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On Evaluating the Volatility of Nigerian Gross Domestic Product Using Smooth Transition Autoregressive-GARCH (STAR - GARCH) Models

Akintunde Mutairu Oyewale*

Department of Statistics, Federal Polytechnic, Ede, Osun State, Nigeria
*Corresponding author: waleakintunde2004@gmail.com

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Abstract STAR-GARCH models are hybrid models that combine the functional form of smooth transition autoregressive models and Generalized autoregressive conditional heteroscedasticity models. The two classes of STAR models considered in this paper are Exponential and Logistic Smooth transition autoregressive models (ESTAR and LSTAR). The functional form of each of this was combined with that of GARCH model and the resulting models becomes ESTAR-GARCH and LSTAR-GARCH models. The derived equations were applied to Nigerian gross domestic product (Real estate) for empirical illustration. Statonarity tests (Unit root test Graphical and correlogrom methods) conducted revealed that the series was stationary at Second difference. The hybrid models equations so derived were used to determine the model that performed better using the information criteria (AIC, SIC and HQIC), variances obtained from the data, performance measure indices (RMSE, MAE, MAPE THEIL U, Bias proportion, variance Bias proportion and covariance Bias proportion) analysis and in - sample forecast accuracy for the models. From all the criteria used it was observed that the duo of LSTAR-GARCH and ESTAR-GARCH models performed far better than classical GARCH model. However, LSTAR-GARCH performs slightly better than ESTAR-GARCH. From these results it is evident that volatility in Nigerian gross domestic product (Real estate) is best captured using Logistic smooth transition GARCH (LSTAR-GARCH) models, it is therefore, recommended for would be forecasters, investors and other end users to make use of LSTAR-GARCH models.

Keywords: *GARCH model, STAR model, ESTAR- GARCH, LSTAR-GARCH, performance measure indices, volatility*

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1. Introduction

Nigeria is regarded as the largest economy in West Africa and is currently at per or has surpassed South Africa as the largest economy in Africa (previously regarded as the second largest economy in Africa), African Development Bank, [1] asserted that Nigerian economy is believed to be about 55 percent of West Africa's Gross domestic product so also Ignite, [2] said Nigeria economy accounted for 64 percent of GDP using purchasing power parity (PPP) indicators of the fifteen member countries in the ECOWAS sub-region. The country projected population is about 200 million people of highly industrious inhabitants. Osagie, [3] said Nigeria natural resources made up of over 80 million hectares of arable land, several solid minerals resources and abundant crude oil and gas reserves.

Nwachukwu V.O. [4] refers to the gross domestic product (GDP) asa measure of national income and output of a particular country's economy. The gross domestic product (GDP) is equivalent to the sum total of all expenditures for all final goods and services produced within the country's under study at a given period of time. In 2019, according to the World Bank official statistics, the Gross Domestic Product (GDP) of Nigeria was valued to be 448.10 billion US dollar, accidentally the above quoted value of GDP is equivalent to 0.37 percent of the total world economy.

Many financial and economic time series data in most cases are noisy, chaotic and volatile because of these behaviors the markets show marked breaks in their characteristics, in a way that the series changes markedly compared to what they exhibited initially. The changes that come along the features could be temporary (If the changes are for a while before returning to its original behaviour or switching to yet another style of behaviour, this is termed a regime shift or regime switch) but if such characteristics become permanent then the problem of structural break is then established. Such features are associated with events such as financial crises [5,6,7] or sudden changes in government policy [8,9,10]. Of interest to financial econometrician or time series practitioners is the seeming behaviour of many economic variables to change differently during economic recession and posterity [11,12]. These unexpected changes are common features of financial and economic data.

Smooth Transition Autoregressive (STAR) models are applied to time series data as an extension of autoregressive models in order to allow for higher degree of easiness in model parameters through a smooth transition. Also, STAR models are introduced, according to Terasvirta and Anderson [13] and Terasvirta [14], because of the existence of two distinct regimes with potentially different dynamic properties and because the transition between the regimes is smooth. STAR models allow economic variables to follow a given number of regimes with switches between regimes achieved in a smooth and continuous fashion and governed by the value of a particular variable or group of variables. The transition parameter denoted by $\{s_t, \gamma, c\}$ is a slope of parameter that determines the speed of transition between the two extreme regimes with low absolute values resulting in slower transition. It should be noted that $\{y_t, \gamma, c\}$ are generated by data series.

The model presented in this study is inspired by the paper published by Akintunde et.al. [15] in which STAR-GARCH models was used to forecast exchange rate data of Nigeria, Botswana, Britain and Japan using American dollar as a benchmark. The results were related to GARCH models.

