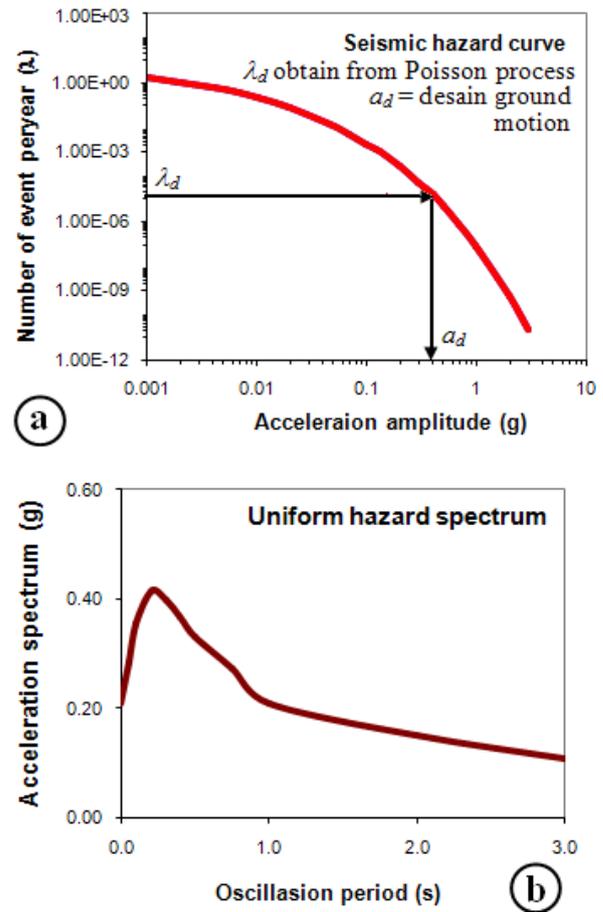


Figure 8. Example of seismic hazard curve (a) and uniform hazard spectrum (b) (drawn based on equation 13 and 14)

Relation between $\lambda(A>a)$ and a , is called seismic hazard curve (Figure 8a). A combination of seismic hazard curve with Poisson distribution equation (13) will give ground motion design (Figure 8a).

$$P = 1 - \exp(-\lambda t) \quad (13)$$

According to ground motion prediction relation and its period (*the attenuation function*) so based on equation (12) and (13) can be accounted for a spectrum of ground motion parameter (e.g. acceleration ground motion) in probabilistic framework as a curve. The curve was called uniform hazard spectrum (Figure 8b).

4. Seismic Hazard Calculation and Program Verification

Seismic hazard calculation is executed by the program that is developed based on the theory above. This calculation is needed to acquire the hazard result that will be verified by other study. The two previous study results i.e. (1) Irsyam *et al.*, [13] developed seismic hazard map for Sumatra, and (2) Aldiamar, [1] developed uniform hazard spectrum for Surabaya Indonesia. The two results will be used to verify the result of the program.

To calculate the hazard Irsyam *et al.*, [13] used attenuation relations of Youngs *et al.*, [22] for subduction zone and Boore *et al.*, [3] and Sadigh *et al.*, [19] for shallow crustal event. In the other hand to calculate the hazard Aldiamar, [1] used Youngs *et al.*, [22] and Atkinson - Boore [2] for subduction zone and Boore -

Atkinson [3], Campell – Bozorgnia [4], Chiou – Youngs [5] for shallow crustal event. Result of hazard computation by two authors can be seen in Figure 11 and Figure 12.

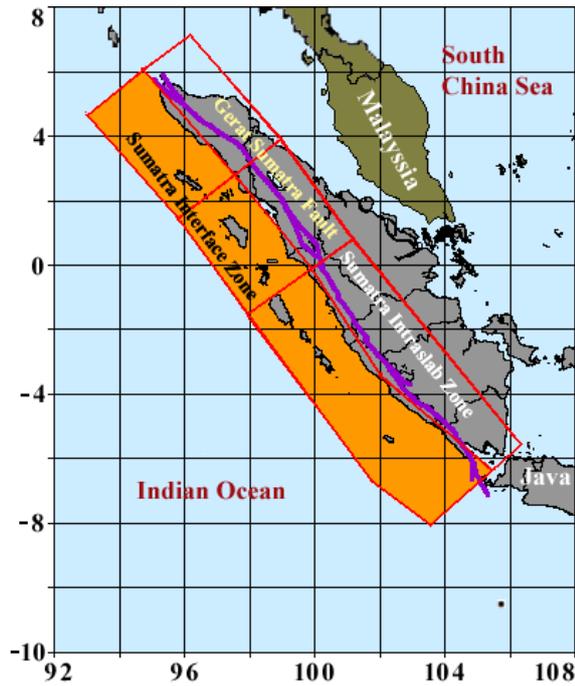


Figure 9. Seismic source used by Irsyam et al., [13]

