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Forecast of Sarima Models: An Application to Unemployment Rates of Greece

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Abstract The low unemployment rate is one of the main targets of macroeconomic policy for each government. Forecasting unemployment rate is of great importance for each country so as the government can draw up strategies for fiscal policy. The aim of the paper is to find the most suitable model which is adjusted on unemployment rates of Greece using Box-Jenkins methodology and to examine the precision of forecasting on this model. Models' estimation was made using the non-linear Maximum likelihood optimization methodology (maximum likelihood–ML), whereas covariance matrix is estimated with OPG method using the numerical optimization of Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. Forecasting unemployment rate was made both with dynamic and static process using all criteria of forecasting measures.

Keywords: *unemployment, SARIMA, Box-Jenkins methodology, forecasting, Greece*

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1. Introduction

Every economy has a particular population size. Population is distinguished between economically active and non active for economic reasons. Economically active population is the labor force of economy. Thus, labor force considers those that can and want to work. Labor force is divided in two categories: those who already work and called employed and those who don't work and are called unemployed. Unemployed are those who can and want to work but cannot find a job.

Unemployment has three basic economic consequences:

- It is regarded as a loss of productive powers
- It means wage loss
- Puts a strain on government's budget due to unemployment benefits.

The consequences of unemployment are certainly wider because not only it reduces wages but also decreases social position, creates self-respect problems and family matters. In other words, unemployment leads to serious social problems apart from economic ones.

The opposition of unemployment is extremely difficult. The measures taken from various governments are divided in two general categories:

1) The measures taken for the increase of total demand and
2) The measures for professional training and re-education on labor force.

The measures for total demand are fiscal and monetary. Fiscal measures consist of the increase of government expenditure for public works and the promotion for investments. The aim of these works is the direct growth of employment and wages. Monetary measures aim at interest rate reduction in order to strengthen private investments, production and therefore employment. Fiscal and monetary measures aim at the increase of total demand, thus unemployment decrease which is due to insufficient demand. The insufficient demand is the Keynesian unemployment which comes from the fall of economic activity during the recession stage of economic cycle.

Measures of professional formation and re-education facilitate unemployed in the acquisition of professional knowledge and specialization which are necessary in order to be occupied in existing vacancies. It is obvious that these measures aim at the reduction of structural unemployment, created by the disproportion between supply and demand of various specialization. This reduction demands re-education of unemployed in order to acquire the necessary skills where there is scarcity.

The measure of unemployment depends on the size of labor force. So, unemployment is measured as percentage (%) of labor force. The unemployment rate is:

$$Unemployment\ rate = \frac{Number\ of\ Unemployed}{Labor\ Force} * 100$$

The progress of unemployment rate has been the central issue of political discussion for many developed countries. However, the behavior of unemployment rate during recession and recovery that followed puzzled researchers and policy makers if there is a change on long run trend of unemployment rate. Given this new situation, policy focuses on the dynamics of unemployment rate after the recession so that they can forecast unemployment rate. Forecasting unemployment rate came to the front from policy makers using autoregressive models as well as structural econometric forecasting models.

Greek economy, reached high rates of growth until 2008 but on 2009 there was a downturn, as a result of the international financial crisis whereas from 2010 and afterwards the downturn grew worse due to fiscal imbalances. The need of reform led the country to a mechanism of economic support which consisted of European Union, International Monetary Fund and European Central Bank. Strict income policy and drastic constraints on public expenses, during the last five years, affected negatively GDP's progress. As a consequence, GDP reduced by 5,4% on 2010, by 8,9% on 2011, by 6,6% on 2012 and by 3,9% on 2013 (constant prices 2010).

Until 2008, unemployment in Greece was relatively low and moved at about 7,8% on average in Eurozone. On 2009, unemployment in Greece increased as a result of international crisis and reached 9,6% while for 2010 it increased further on 12,7% as a result of restrictive fiscal policy that implemented because of debt crisis. During 2011, unemployment rate reached 17,9%, as a consequence of the crisis on Greek economy and the measures taken for fiscal smoothing, while during 2012 exceeded 24% and during 2013 was 27,5%. During 2014 it was noted, for the first time, a slight decrease, even if unemployment remained on high levels about 26,5%. Finally, in September 2015, unemployment reduced and reached 24,6% according to the Hellenic Statistical Authority.

The aim of this paper is to construct the most suitable model in order to investigate and forecast unemployment rates. For this reason the SARIMA models and Box-Jenkins methodology were used, while the forecasting of the models is examined both on dynamic and static process, employing all the criteria forecasting measures. The rest of the paper is organized as follows: the second chapter presents literature review. Following, the Box-Jenkins methodology is provided. On chapter four data and empirical results are presented and on the last chapter the conclusions of the paper are given.

2. Review of Literature

[2] created a family of models known as AutoRegressive Integrated Moving Average (ARIMA) models. These models are applicable to a wide variety of situations. Box and Jenkins have also developed a practical process for the selection of the most suitable ARIMA model out of this family of ARIMA models. Many researchers claim that the creation of an ARIMA model needs judgement and experience.

ARIMA models are suitable for short run forecasts. This is due to the fact that ARIMA models give more emphasis on the recent past rather than distant past. According to [26], long run forecasts on ARIMA models are less reliable than short run. Seasonal AutoRegressive Integrated Moving Average (SARIMA) model is an expansion of simple ARIMA models and contains seasonal and non-seasonal data. SARIMA models have been applied for inflation forecasting ([13,16,21]), for exchange rate forecasting ([10,11]), for tourist arrivals and revenues forecasting ([4,31]) as well as unemployment forecasting.

Few papers related with unemployment forecasting and Box-Jenkins methodology or ARIMA and SARIMA models have been published. Some of them are the following:

[15] examines the unemployment in Germany using monthly data from January 1965 until November 1989. He uses both ARIMA and VAR models. The comparison of the results for the forecasting of the two examined models shows the advantages and disadvantages of the two methods.

[8] use monthly data for Romania for the period 1998-2007. On their paper, employing Box-Jenkins methodology, they present that the most suitable model is ARIMA (2,1,2). Following with this model they forecast Romania's unemployment for the following months of 2008.

[12] following Box-Jenkins methodology and using monthly data for the unemployment of Nigeria find that the best model is ARIMA (1,2,1) for the data used. With this model they forecast the unemployment for the following months of Nigeria.

[24] uses quarterly data for the period 1976Q1 – 2011Q4 to examine the forecast of unemployment in Nigeria. Among other models used, he proved that the most suitable for unemployment forecasting in Nigeria is the ARIMA(1,1,2)-ARCH(1) model instead of that of Etuk et al. (2012) that supported on their paper.