2. Mathematical Specification

The GARCH model used for the study is represented by

$$y_t = \sigma_t \varepsilon_t + \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1} + \sum_{j=1}^{q} \beta_j \sigma_{t-j} \qquad (1)$$

The general STAR model can be represented as:

$$y_t = \Phi_1' x_t \left(1 - G\left(y_t, \gamma, c\right)\right) + \Phi_2' x_t G\left(y_t, \gamma, c\right) + \varepsilon_t \qquad (2)$$

where $x_t = \left(1, y_{t-1}, y_{t-2}, \ldots, y_{t-p}\right)$, $\left(\phi_{i0}, \phi_{i1}, \ldots, \phi_{ip},\right)\left(i = 1, 2\right)$, ε_t is the error term distributed independently and identically with mean zero and variance $\left(\sigma^2\right)$ one. $G\left(y_t, \gamma, c\right)$ is the transition function bounded between zero and unity (0,1).

The Logistic Smooth Transition (LSTAR) model is as follows:

$$\begin{aligned} y_t &= \Phi_1' y_{t-j} \left\{1 - \left(\left(1 + \exp - \left(\gamma\left(y_{t-j} - c\right)\right)\right)^{-1}\right\} \\ &+ \left\{\Phi_2' y_{t-j} \left(1 + \exp - \left(\gamma\left(y_{t-j} - c\right)\right)\right)^{-1}\right\} + \varepsilon_t \\ &= \Phi_1' y_{t-j} \left(1 - G_t^L\right) + \Phi_2' y_{t-j} G_t^L + \varepsilon_t \end{aligned} \qquad (3)$$

The Exponential Smooth Transition (ESTAR) model is as follows:

$$\begin{aligned} y_t &= \Phi_1' y_{t-j} \left(-\exp - \left(\gamma\left(y_{t-j} - c\right)^2\right)\right) \\ &+ \Phi_2' y_{t-j} \left(1 - \exp - \left(\gamma\left(y_{t-d} - c\right)^2\right)\right) + \varepsilon_t \\ &= \Phi_1' y_{t-j} \left(1 - G_t^E\right) + \Phi_2' y_{t-j} G_t^E + \varepsilon_t. \end{aligned} \qquad (4)$$

Based on the conditions stated above, the STAR model offers the possibility to investigate the presence of non-linearity in time series data which may account for the weakness of GARCH model mentioned in Chapter Four. Without loss of generality, we can strengthen the GARCH model with STAR models by adjusting the error terms.

The LSTAR-GARCH model is

$$y_t = \Phi_1' y_{t-j} \left(1 - G_t^L\right) + \Phi_2' y_{t-j} G_t^L + \sigma_t \varepsilon_t \qquad (5)$$

The ESTAR-GARCH model is

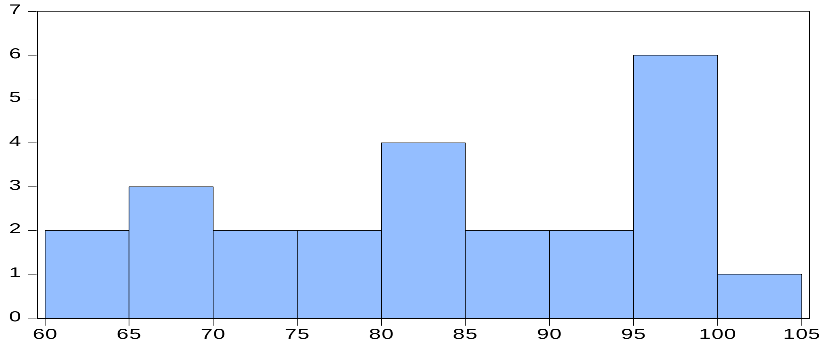
$$y_t = \Phi_1' y_{t-j} \left(1 - G_t^E\right) + \Phi_2' y_{t-j} G_t^E + \sigma_t \varepsilon_t \qquad (6)$$

Generally, the STAR-GARCH model used for the study is of the form

$$y_t = \Phi_1' y_{t-j} \left(1 - G_t^\tau\right) + \Phi_2' y_{t-j} G_t^\tau + \sigma_t \varepsilon_t \qquad (7)$$

3. Empirical Analysis with Nigerian Gross Domestic Product Data

Data was analysed using Econometrics view software and set of programme was written to accomplish the goal using monthly Nigerian Gross domestic product data from 1997 to 2019. The data was obtained from National Bureau of Statistics website. The results obtained from the analysis are shown below.



