

# Why Segregating Cointegration Test?

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**Abstract** This paper resolves the conflicts that exist between various cointegration tests for cases when different tests for cointegration provide different answers under the same data set. The tests considered are, Augmented Dickey Fuller (ADF) test, Hansen  $L_c$  test, Johansen's test, and Stock and Watson (SW) test. The Monte Carlo experiments conducted show that the Stock Watson and Johansen tests can be grouped together while ADF test significantly shows different performances from that of Hansen  $L_c$  test.

**Keywords:** unit root, cointegration

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## 1. Introduction

The concepts of cointegration are being widely used in empirical studies in various areas of economics in order to analyze relationships among non-stationary time series. The cointegration hypothesis states that: among variables that are individually integrated of order one  $I(1)$ , at least one linear combination of the variables exists that is stationary or integrated of order zero  $I(0)$ .

Since Granger (1981) introduced the concept of cointegration, a number of authors have suggested various tests to determine the number of cointegrating vectors in a system of  $I(1)$  variables. Engle and Granger [1] suggested an Augmented Dickey Fuller (ADF) test on ordinary least squares (OLS) residuals from regression of one variable upon the others. The asymptotic theory for this OLS "residual-based" approach has been developed by Engle and Yoo [2], Phillips and Ouliaris [3] and Hansen [4]. A likelihood ratio (LR) test using full information maximum likelihood estimation (MLE) was proposed and developed by Johansen [5,6]. Alternative test have been proposed by Phillips and Solo [7], Stock and Watson [8], Phillips and Ouliaris [3], and Park and Phillips [9] among others.

Gonzalo [10] and Stock and Watson [8], among others addressed the issue of efficient estimation of cointegrating vectors with Monte Carlo method. Gonzalo recommended estimating the cointegrating vectors with Johansen's method. However, Phillips et al [3] derived exact finite-sample distributions of maximum likelihood estimators of cointegrating vectors and showed how Johansen's procedure may produce unreliable estimates. Stock and Watson [8] gave an empirical example. Gregory (1990) compared the performance of two cointegrating tests in a specific linear quadratic model. Stock [11] conducted a limited Monte Carlo study of some cointegrating tests. Also, Kremers et al [12] demonstrated theoretically and with Monte Carlo method cases for which ADF test may

have extremely low power, and Egn Zakrgsek [13], Boswik and Frances [14] reported a few Monte Carlo results for weakly independent regressors and two parameters settings for the ADF lamda max, and a Wald-type test.

Despite the large number of papers, the theory is not yet fully developed. Applied researchers are given little or no guidance in the literature as to which test to apply. The large number of tests, each with their own non-standard distribution, is confusing to many applied researchers. Hence this paper addresses this problem by segregating the test choices as a basic example.

The rest of the paper is organized as follows: section 2 outlines the theoretical distribution of the tests and the modelling assumptions. In section 3, we present the result using Monte Carlo. In section 4, we give the conclusion remarks. The appendix contains the tables of critical values for the test statistics.

## 2. Data Generating Process

### 2.1. Theoretical Distribution of the Tests

#### 2.1.1. Engle and Granger's Augmented Dickey – Fuller Test

One of the most commonly used cointegration tests is Engle and Granger's augmented Dickey-Fuller (ADF) test. In statistics, an augmented Dickey-Fuller test is a test for a unit root in a time series sample. It is a version of the Dickey-Fuller test for a larger and more complicated set of time series models. The augmented Dickey-Fuller (ADF) statistic, used in the test, is a negative number. The more negative it is, the stronger the rejections of the hypothesis that has a unit root at some level of confidence.