[29] on her paper deals with the modeling of employment market on Czech Republic. Box-Jenkins methodology or ARIMA model, are the approaches which uses for modeling time series. Particularly, for unemployment's model, data from January 2004 until April 2012 are used and with SARIMA model $(1,1,0)\ (1,1,0)_{12}$ she forecasts unemployment rate until December 2012.

[18] in order to examine unemployment rate in Thailand they use two techniques: Box-Jenkins and Neural Networks. Their results showed that Box-Jenkins technique proved more efficient for the estimation of unemployment rate in Thailand. Forecasted values that were estimated were consistent with the actual values for unemployment rate.

Finally, [25] using Phillips curve examines unemployment rates and inflation for USA from January 1980 to April 2015. Examining these variables with ARIMA and VAR models, she concluded that VAR models give better forecast than ARIMA.

3. Theoretical Background

[2] and [3] on their papers referred the procedures for the construction of ARIMA models. Seasonal ARIMA models consist of both seasonal and non-seasonal factors in a multiplicative model. ARIMA models, which were first introduced by Box-Jenkins, aimed at time-series forecasting when they became stationary by differencing. A time-series can have seasonal and non-seasonal characteristics. A series has seasonal characteristics when these are repeated over s time periods. Furthermore, in a seasonal series there is often a different mean value between seasonal intervals. Thus, in most cases, seasonal time series are non-stationary.

3.1. Non-seasonal ARIMA Model

A non-seasonal ARIMA model is symbolized as ARIMA (p,d,q) where p is the number of autoregressive lag, d is the differencing lag and q is the moving average lag and can be written as:

$$Y_t = \sum_{k=1}^{p} \alpha_k Y_{t-k} + \sum_{k=1}^{q} \theta_k e_{t-k} + \mu + e_t \quad (1)$$

Equation (1) can be also written as:

$$\alpha(B)Y_t = \theta(B)e_t + \mu \quad (2)$$

where

$$\alpha(B) = 1 - \alpha_1 B - \alpha_2 B - \ldots - \alpha_p B^p$$

and

$$\theta(B) = 1 - \theta_1 B - \theta_2 B - \ldots - \theta_q B^q.$$

If Z_t is a stationary series obtained after d differencing from Y_t series then we get:

$$Z_t = \forall^d Y_t = (1-B)^d Y_t \quad (3)$$

so the final form of ARIMA (p,d,q) model can be shaped as:

$$(1-B)^d \alpha(B)Y_t = \theta(B)e_t. \quad (4)$$

The above model is a popular time-series forecasting technique used on a large scale from analysts. In other words, this technique takes into account the historical data and is decomposed in an Autoregressive (AR) process where there is a memory from previous values, an integrated procedure that represents data stationarity and a Moving-Average (MA) procedure which represents the terms of previous error in order to make forecasting easier.

3.2. Seasonal ARIMA Model

A time-series is called seasonal if there is at least one seasonal autoregressive parameter P (SAR) or at least one seasonal moving average parameter Q (SMA) or both parameters (P,Q). Seasonal ARMA (P,Q) is used when seasonal (hence non stationary) behavior is present in the time series. ARMA (P,Q) model can be written as follows:

$$\Phi_P(B^s)Z_t = \Theta_Q(B^s)e_t \quad (5)$$

where
s=number of periods per season.

Seasonal differencing may be in order if the seasonal component follows a random walk, as in:

$$\forall Z_t = Z_t - Z_{t-s} = (1-B^s)Z_t. \quad (6)$$

The seasonal difference of order D is defined as:

$$\forall_s^D Z_t = (1-B^s)^D Z_t. \quad (7)$$

So the final form of SARIMA (p,d,q) model X $(P,D,Q)_s$ can be formed as:

$$\Phi_P(B^s)\varphi(B)\forall_s^D \forall^d Z_t = \Theta_Q(B^s)\theta(B)e_t \quad (8)$$

where
$\forall_s^D \forall^d Z_t$ is an ARMA model with lots of coefficients set to zero.

$$\Phi_P(\mathrm{B}^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_p B^{Ps}$$

$$\varphi(\mathrm{B}) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \varphi_p B^P$$

$$\Theta_Q(\mathrm{B}^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \ldots - \Theta_q B^{Qs}$$

$$\theta(\mathrm{B}) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^Q.$$

3.3. Procedure for SARIMA Modeling

- We test diagrammatically the data for the presence of seasonal fluctuations as well as for a possible trend.
- We observe the data correlogram. The coefficients ρ_k can present slow or quick drop in a expontential or corrugated way**.**
- If for any lag k=s, the respective coefficient is quite strong in relation to its neighbouring, we consider that the model has seasonality s. Then we isolate correlations coefficients ρ_k for k=s, 2s, 3s and if they diminish in a slow measure then we get seasonal differences $\Delta_s^D Y_t$ in order to determine the number D of ARIMA seasonal model $(P,D,Q)_s$ which is adjusted on data.
- If the existence of trend is obvious, we get the differences ΔY_t on Y_t observations till we achieve stationarity. If autocorrelation on first data of the series is strong, then seasonal correlations are becoming obvious on autocorrelation diagrams after differencing or generally on differences order d.
- When we get the required differences (seasonal and non-seasonal), we examine the new autocorrelation and partial autocorrelation diagrams of data on differences for identification on p,q and P,Q order on multiplicative ARIMA model $(p,d,q)(P,D,Q)_s$.
- For facilitation, we can isolate the autocorrelation coefficients with seasonal lags s, 2s, 3s in order to determine the values of P and Q of seasonal ARIMA $(P,D,Q)_s$.

A seasonally SARIMA model is symbolized as SARIMA (P,D,Q) where P is the number of autoregressive lag, D is the differencing lag and Q is the moving average lag and can be written as follows:

$$Y_t = \sum_{i=1}^{P} \alpha_{is} Y_{t-is} + \sum_{i=1}^{Q} \theta_{is} e_{t-is} + e_t. \quad (9)$$

3.4. Estimation of the Model SARIMA

For the estimation of SARIMA models we use the Maximum Likelihood –ML method, where $\hat{\theta}_n$ is the estimator of a matrix of parameters θ_0 and can be approximated by a multivariate normal distribution with mean and covariance matrix and is the following:

$$V_n = \frac{1}{n}\left(Var\left[\forall_\theta \ln\left(f_X\left(\mathrm{X};\theta_0\right)\right)\right]\right)^{-1} \quad (10)$$

where $\ln\left(f_X\left(X;\theta_0\right)\right)$ is the log-likelihood of one observation from the sample, evaluated at the parameter θ_0 , and $\forall_\theta \ln\left(f_X\left(X;\theta_0\right)\right)$ is the vector of first derivatives of the log-likelihood.