Series: GDP	
Sample 1997 2020	
Observations 24	
Mean	83.47813
Median	82.66667
Maximum	100.2583
Minimum	62.60000
Std. Dev.	12.89748
Skewness	-0.136838
Kurtosis	1.706790
Jarque-Bera	1.747291
Probability	0.417427

Figure 1. DESCRIPTIVE STATISTICS

Figure 1 above shows the distributional properties of the data used in the study. There is evidence of fluctuation in data as shown by the high standard deviation obtained. The distribution of the data is almost symmetric. The data is platykurtic in nature (1.7 < 3). The hypothesis of normality is rejected as evidence by Jarque-Bera test shown above. So before proceeding to the analysis of the data there is the need to make the data stationary.

Table 1. The GARCH model fitted for the series

SERIES	COEFFICIENT (S.E) α_0	α_1	β_1	VARIANCE
GDP	1.2314 (0.2451)	0.3210 (0.2117)	−0.5231 (0.0021)	9.7312

$$y_t = \sigma_t \varepsilon_t + \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}$$

$$y_t = \sigma_t \varepsilon_t + 1,2314 + 0.321 y_{t-1} - 05231 \sigma_{t-j}$$

The general form of our STAR-GARCH is

$$y_{t(S-G)} = \underline{\phi}_1{}' y_{t-j} \left(1 - G_t\right) + \underline{\phi}_2{}' y_{t-j} G_t + \sigma_t \varepsilon_t$$

ESTAR-GARCH

$$y_{t(ESTAR-GARCH)} = \sigma_t \varepsilon_t + \underline{\phi}_1' y_{t-1} \left(-\exp-\left(\gamma\left(y_{t-1} - c\right)^2\right)\right) + \underline{\phi}_2' y_{t-1} \left(1 - \exp-\left(\gamma\left(y_{t-1} - c\right)^2\right)\right)$$

$$y_{t(ESTAR-GARCH)} = \sigma_t \varepsilon_t + \underset{(0.0021)}{1.2319} \times y_{t-1} \left(1 - G_t\right) - \underset{(0.9141)}{3.2112} \times y_{t-1} \left(G_t\right)$$

with var. of 3.2435

LSTAR-GARCH

$$y_{t(LSTAR-GARCH)} = \sigma_t \varepsilon_t + \underline{\phi}_1' y_{t-1} \left(1 - \left(1 + \exp-\left(\gamma\left(y_{t-1} - c\right)\right)\right)^{-1}\right) + \underline{\phi}_2' y_{t-1} \left(1 + \exp-\left(\gamma\left(y_{t-1} - c\right)\right)\right)^{-1}$$

$$y_{t(LSTAR-GARCH)} = \sigma_t \varepsilon_t + \underset{(1.2451)}{4.4321} \times y_{t-1} \left(1 - G_t\right) + \underset{(0.0421)}{1.0021} \times y_{t-1} \left(G_t\right)$$

with var. 2.1311

FITTED VALUES FOR STAR-GARCH MODELS

Table 2. Fitted model for STAR-GARCH

MODE;	COEFFICIENT (SE) C(1)	C(2)	VARIANCE
ESTAR-GARCH	1.2314 (0.0021)	−3.2112 (0.9141)	3.2435
LSTAR-GARCH	4.4321 (1.2451)	0.0021 (0.0421)	2.1311

3.2. Empirical Comparison of the Models

The models under consideration are GARCH and ESTAR-GARCH and LSTAR-GARCH models. Table 3 below shows the variances of models as obtained from the analysis. The performance of ESTAR-GARCH and LSTAR-GARCH is comparable (3.2435 for ESTAR-GARCH and 2.1311 for LSTAR- GARCH), however the two models performed far better than classical GARCH model with variance 9.7312. This implies that to would be analyist, investors and other would-be users LSTAR model is recommended, this is closely followed by ESTAR-GARCH. The policy implication of this is that the users can make use of ESTAR-GARCH in the absence of LSTAR-GARCH, while the performance of classical is nothing to write home about.

Table 3. Variances of all models

MODEL	VARIANCE
GARCH	9.7312
ESTAR- GARCH	3.2435
LSTAR- GARCH	* 2.1311

From Table 3 above, the variances obtained for the three models were considered. LSTAR-GARCH produced the least variance, making it the best variance wise closely followed by ESTAR-GARCH and GARCH models in that order. Variance is a measure of error, and the model that has the least is preferred over and above the model(s) with higher error.

Table 4. Information criteria of all models

MODEL	**INFORMATION CRITERIA** **AIC**	**SIC**	**HQIC**
GARCH	3.5311	3.9284	3.5328
ESTAR- GARCH	3.3490	3.4477	* 3.3738
LSTAR- GARCH	* 3.3146	* 3.4139	3.3812

Table 4 above revealed the performance of the three models on the basis of information criteria used. Considering the first column which is Akaike information criteria (AIC), LSTAR-GARCH produced the best information closely followed by ESTAR-GARCH. Column 2 is the column of Schwarz information criteria, here also LSTAR-GARCH produced the best results followed by ESTAR-GARCH and GARCH models in that order. The last column is the column of Hannan-Quinn information criteria, here ESTAR-GARCH had the best results followed by LSTAR-GARCH and GARCH models in that order. In all the three criteria used LSTAR-GARCH performed best in two (AIC and SIC) while ESTAR-GARCH model performed best in one (HQIC). It should be noted that the asterisk sign used here shows the column by column of the model that performed best using information criteria.