It is assumed that the variables  $y_t$  and  $x_{it}$ ,  $i = 1, \dots, m$ , are individually  $I(1)$ . The null hypothesis is that at least one linear combination is stationary or  $I(0)$ . The variables are then cointegrated under the alternative hypothesis. The

ADF test involves the ordinary least squares (OLS) residuals  $u_t$  from the cointegration regression.

$$y_t = \beta_0 + \sum_{i=1}^m \beta_i x_{it} + u_t, t = 1, \dots, T. \tag{1}$$

Under the alternative hypothesis of cointegration,  $\mu_t$  is integrated of order zero and  $\mu_t$  is hence stationary. The next step is to apply OLS to equation 1 get

$$\Delta u_t = \alpha u_{t-1} + \psi_t \mu_t + v_t. \tag{2}$$

and  $n$  should be chosen, so that, the residuals are serially uncorrelated. In order to choose the lag length  $n$  in the Monte Carlo study, a data-dependent lag selection criterion is applied: Akaike's Information Criterion (AIC). This is a commonly used criterion in empirical research in connection with the ADF test.

**2.1.2. Stock and Watson's SW test**

The  $p$  ( $=m + 1$ )- dimensional vector  $X_t$  is defined to contain the variables  $y_t$  and  $x_{it}$ . The Stock-Watson method allows, unlike non-system-based tests, to determine how many cointegrating vectors exist in the vector autoregressive system

$$X_t = \Psi X_{t-1} + \zeta_t. \tag{3}$$

The covariance matrix estimate is given by

$$\hat{\Omega}_\zeta = \sum_{s=1}^T w(s/\hat{M}) \sum_{t=s+1}^T \zeta_t \zeta_{t-s}' \tag{4}$$

using again the quadratic spectral kernel. In comparison to the single-equation model, the systems approach requires a first-order autoregressive coefficient estimate for each element of the vector  $\zeta_t$  which is  $(p \times 1)$ , where  $\Psi_t = X_t - T^{-1} \sum X_t$ . Under the null hypothesis of no cointegration,  $SW = T(\lambda_{\min} - 1)$  should not be significantly different from zero. The asymptotic distribution of the test statistic depends on  $p$  and critical values are given in Stock and Watson (1994).

**2.1.3. Johansen's  $\lambda_{\min}$  Test**

Looking for cointegration between variables is currently widespread in empirical economics, namely: to find relationships among non-stationary variables, to test for convergence, to look at causality among variables, etc. In such research, the Johansen technique has been accepted as a powerful way to test for cointegration – justified by the works of Phillips [15] and Gonzalo [10], among others. Nevertheless, this technique can sometimes produce results that appear to be counter-intuitive. One of these outcomes is related to the transitivity property.

Johansen [5,16] and Johansen and Juselius [17] developed two test to determine the number of cointegrating vectors in the  $p$ -dimensional vector  $X_t$ :

$$X_t = \Pi_1 X_{t-1} + \dots + \Pi_k X_{t-k} + \varphi + v_t, \tag{5}$$

where  $v_t$  represents vector of i.i.d normal errors. This equation may be expressed in error correction form:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_k \Delta X_{t-k+1} + \Pi X_{t-k} \varphi + v_t \tag{6}$$

with

$$\Gamma_i = -(1 - \Pi_1 - \dots - \Pi_i), i = 1, 2, \dots, k-1 \tag{7}$$

and

$$\Pi = -(1 - \Pi_1 - \dots - \Pi_k). \tag{8}$$

The matrix  $\Pi$  gives information about a possible cointegrating vector among the variables in  $X_t$ . The number of cointegrating vectors, are identical to the number of stationary relationships in the  $\Pi$ -matrix. The rank of  $\Pi$  matrix determines the number independent rows in  $\Pi$ , and therefore also the number of cointegrating vectors. The rank of  $\Pi$  is given by the number of significant eigenvalues found in  $\hat{\Pi}$ . Each significant eigenvalue represent a stationary relation. Under the null hypothesis of  $\{x_t\} \sim I(d)$ , with  $d > 1$ , the test statistics for determine the significance of the eigenvalues is non-standard and hence must be simulated.

If  $\Pi$  has reduced rank, there are co-integrating relations among the  $x$ 's. Thus  $\text{rank}(\Pi) = 0$ , implies that all  $x$ 's are non-stationary. There is no combination of variables that lead to stationarity, if  $\text{rank}(\Pi) = p$ , so  $\Pi$  has full rank, then all variables in  $x_t$  must be stationary.