For the estimation of the asymptotic covariance matrix (10) the Outer Product of Gradients (OPG) estimate is used and is computed as:

$$\hat{V}_n = \left(\frac{1}{n}\sum_{i=1}^{n} \forall_\theta \ln\left(f_X(x_i;\hat{\theta}_n)\right)\forall_\theta \ln\left(f_X(x_i;\hat{\theta}_n)\right)\right)^{-1}. \quad (11)$$

Provided some regularity conditions are satisfied, the OPG estimator $\hat{V}_n$ is a consistent estimator of V_n. (see [15]).

Also, for the optimization of matrix $\hat{V}_n$ we use the algorithm of Broyden–Fletcher–Goldfarb–Shanno (BFGS). On numerical optimization, the algorithm BFGS is an iterative method for solving unconstrained nonlinear optimization problems and was developed by [5,14,17] and [28].

3.5. Diagnostic Checking of the Model SARIMA

There are several diagnostic tests for the analysis of models. A statistical tool which can be used to determine whether the series present autocorrelation or heteroscedasticity is Q statistic of [19].

$$Q_m = n(n+2)\sum_{k=1}^{m}\frac{e_k^2}{n-2} \quad (12)$$

where e_k is the residual autocorrelation at lag k, n is the number of residuals, m is the number of time lags including in the test. The model is considered adequate only if the p value associated with the Ljung-Box Q statistic is higher than a given significance.

The correlogram of the residuals can be used to check residuals' autocorrelation.

If there is no serial correlation, the autocorrelations and partial autocorrelations at all lags should be nearly zero, and all Q-Statistics should be insignificant with large probability-values.

The correlograms of the squared residuals can be used to check autoregressive conditional heteroskedasticity (ARCH) in the residuals. If there is no ARCH in the residuals, the autocorrelations and partial autocorrelations should be zero at all lags and the Q-statistics should not be significant.

3.6. Forecasting Performance

The forecasting on seasonal ARIMA models is computed for both in sample and out sample values. The optimum forecast value is evaluated from mean squared error (MSE) which measures the average of squared error over the sample period. Other measures (indices) often used for the return of forecasting are the Root Mean Square Error (RMSE), the Mean Absolute Percentage Error (MAPE) and Theil's inequality index [30].

These indices are taken from the following functions:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{Y}_t - Y_t\right)^2 \quad (13)$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{Y}_t - Y_t\right| \quad (14)$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{Y}_t - Y_t\right)^2} \quad (15)$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\hat{Y}_t - Y_t}{Y_t}\right|. \quad (16)$$

Theil's inequality index is taken from the following function:

$$U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{Y}_t - Y_t\right)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{Y}_t\right)^2} + \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(Y_t\right)^2}}, \quad 0 \le U \le 1 \quad (17)$$

where

Y_t : Actual value of endogenous variable Y at time t.

$\hat{Y}_t$: Redacted value of endogenous variable Y at time t.

T: Number of observations in the simulations (of the sample).

If Theil's unequal index is U=0, then actual values of the series will be equal with the estimated $Y_t = \hat{Y}_t$ for all t, so in this case we can consider that there is a "perfect fit" between actual and predicted data. On the contrary, if coefficient U=1, there is wrong forecasting for the examined model. Afterwards, we present individual Theil's indices called "unequal ratios" and are the following:

- Bias proportion: indicates the systematic differences in actual and forecasted values.

$$UM = \frac{\left(\bar{\hat{Y}} - \bar{Y}\right)^2}{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{Y}_t - Y_t\right)^2} \quad (18)$$

where $\bar{\hat{Y}}$ and $\bar{Y}$ are the means of the series of $\hat{Y}_t$ and Y_t respectively. Bias proportion measures the distance between the mean of simulated series from the mean of actual series.

Variance proportion: indicates unequal variances of actual and forecasted values.

$$US = \frac{\left(\hat{s}_{\hat{Y}} - s_Y\right)^2}{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{Y}_t - Y_t\right)^2} \quad (19)$$

where $\hat{s}_{\hat{Y}}$ and s_Y are the standard deviations of the series of $\hat{Y}_t$ and Y_t respectively. Variance proportion measures the distance between the variance of simulated series from the variance of actual series.

- Covariance proportion: indicates the correlation between the actual and forecasted values (zero=perfect correlation between actual and forecasted values).

$$UC = \frac{2(1-\rho)\hat{s}_{\hat{Y}} s_Y}{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{Y}_t - Y_t\right)^2} \quad (20)$$

where ρ is the correlation coefficient between $\hat{Y}_t$ and Y_t. Covariance proportion measures the rest of non-systematic error of simulating.

The forecasting ability of a model is satisfying when bias proportions and variance proportions are small. The relationship among the above proportions are UM+US+UC=1 (see [9]).

4. Data and Empirical Results

The variable used in the analysis of the paper is unemployment rate and covers the period from April 1998 until September 2015, total 210 monthly observations. Data derived from OECD database.

4.1. Testing for Non-stationarity

Figure 1 and Figure 2 show Greece's monthly unemployment rates and the trend analysis respectively. Diagrammatic test is made in order to examine the existence of seasonal fluctuations as well as a possible trend. Following diagram 3 describes the function of autocorrelation and partial autocorrelation respectively.

From Figure 1 we can see that the original data show changeable variance. Also, trend analysis from Figure 2 shows an upward trend. However, on Figure 3, the coefficients on autocorrelation function have a slow fall confirming that the series is non-stationary. Afterwards, we get the first differences of the series and examine stationarity. Figure 4 and Figure 5 present monthly unemployment rates and trend analysis on first differences respectively.