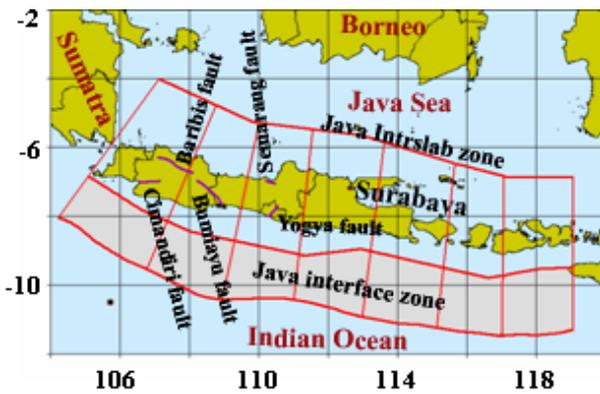


Figure 10. Seismic source used by Aldiamar, [1]

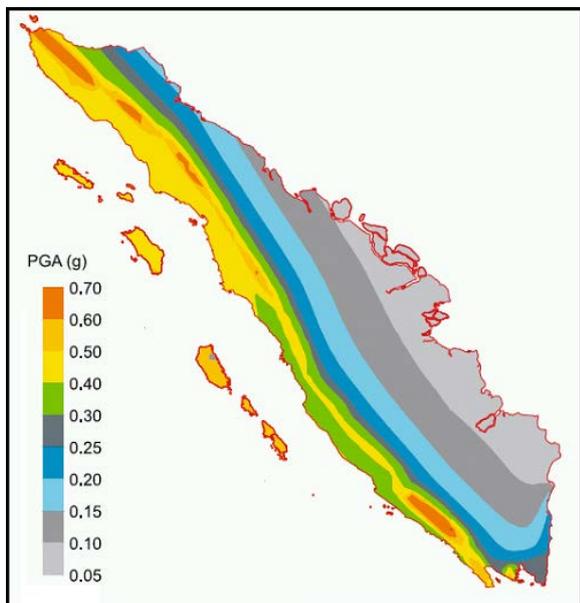


Figure 11. Seismic hazard map (PGA) of Sumatra with hazard level 10% probability of exceedance in 50 years, developed by Irsyam et al., [13]

The two authors used hazard level 10% in 50 years as basis to calculate and employ *EZ Frisk* program [15] to compute the hazard.

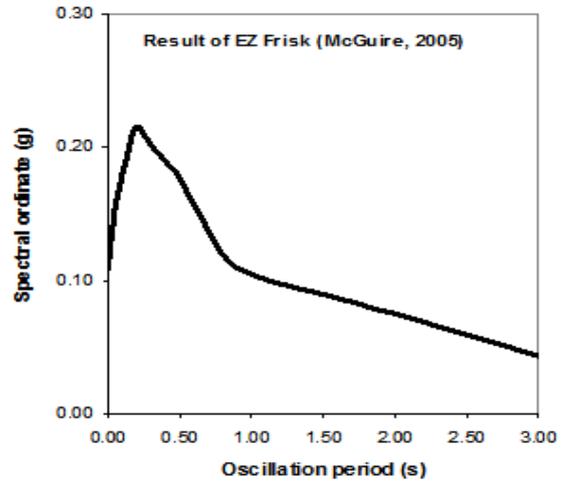


Figure 12. Uniform hazard spectrum of Surabaya with hazard level 10% probability of exceedance in 50 years, developed by Aldiamar [1]

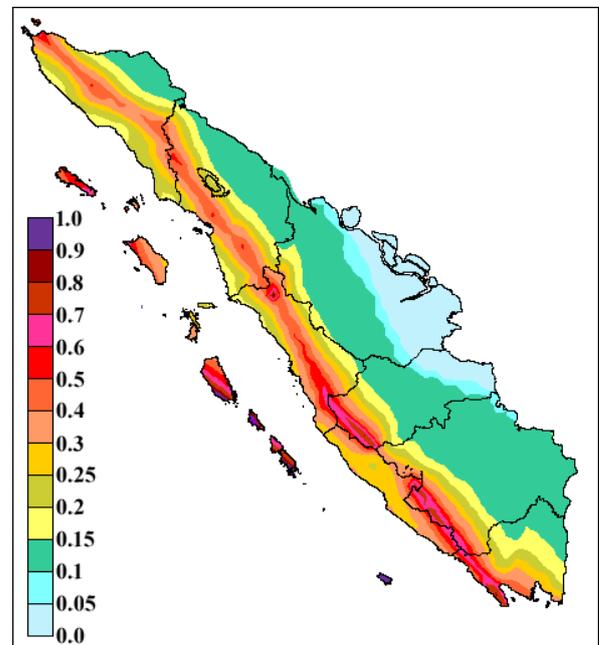


Figure 13. Seismic hazard map (PGA) of Sumatra with hazard level 10% probability of exceedance in 50 years, developed in this study (2012)