If  $\text{rank} \Pi$  has full rank and then,  $X_t$  is stationary. If  $\text{rank} \Pi = 0$ , then the model is the traditional first-differenced vector autoregression. But if  $0 < \text{rank} \Pi = r < p$ , then  $\Gamma_i = \Pi_i X_t$  where  $\Gamma$  is a  $p \times r$  matrix of error correction vectors and  $\Pi$  is a  $p \times r$  matrix of cointegrating vectors so that  $\Pi_i X_t$  is stationary even though  $X_t$  itself is nonstationary.

If  $\Pi$  has reduced rank,  $0 < r < p$ , the cointegrating vectors are given as  $\Pi = \alpha \beta'$  where  $\beta_i$  represents the  $i$ :th co-integrating vector, and  $\alpha_j$  represents the effect of each co-integrating vector on the  $\Delta x_{p,t}$  variables in the model.

Once the rank of  $\Pi$  is determined and imposed on the model, the model will consists of stationary variables or expression, and estimated parameters follows standard distributions.

The next step is to compute the residuals  $R_{kt}$  and  $R_{ot}$  from OLS regressions o  $[X_{2t-k}]'$  on  $X_{t-1}$ , on the same set of regressors. The product moment matrices are then given by

$$S_{ij} = T^{-1} \sum_{i=1}^T R_{it} R_{jt}', \tag{9}$$

where  $I, j = 0, k$  and the eigenvalues  $\hat{\lambda}_1 > \dots > \hat{\lambda}_p$  are the solutions from

$$\left| \lambda S_{kk} - S_{ko} S_{00}^{-1} S_{0k} \right| = 0. \tag{10}$$

The likelihood ratio test statistic (trace test) for the hypothesis that there are at most  $r$  cointegrating vectors is given by

$$\lambda_{tr} = -T \sum_{i=r+1}^p \ln(1 - \lambda_i). \tag{11}$$

Johansen and Juselius suggested an additional, possibly more powerful test  $\lambda_{\max}$  (maximum eigenvalue test):

$$\lambda_{\max} = -T \ln(1 - \hat{\lambda}_r). \tag{12}$$

For testing the null hypothesis of r-1 cointegrating vectors against the alternative of r cointegrating vectors. Both tests have nonstandard distributions, which are functionals of Wiener processes and are generalizations of scalar Dickey-Fuller Wiener processes. Critical values for both tests are tabulated in Johansen and Juselius. The  $\lambda_{\max}$  test is directly comparable to the other tests when setting  $r = 1$  and therefore had been use in the Monte Carlo study this test.

Where  $\hat{\lambda}_i$  denotes the estimated values of the characteristic roots obtained from the estimated  $\pi$ , and T is the number of observations.

The first statistics tests the null that the number of distinct cointegrating vectors is equal to or less than r, against a general alternative.  $\lambda_{trace} = 0$  when all  $\lambda_i = 0$ . The further the estimated characteristic roots are from zero, the more negative is  $\ln(1 - \lambda_i)$ , and the greater is  $\lambda_{trace}$ . The second tests, the null that the number of cointegrating vector is r, against the alternative of r+1. Critical values for both of these have been calculated by Johansen & Juselius [17]. One of the most interesting aspect of the Johansen procedure is that it allows for testing restricted forms of the cointegrating vector.

**2.1.4. Hansen’s L<sub>c</sub> Test**

The Hansen method introduced in Hansen [4] is a generalization of the semiparametric method for a model with changing parameters.

$$y_t = \beta_t x_t + \mu_{1t} \tag{13}$$

$$x_t = [x_{1t}, x_{2t}] \tag{14}$$

$$x_{1t} = k_{1t} \tag{15}$$

$$x_{2t} = \Pi_1 k_{1t} + \Pi_2 k_{2t} + x_{2t}^0 \tag{16}$$

$$x_{2t}^0 = x_{2t-1}^0 + \mu_{2t} \tag{17}$$

where  $\beta_t$  is a vector of time-dependent parameters,  $\Pi_1$  and  $\Pi_2$  are matrices of (constant) parameters  $k_{1t}$  and  $k_{2t}$ .  $k_{1t}$  is a deterministic polynomial, contains a constant and a linear trend.  $k_{2t}$  is also a polynomial in t ( but it does not appear in the regression  $x_{2t}$  ).