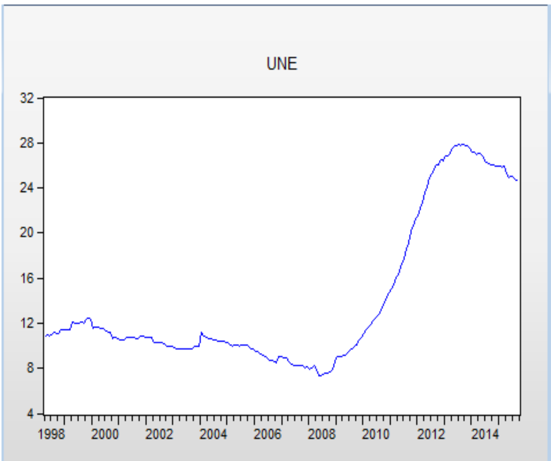


Figure 1. Time series plot of Greece monthly unemployment rate (Linear Trend Model UNE_t=5.860+0.078*t)

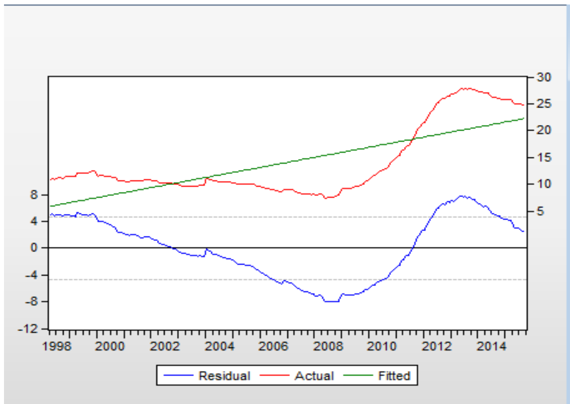
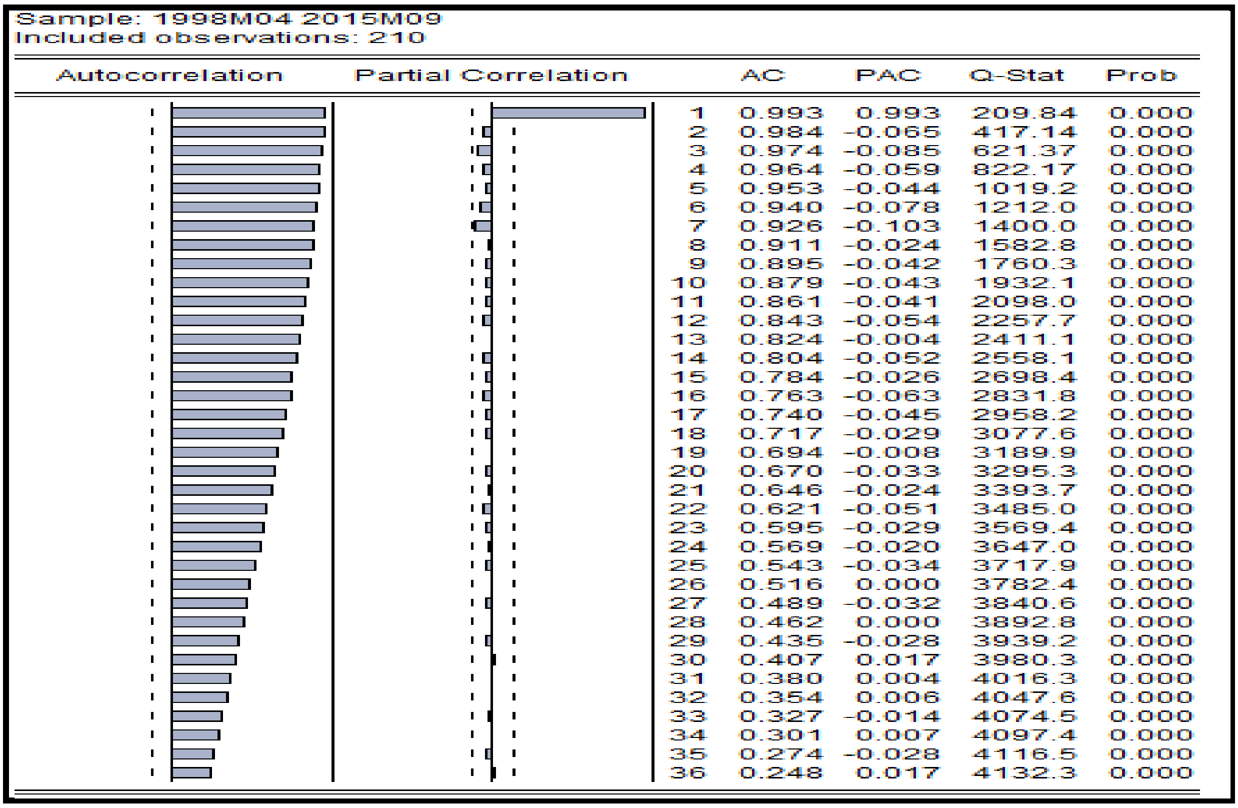


Figure 2. Trend plot analysis of Greece monthly unemployment rate (Linear Trend Model UNE_t=5.860+0.078*t)


Sample: 1998M04 2015M09
Included observations: 210

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.993	0.993	209.84	0.000
		2	0.984	-0.065	417.14	0.000
		3	0.974	-0.085	621.37	0.000
		4	0.964	-0.059	822.17	0.000
		5	0.953	-0.044	1019.2	0.000
		6	0.940	-0.078	1212.0	0.000
		7	0.926	-0.103	1400.0	0.000
		8	0.911	-0.024	1582.8	0.000
		9	0.895	-0.042	1760.3	0.000
		10	0.879	-0.043	1932.1	0.000
		11	0.861	-0.041	2098.0	0.000
		12	0.843	-0.054	2257.7	0.000
		13	0.824	-0.004	2411.1	0.000
		14	0.804	-0.052	2558.1	0.000
		15	0.784	-0.026	2698.4	0.000
		16	0.763	-0.063	2831.8	0.000
		17	0.740	-0.045	2958.2	0.000
		18	0.717	-0.029	3077.6	0.000
		19	0.694	-0.008	3189.9	0.000
		20	0.670	-0.033	3295.3	0.000
		21	0.646	-0.024	3393.7	0.000
		22	0.621	-0.051	3485.0	0.000
		23	0.595	-0.029	3569.4	0.000
		24	0.569	-0.020	3647.0	0.000
		25	0.543	-0.034	3717.9	0.000
		26	0.516	0.000	3782.4	0.000
		27	0.489	-0.032	3840.6	0.000
		28	0.462	0.000	3892.8	0.000
		29	0.435	-0.028	3939.2	0.000
		30	0.407	0.017	3980.3	0.000
		31	0.380	0.004	4016.3	0.000
		32	0.354	0.006	4047.6	0.000
		33	0.327	-0.014	4074.5	0.000
		34	0.301	0.007	4097.4	0.000
		35	0.274	-0.028	4116.5	0.000
		36	0.248	0.017	4132.3	0.000

Figure 3. Autocorrelation and Partial Correlation Plot of Greece's monthly unemployment rate

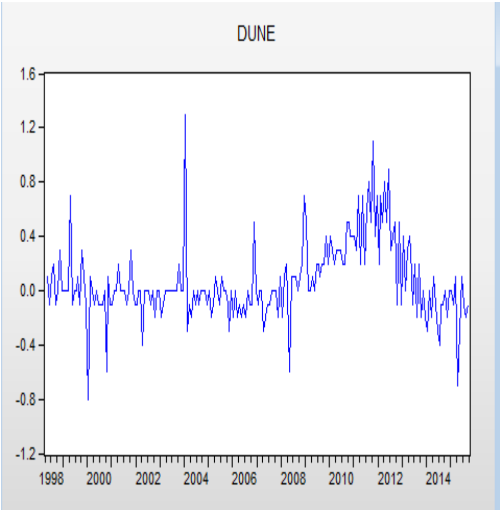


Figure 4. Time series plot of first difference of the original data (Linear Trend Model ΔUNE_t=-0.041+0.001*t)

From Figure 4 and Figure 5 we notice that stationarity has not been achieved and seasonality is not obvious.