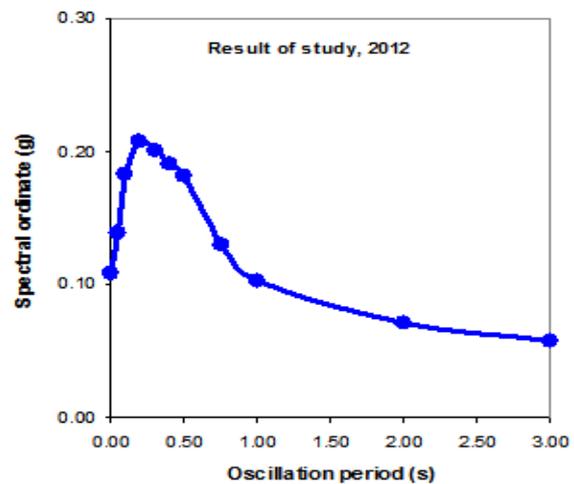


Figure 14. Uniform hazard spectrum of Surabaya with hazard level 10% probability of exceedance in 50 years, developed in this study

All of sources and sites, including seismic parameters used by two authors utilized to develop seismic hazard map of Sumatra and uniform hazard spectrum of Surabaya Indonesia respectively for verification purpose. Result of hazard calculation by the program of this study shown below in Figure 13 and Figure 14.

5. Comparison of Result

Comparison of the study results of Irsyam *et al.*, [13] and Aldiamar, [1] versus the study results is shown in Figure 12 and Figure 13.

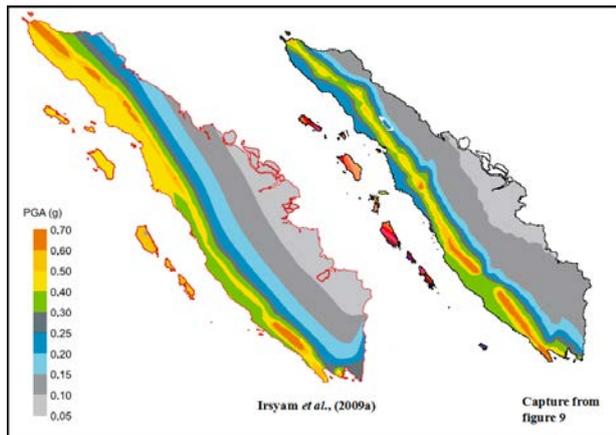


Figure 15. Comparison of PGA map of Sumatra with 10% probability in 50 years between Figure 9 and Figure 11

Based on those comparisons can be seen that the map of this study result very similar to that map of Irsyam *et al.* [13] study, and uniform hazard spectrum of this study result very similar to that uniform hazard spectrum of Aldiamar [1] study.

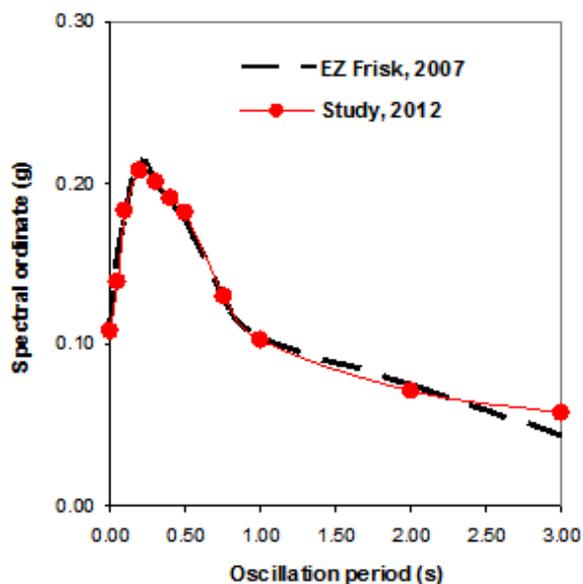


Figure 16. Comparison of uniform hazard spectrum of Surabaya between Figure 12 and Figure 14

6. Conclusion

Computer program has been developed to calculate probabilistic seismic hazard. The program has been used

to develop Sumatra seismic hazard map and Surabaya's Uniform Hazard Spectrum (UHS). The result map and UHS has been compared to other studies. Those comparisons shown the map and UHS obtained from this study very similar to both of other studies respectively. Therefore the computer program can be employed for probabilistic seismic hazard analysis

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