The long run covariance matrix  $\Omega$  is defined by

$$\Omega = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^T E(\mu_j \mu_t'), \tag{18}$$

and then  $\Lambda$  matrix, which represents bias due to endogeneity of  $x_t$ , is defined as

$$\Lambda = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^T E(\mu_j \mu_t') \tag{19}$$

where  $[\mu_{1t}' \ \mu_{2t}']'$ .

Matrices  $\Omega$  and  $\Lambda$  are nuisance parameter matrices.  $\Omega$  is approximately equal to the long-run variance of  $\mu_{1t}$  conditional on  $\mu_{2t}$ . For a weakly stationary innovations vector  $\mu_t$  is proportional to the spectral density matrix evaluated at zero frequency.

The equation (11) estimated with use of OLS gives estimate of the parameter  $\hat{\beta}_t$  and OLS residual  $\hat{\mu}_{1t}$ . From equation (13), we obtain OLS residual  $\hat{\mu}_{2t}$ .

Let

$$\hat{\mu}_t = \begin{bmatrix} \hat{\mu}_{1t} \\ \hat{\mu}_{2t} \end{bmatrix}. \tag{20}$$

The matrices  $\Omega$  and  $\Lambda$  are estimated with use of spectral density method applied to the series  $\hat{\mu}_t$ .

Hansen [4] suggests that when serial correlation is present, then the approach of Andrews and Monahan (1992) should be adopted, namely: first  $\hat{\mu}_t$  should be prewhitened, after estimation of VAR

$$\hat{\mu}_t = \rho \hat{\mu}_{t-1} + \hat{e}_t. \tag{21}$$

The residual  $\hat{v}_t$  are used in subsequent steps of the estimation of  $\Omega$  and  $\Lambda$ .

The next step of the Hansen method is based on spectral density estimation of the residuals, and estimation of  $\Omega$  and  $\Lambda$  matrices. The covariance matrices  $\Omega$  and  $\Lambda$  could be estimated directly from the residual  $\hat{\mu}_t$  via a kernel. In most applications, the cointegration residual  $\hat{\mu}_{1t}$  has a significant degree of serial correlation. In this event, the kernel estimate will be highly biased, unless a large bandwidth parameter is used, which increases the variance of the estimator. In such cases, an estimator based on prewhitening is often preferable in moderate sample sizes. VAR (vector autoregressive), a higher order can be used. One first fit a VAR to the residual  $\hat{\mu}_t = \rho \hat{\mu}_{t-1} + \hat{e}_t$ . A kernel estimator is the applied to the whitened residual  $\hat{e}_t$ . These take the form

$$\hat{\Lambda}_e = \sum_{j=0}^T w(j/M) \frac{1}{T} \sum_{t=j+1}^T \hat{e}_{t-j} \hat{e}_t' \tag{22}$$

and

$$\hat{\Omega}_e = \sum_{j=-t}^T w(j/M) \frac{1}{T} \sum_{t=j+1}^T \hat{e}_{t-j} \hat{e}_t' \tag{23}$$

where  $w(\cdot)$  is a weight function (or kernel) that yields positive semi-definite estimates and M is a bandwidth parameter. The estimator  $\hat{\Omega}_e$  can be seen as a scaled estimate of the spectral density of  $e_t$  (when  $e_t$  is covariance stationary) and has its origin in the literature on spectral density estimation.

**2.2. Data Generating Process**

The Monte Carlo study is based on the modified data generating process of Dolado et al [18], David Bernstein

and Bent Nielson [19], which has been used in several papers in the literature. The canonical form of the process allows us to see the dependence of the test performance on some key parameters.