Furthermore, trend analysis on Figure 5 show that there is an upward trend. On Figure 6 we present autocorrelation and partial autocorrelation functions that correspond on the first differences of the series.

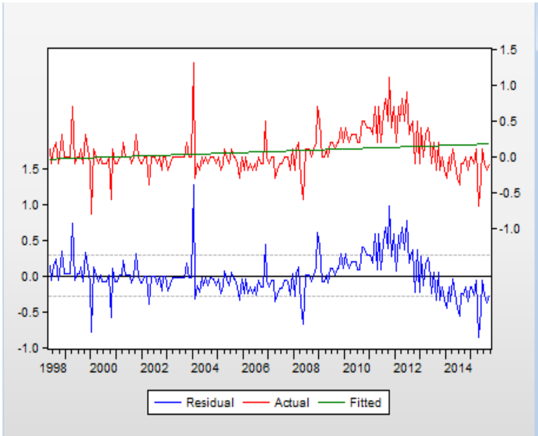
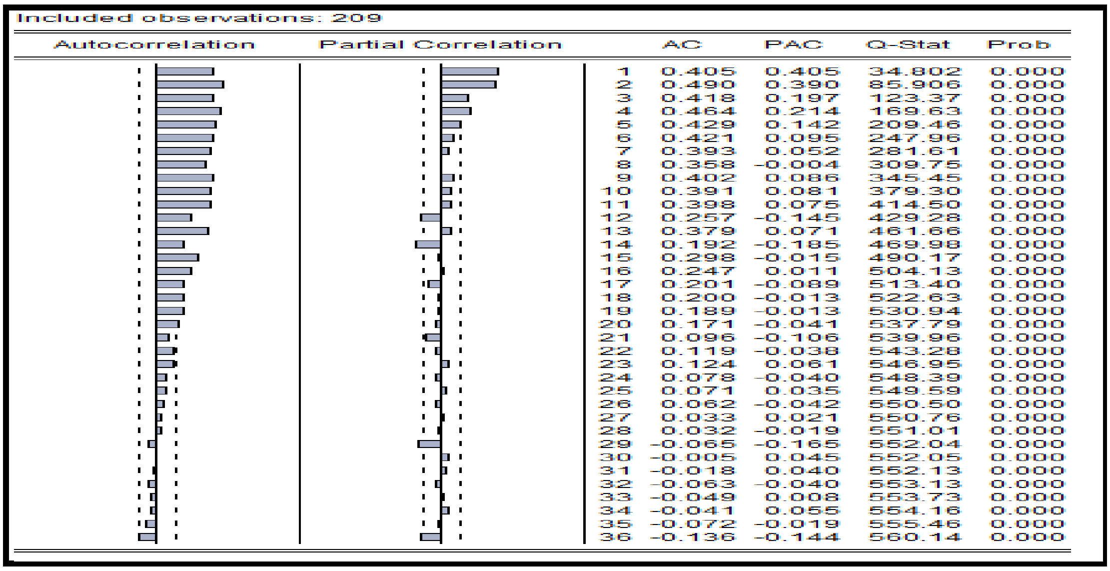


Figure 5. Trend analysis of first difference of the original data (Linear Trend Model ΔUNE_t=-0.041+0.001*t)


Included observations: 209

	AC	PAC	Q-Stat	Prob
1	0.405	0.405	34.802	0.000
2	0.490	0.390	85.906	0.000
3	0.418	0.197	123.37	0.000
4	0.464	0.214	169.63	0.000
5	0.429	0.142	209.46	0.000
6	0.421	0.095	247.96	0.000
7	0.393	0.052	281.61	0.000
8	0.358	-0.004	309.75	0.000
9	0.402	0.086	345.45	0.000
10	0.391	0.081	379.30	0.000
11	0.398	0.075	414.50	0.000
12	0.257	-0.145	429.28	0.000
13	0.379	0.071	461.66	0.000
14	0.192	-0.185	469.98	0.000
15	0.298	-0.015	490.17	0.000
16	0.247	0.011	504.13	0.000
17	0.201	-0.089	513.40	0.000
18	0.200	-0.013	522.63	0.000
19	0.189	-0.013	530.94	0.000
20	0.171	-0.041	537.79	0.000
21	0.096	-0.106	539.96	0.000
22	0.119	-0.038	543.28	0.000
23	0.124	0.061	546.95	0.000
24	0.078	-0.040	548.39	0.000
25	0.071	0.035	549.59	0.000
26	0.062	-0.042	550.50	0.000
27	0.033	0.021	550.76	0.000
28	0.032	-0.019	551.01	0.000
29	-0.065	-0.165	552.04	0.000
30	-0.005	0.045	552.05	0.000
31	-0.018	0.040	552.13	0.000
32	-0.063	-0.040	553.13	0.000
33	-0.049	0.008	553.73	0.000
34	-0.041	0.055	554.16	0.000
35	-0.072	-0.019	555.46	0.000
36	-0.136	-0.144	560.14	0.000

Figure 6. Autocorrelation and partial function of first differences of the original data

From the above figure, the coefficients on autocorrelation function present a slow downturn confirming that the series is not stationary on first differences. Thus, we get second differences.