Table 5. Performance measure indices

MODEL	RMSE	MAE	MAPE	THEIL U	BIAS PROP	VAR.PROP	COV PROP
GARCH	106967.1	1378259.5	93256.7	0.0833	0.0045	0.9824	0.00032
ESTAR- GARCH	106832.1	1257963.4	92189.7	* 0.0721	0.0032	0.9871	* 0.00030
LSTAR- GARCH	* 106820.4	* 1257984	* 92186.1	0.0732	* 0.0022	* 0.9875	0.00031

Table 5 is a table of performance measure indices, in all seven criteria were used out of which LSTAR-GARCH models is best in five, ESTAR-GARCH is best in two. The asterisk is used to denote the best performance measure indices per column as shown above. it could be remarked clearly that if the performance measure indices as criteria were used there is no gainsaying in the fact that LSTAR-GARCH is best and it is followed by ESTAR-GARCH.

3.3. Forecast Efficacy

The in-samples forecast performance for STAR-GARCH models (lower and upper forecast limits) for all models under show that ESTAR-GARCH and LSTAR-GARCH models forecast excellently well as shown in the Table 7 and Table 8 below. It is observed that the actual values of data obtained from record are at par with the upper forecast values. The implication of this is that, its performance asserts its superiority LSTAR-GARCH over and above GARCH, and ESTAR-GARCH models, as could be seen from 6 through 8. However, the performance of ESTAR-GARCH is very close to that of LSTAR-GARCH.

Table 6. In -samples forecast performance for GARCH

Date	Actual	Lower forecast limit	upper forecast limit
2019 (1)	99.8	78.2491	91.7196
2019 (2)	99.9	82.6285	95.7506
2019 (3)	100	84.0997	96.1532
2019 (4)	100.1	84.2341	96.2532
2019 (5)	100.3	84.4432	96.4321
2019 (6)	100.2	84.2721	96.3211
2019 (7)	100.2	84.2721	96.3211
2019 (8)	100.3	84.4432	96.4321
2019 (9)	100.4	84.4549	96.4539
2019 (10)	100.6	84.7632	96.6432
2019 (11)	100.5	84.6932	96.5575
2019 (12)	100.8	84.9932	96.6998

Table 7. In -samples forecast performance for ESTAR-GARCH

Date	Actual	Lower forecast limit	upper forecast limit
2019 (1)	99.8	87.0828	98.9302
2019 (2)	99.9	87.9928	98.4212
2019 (3)	100	88. 5998	99.2138
2019 (4)	100.1	88.79086	99.6010
2019 (5)	100.3	89.8348	99.6054
2019 (6)	100.2	89.7898	99.5220
2019 (7)	100.2	89.7898	99.5220
2019 (8)	100.3	89.8348	99.6054
2019 (9)	100.4	89.9822	99.6401
2019 (10)	100.6	92.4388	99.6745
2019 (11)	100.5	92.2028	99.6258
2019 (12)	100.8	93.3278	99.9491

Table 8. In-samples forecast performance for LSTAR-GARCH

Date	Actual	Lower forecast limit	upper forecast limit
2019 (1)	99.8	96.6428	98.9734
2019 (2)	99.9	96.8456	99.6773
2019 (3)	100	97.2314	99.7532
2019 (4)	100.1	97.3219	99.7653
2019 (5)	100.3	97.7483	99.8796
2019 (6)	100.2	97.5646	99.7653
2019 (7)	100.2	97.5646	99.7653
2019 (8)	100.3	97.7483	99.8796
2019 (9)	100.4	97.8010	99.8671
2019 (10)	100.6	97.9674	99.9001
2019 (11)	100.5	97.8996	99.8753
2019 (12)	100.8	98.1011	100.6872

4. Conclusion

From Table 5 through Table 7, it is clear that the STAR-GARCH models are better than the classical GARCH model for all models in terms of their variances compared to classical GARCH models. However, within the group of STAR-GARCH, LSTAR-GARCH performed excellently better than others. This is closely followed by ESTAR-GARCH and LSTAR-GARCH in that order. Also, the forecast efficacy of STAR-GARCH model could not be compared with classical GARCH, as its forecast is excellent and could be compared to the original data.

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