Let  $y_{it}$  be a  $p$ -dimensional vector, where  $i$  is the index for the cross-section,  $t$  is the index for the time dimension and  $p$  denotes the number of variables in the model. The data generating process has the form of a VAR(1) process. The general form of the modified Dolado process for a system of three variables in the absence of a linear trend in the data is,

$$y_{it} = \begin{pmatrix} \phi_a & 0 & 0 \\ 0 & \phi_b & 0 \\ 0 & 0 & \phi_c \end{pmatrix} y_{it} + \varepsilon_{it}, \quad \begin{matrix} t = 1, \dots, T \\ i = 1, \dots, N \end{matrix} \quad (24)$$

where the initial values of  $y_{it}$  which can be represented as  $y_{it}$  are zeros. The error terms for each cross-section has the following structure:

$$\varepsilon_{i,t} = \begin{pmatrix} \varepsilon_{1it} \\ \varepsilon_{2it} \end{pmatrix} = iid \left[ \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \eta\sigma \\ \eta\sigma & \sigma^2 \end{bmatrix} \right]. \quad (25)$$

The true cointegrating rank of the process is denoted by  $r$  and  $\varepsilon_{1it}, \varepsilon_{2it}$  are the disturbances to the stationary and non-stationary parts of the data generating process, respectively.  $\varepsilon$  represents the vector of instantaneous correlation between the stationary and non-stationary components of the relevant cross-section.

Taking into account (equation 1), when  $\phi_a = \phi_b = \phi_c = 1$ , a cointegrating rank of  $r = 0$  is obtained. Thus, the data generating process becomes,

$$y_{it} = \sigma_3 y_{it-1} + \varepsilon_{it}, \quad (26)$$

where  $\varepsilon_{it} \sim iid N(0, I_3)$ , which means that the process consists of three non-stationary components and these components are instantaneously uncorrelated. The VEC representation of (eq2) is:

$$\Delta y_t = \Pi_{it} y_{it-1} + \varepsilon_{it}. \quad (27)$$

Here,  $\Pi_{it} = (I_3 - A_{i1})$  and  $A_{i1} = I_2$  represents the

coefficient matrix of the VAR(1). As  $\Pi_{it}$  is a null matrix, (3) turns into:  $\Delta y_t = \varepsilon_{it}$ .

### 2.3. Derivation of Model

Generating the data  $X_t$  by a random walk without a drift:

$$X_t = X_{t-1} + \eta_t, \quad \eta_t \sim N(0,1) \quad (28)$$

$Y_t$  is defined as

$$Y_t = X_t + V_t, \quad (29)$$

where  $V_t$  is an AR(1) process,

$$V_t = \rho V_{t-1} + w_t, \quad w_t \sim N(0, \sigma_w^2) \quad (30)$$

$X_t$  and  $Y_t$  are cointegrated if  $\rho < 1$ , and are not cointegrated if  $\rho = 1$ .

Consider the models in equations (1),(3),(5), and (11), therefore, the bivariate case of the general model is given by

$$\begin{aligned} y_t - x_{1t} &= v_t \\ a_1 y_t - a_2 x_{1t} &= \psi_t \\ V_t &= \rho v_{t-1} + w_t \\ \psi_t &= \psi_{t-1} + \rho_t \\ \rho_t &= \phi_t + \theta \phi_{t-1} \end{aligned} \quad (31)$$

$$\begin{bmatrix} w_t \\ \phi_t \end{bmatrix} = iid \begin{bmatrix} 1 & \eta\sigma \\ \eta\sigma & \sigma_{\phi}^2 \end{bmatrix}.$$

### 3.1. Simulation Results

The simulation results of the experiments and selected tests for cointegration equation are described below. The choice of values for  $\rho$  and  $\theta$  is motivated by earlier studies conducted by Hansen and Phillips [20]. All results reported below are for a 5% and 1% significance level. The results reported in Table 1 to Table 3 are based on explanatory variable  $x$  for (for the above DGP with  $m = 1, a = 1$ ).