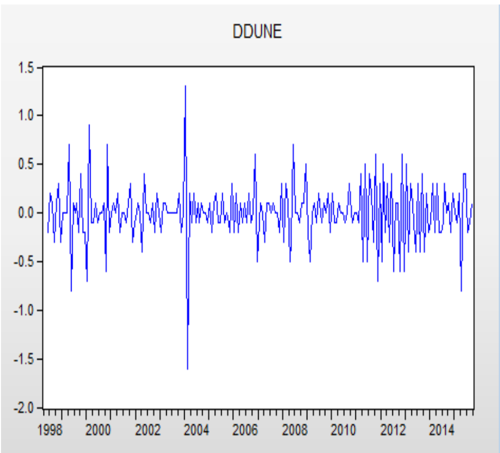


Figure 7. Time series plot of second differences of the original data (Linear Trend Model $\Delta^2 UNE_t$=0.0009-1.8E-0.5*t)

Figure 7 and Figure 8 show monthly unemployment rates and trend analysis on second differences respectively.

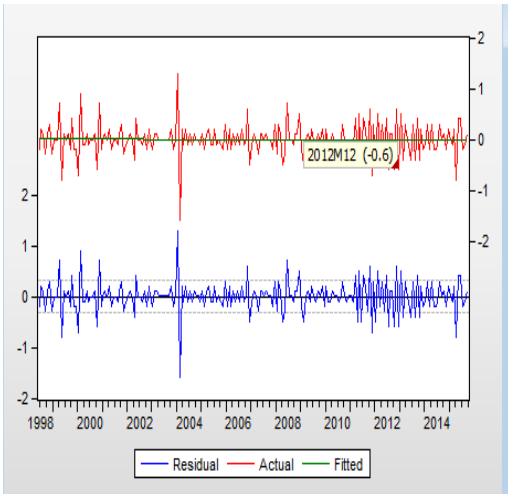


Figure 8. Trend analysis for second differences of the original data (Linear Trend Model $\Delta^2 UNE_t$=0.0009-1.8E-0.5*t)

From Figure 7 and Figure 8 we can see that stationarity has been achieved as there is no trend, and seasonality is obvious. On Figure 9, autocorrelation and partial autocorrelation functions are shown on second differences of series respectively.

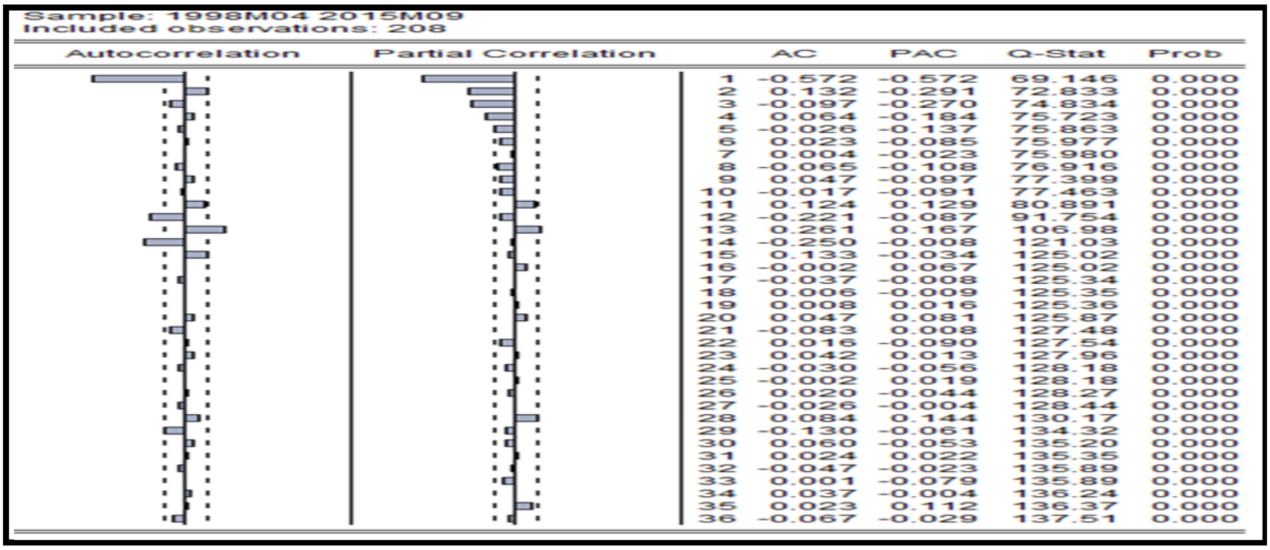


Figure 9. Autocorrelation and partial function of second difference of the original data

Coefficients of autocorrelation function show a quick fall on Figure 9, thus series is stationary on second differences. Afterwards, we test for series stationarity using [6,7] test and [27] unit roots tests.

The results of Augmented Dickey–Fuller (ADF) test and Phillips-Perron (PP) test on unemployment rate series are represented on Table 1.

Table 1. ADF and Phillip-Perron Test on Unemployment Series

	Level		First Differences		Second Differences	
	C	C,T	C	C,T	C	C,T
ADF	-0.748(4)	-1.798(4)	-1.985(3)	-3.075(3)	-12.07(3)*	-12.05(3)*
PP	0.125[10]	-1.192[10]	-1.489[9]	-1.795[9]	-19.94[8]*	-24.49[8]*

Notes: 1. *, **, *** imply significance at the 1%, 5%, 10% level, respectively. 2. The numbers within parentheses for the ADF, represents the lag length of the dependent variable used to obtain white noise residuals. 3. The lag length for the ADF equation was selected using [1]. 4. The numbers within brackets for the P-P statistics represent the bandwidth selected based on [23] method using Bartlett Kernel. 5. [20] critical value for rejection of null hypothesis of a unit root a significant at the 1% level.

The results on Table 1 indicate that unemployment rate is stationary in second differences. Therefore for our model ARIMA (p,d,q) we will have the value d=2.

4.2. Identification of the Model

After the detection of stationarity of the series, we define the form of ARIMA (p,q) models from the correlogram on Figure 9. Parameters p and q can be assessed from partial autocorrelation and autocorrelation coefficients respectively, comparing them with $\pm\frac{2}{\sqrt{n}}$ critical value. The limits for both functions (ACF, PACF) are $\pm\frac{2}{\sqrt{210}}=\pm 0.138$. From the column of autocorrelation in figure 9 we can notice that only the value of the coefficient ρ_1 (autocorrelation coefficient) is greater from the value ± 0.138, while from the column of the coefficients of partial autocorrelation the values $\hat{\phi}_4$ (partial autocorrelation coefficients) is greater than the value ± 0.138 . Therefore, the value of p will be $0 \le p \le 4$, and respectively, the value of q will be $0 \le q \le 1$. Thereafter we create Table 2 with the values of p and q as follows:

Table 2. Comparison of models within the range of exploration using AIC, SIC and HQ

ARIMA model	AIC	SC	HQ
(1,2,0)	0.119	0.152	0.132
(2,2,0)	0.041	0.090	0.061
(3,2,0)	-0.022	0.041	0.032
(4,2,0)	-0.046	0.033	-0.014
(0,2,1)	**-0.086**	**-0.046**	**-0.073**
(1,2,1)	**-0.094**	**-0.054**	**-0.075**
(2,2,1)	-0.085	-0.021	-0.059
(3,2,1)	-0.081	-0.015	-0.049
(4,2,1)	-0.072	0.023	-0.033

The results from Table 2 indicate that according to Akaike (AIC), Schwartz (SIC) and Hannan-Quinn (HQ) criteria, ARIMA(1,2,1) and ARIMA(0,2,1) models are the most suitable.