Table 1. Power of 5% level test with the null hypothesis of no cointegration when ( $\rho = 0.85$ )

	$\sigma = 0.25$																
$\eta$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
ADF	0.024	0.035	0.028	0.029	0.034	0.035	0.052	0.054	0.068	0.062	0.063	0.064	0.062	0.061	0.067	0.066	0.065
SW	0.259	0.324	0.331	0.338	0.345	0.359	0.368	0.374	0.388	0.386	0.366	0.349	0.341	0.337	0.332	0.315	0.302
$\lambda_{max}$ (SC)	0.511	0.432	0.441	0.422	0.384	0.376	0.372	0.371	0.176	0.174	0.175	0.184	0.245	0.251	0.278	0.293	0.341
$L_c$	0.046	0.048	0.052	0.053	0.054	0.054	0.057	0.058	0.061	0.058	0.060	0.059	0.057	0.032	0.031	0.030	0.029

Table 2. Power of 5% level test with the null hypothesis of no cointegration when ( $\rho = 0.85$ )

	$\sigma = 1$																
$\eta$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
ADF	0.136	0.138	0.146	0.156	0.158	0.169	0.184	0.212	0.284	0.263	0.301	0.304	0.305	0.324	0.324	0.425	0.462
SW	0.258	0.289	0.312	0.339	0.343	0.362	0.369	0.374	0.385	0.366	0.354	0.346	0.341	0.337	0.325	0.318	0.309
$\lambda_{max}$ (SC)	0.335	0.328	0.322	0.311	0.304	0.286	0.265	0.221	0.176	0.182	0.194	0.201	0.213	0.229	0.241	0.253	0.256
$L_c$	0.058	0.059	0.061	0.063	0.065	0.066	0.068	0.071	0.073	0.071	0.069	0.064	0.061	0.059	0.054	0.052	0.042

**Table 3. Power of 5% level test with the null hypothesis of no cointegration when ( $\rho = 0.85$ )**

$\eta$	$\sigma = 3$																
	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
ADF	0.174	0.179	0.187	0.198	0.213	0.245	0.268	0.273	0.287	0.284	0.281	0.275	0.269	0.264	0.254	0.248	0.243
SW	0.224	0.236	0.251	0.267	0.277	0.285	0.301	0.308	0.311	0.282	0.281	0.263	0.261	0.252	0.248	0.235	0.231
$\lambda_{\max}$ (SC)	0.242	0.226	0.214	0.205	0.195	0.187	0.154	0.122	0.111	0.146	0.158	0.185	0.201	0.235	0.244	0.269	0.285
$L_c$	0.038	0.039	0.040	0.042	0.043	0.045	0.046	0.047	0.048	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038

The relative performance of each test and focus are discussed first, for the ADF test, and for the SW test. Then, the effect of choosing different lag lengths for the ADF test and the  $\lambda_{\max}$  test follows. In Table 1, when  $\rho$  equals 0.85, and the equivalent  $\sigma$  is 0.25, the following results were obtained. ADF recorded highest value of 0.068 when  $\eta$  is 0 and lowest value of 0.024 when  $\eta$  is  $-0.8$ . SW has highest value of 0.388 when  $\eta$  is  $-0.1$  and lowest value of 0.259 when  $\eta$  is  $-0.8$ .  $\lambda_{\max}$  (SC) has highest value of 0.511 when  $\eta$  is  $-0.8$  and lowest value of 0.175 when  $\eta$  is 0.2.  $\hat{P}_u$  and  $\hat{P}_z$  have 0.036 and 0.388 as their highest values when  $\eta$  is  $-0.1$  and 0.8 respectively. The lowest values are 0.015 and 0.320 when  $\eta$  is  $-0.8$  for the two. Hansen  $L_c$  has highest value of 0.060 when  $\eta$  is zero and lowest value 0.029 when  $\eta$  is  $-0.8$ .

In summary, for  $\sigma = 0.25$  in Table 1, the power of all tests is relatively low and the null hypothesis of ‘no cointegration’ is not often enough rejected. The  $L_c$  test and ADF test have the lowest power and, on the hand, the SW test and  $\lambda_{\max}$  test have the highest powers. The power of the residual based tests, except for the  $L_c$  test is substantially lower than the power of the systems-based tests SW. When  $\eta$  deviates from zero, the power of all tests decreases (especially for negative values of  $\eta$  except for the (SW) test that shows an increase in power. Hence Johansen and SW can be grouped together while ADF performances significantly differs from  $L_c$ .

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