4.3. Seasonal Autoregressive Models

Continued on Figure 10 and Figure 11, we present the seasonal difference and the trend analysis on second differences of the series.

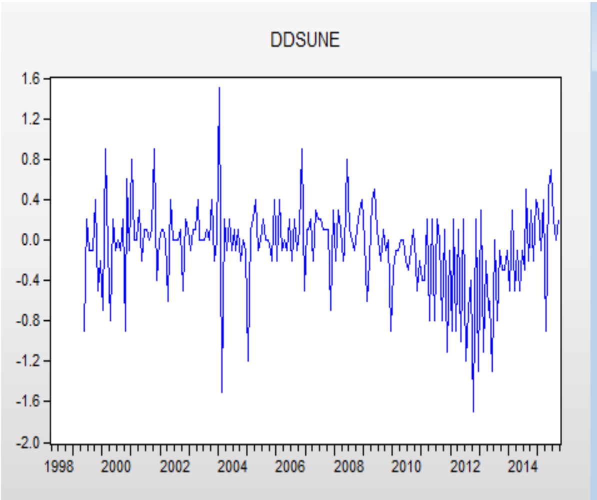


Figure 10. Time series plot of the seasonal difference of the second difference data (lag=12)

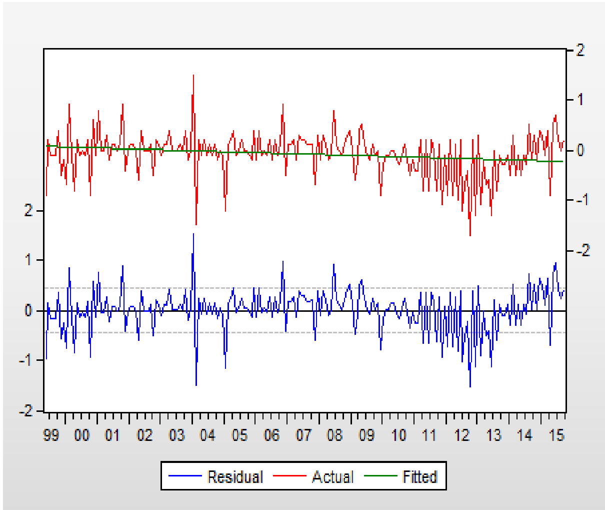
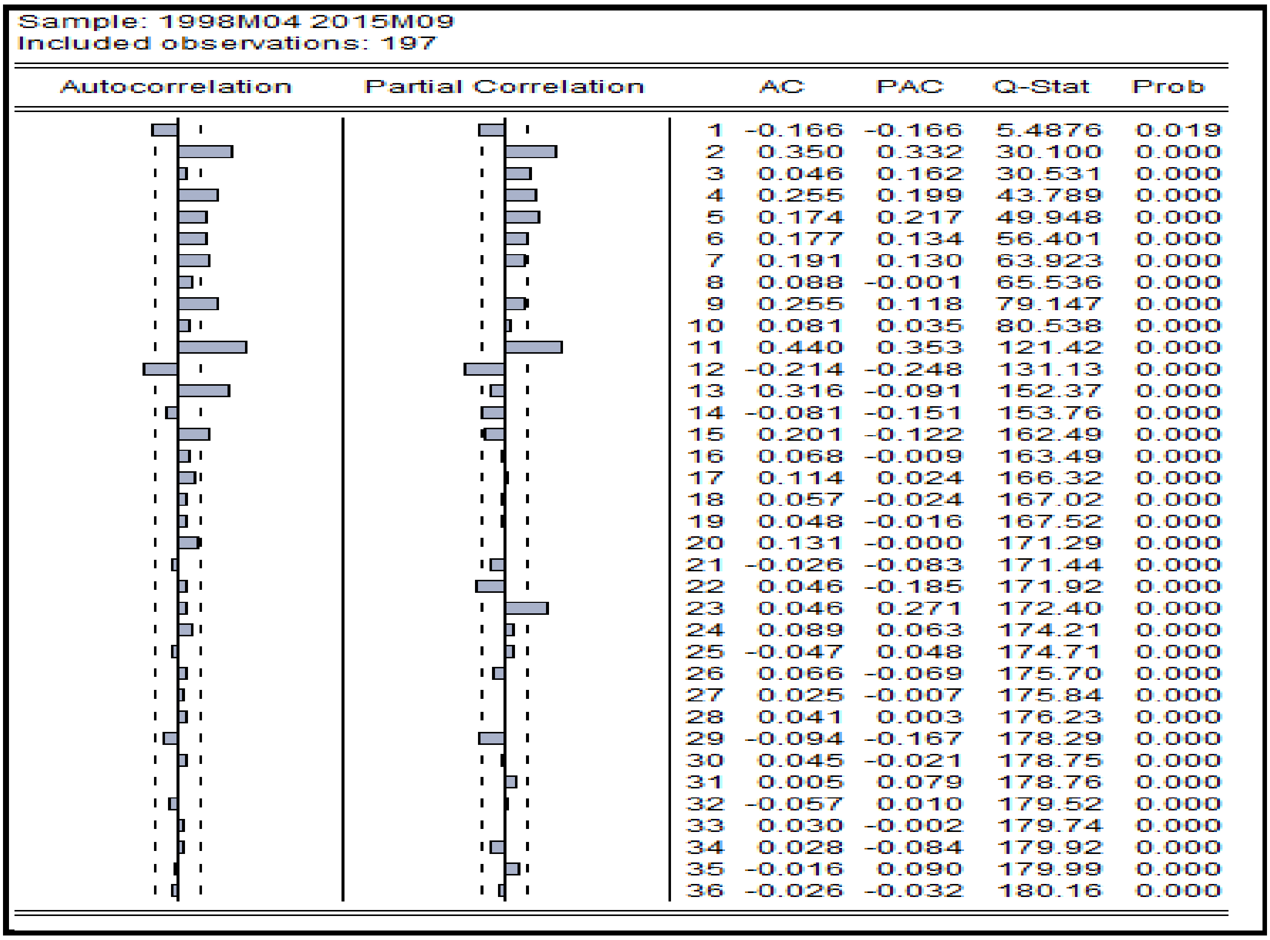


Figure 11. Trend analysis of the seasonal difference of the second difference data (Linear trend model D^2SUNE_t =0.088-0.0015*t)

From the above figures we notice that there is stability both in seasonal and non seasonal level and also the trend is stable (no rise or fall) showing us that there is stationarity on the mean.

On Figure 12 the autocorrelation and partial autocorrelation functions of seasonal difference appear and correspond to the second differences of the series.



Sample: 1998M04 2015M09
Included observations: 197

	AC	PAC	Q-Stat	Prob
1	-0.166	-0.166	5.4876	0.019
2	0.350	0.332	30.100	0.000
3	0.046	0.162	30.531	0.000
4	0.255	0.199	43.789	0.000
5	0.174	0.217	49.948	0.000
6	0.177	0.134	56.401	0.000
7	0.191	0.130	63.923	0.000
8	0.088	-0.001	65.536	0.000
9	0.255	0.118	79.147	0.000
10	0.081	0.035	80.538	0.000
11	0.440	0.353	121.42	0.000
12	-0.214	-0.248	131.13	0.000
13	0.316	-0.091	152.37	0.000
14	-0.081	-0.151	153.76	0.000
15	0.201	-0.122	162.49	0.000
16	0.068	-0.009	163.49	0.000
17	0.114	0.024	166.32	0.000
18	0.057	-0.024	167.02	0.000
19	0.048	-0.016	167.52	0.000
20	0.131	-0.000	171.29	0.000
21	-0.026	-0.083	171.44	0.000
22	0.046	-0.185	171.92	0.000
23	0.046	0.271	172.40	0.000
24	0.089	0.063	174.21	0.000
25	-0.047	0.048	174.71	0.000
26	0.066	-0.069	175.70	0.000
27	0.025	-0.007	175.84	0.000
28	0.041	0.003	176.23	0.000
29	-0.094	-0.167	178.29	0.000
30	0.045	-0.021	178.75	0.000
31	0.005	0.079	178.76	0.000
32	-0.057	0.010	179.52	0.000
33	0.030	-0.002	179.74	0.000
34	0.028	-0.084	179.92	0.000
35	-0.016	0.090	179.99	0.000
36	-0.026	-0.032	180.16	0.000

Figure 12. Autocorrelation and partial function of seasonal difference of the second difference data

From this figure we see that seasonal lags on autocorrelation function is important on lag 11, whereas on partial autocorrelation function is on 11 and 23 lag. This fact denotes that 0<P<2, 0<Q<1.

Thereafter we create Table 3 with the values of P and Q as follows:

Table 3. Comparison of models within the range of exploration using AIC, SIC and HQ

SARIMA model	AIC	SC	HQ
(1,2,1) (0,2,1)	**-0.094**	**-0.030**	**-0.068**
(1,2,1) (1,2,0)	-0.094	-0.029	-0.068
(1,2,1) (1,2,1)	**-0.116**	**-0.036**	**-0.083**
(1,2,1) (2,2,0)	-0.084	-0.004	-0.052
(1,2,1) (2,2,1)	-0.093	-0.010	-0.067

The results from Table 3 indicate that according to the criteria of Akaike (AIC), Schwartz (SIC) and Hannan-Quinn (HQ) the model SARIMA is formulated to SARIMA (1,2,1) $(1,2,1)_{12}$ and SARIMA (1,2,1) $(0,2,1)_{12}$ are the most suitable.

Table 4. Comparison of models within the range of exploration using AIC, SIC and HQ

SARIMA model	AIC	SC	HQ
(0,2,1) (0,2,1)	**-0.107**	**-0.043**	**-0.074**
(0,2,1) (1,2,0)	-0.090	-0.042	-0.070
(0,2,1) (1,2,1)	**-0.116**	**-0.052**	**-0.090**
(0,2,1) (2,2,0)	-0.081	-0.017	-0.055
(0,2,1) (2,2,1)	-0.091	-0.027	-0.071

From Table 4 we notice that according to the criteria of Akaike (AIC), Schwartz (SIC) and Hannan-Quinn (HQ) the model SARIMA (0,2,1) $(0,2,1)_{12}$ and SARIMA (0,2,1) $(1,2,1)_{12}$ are the most suitable.

We then proceed to the next stage of the Box-Jenkins approach which is the estimation of the models.

4.4. Estimation of the Model

Thereafter we can proceed to estimating the above model. The following Table 5, Table 6, Table 7 and Table 8 present the results of these models.

Table 5. Estimation Model SARIMA(1,2,1)$(1,2,1)_{12}$

Dependent Variable: DDUNE
Method: ARMA Maximum Likelihood (BFGS)
Date: 03/04/16 Time: 12:43
Sample: 1998M06 2015M09
Included observations: 208
Convergence achieved after 44 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-0.121443	0.076818	-1.580920	0.1155
SAR(12)	0.789946	0.183517	4.304481	0.0000
MA(1)	-0.760741	0.056077	-13.56598	0.0000
SMA(12)	-0.943543	0.215323	-4.381983	0.0000
SIGMASQ	0.048286	0.004553	10.60625	0.0000

R-squared	0.498322	Mean dependent var	-0.000962
Adjusted R-squared	0.488437	S.D. dependent var	0.310989
S.E. of regression	0.222430	Akaike info criterion	-0.116310
Sum squared resid	10.04349	Schwarz criterion	-0.036081
Log likelihood	17.09623	Hannan-Quinn criter.	-0.083869
Durbin-Watson stat	1.994966		

Inverted AR Roots	.98	.85+.49i	.85-.49i	.49-.85i
	.49+.85i	.00-.98i	-.00+.98i	-.12
	-.49-.85i	-.49+.85i	-.85-.49i	-.85+.49i
	-.98			
Inverted MA Roots	1.00	.86+.50i	.86-.50i	.76
	.50+.86i	.50-.86i	.00+1.00i	-.00-1.00i
	-.50+.86i	-.50-.86i	-.86+.50i	-.86-.50i
	-1.00			

Results on Table 5 show that the coefficient on AR(1) parameter is not statistically significant.

Table 6. Estimation Model SARIMA(1,2,1)$(0,2,1)_{12}$

Dependent Variable: DDUNE
Method: ARMA Maximum Likelihood (BFGS)
Date: 03/07/16 Time: 12:26
Sample: 1998M06 2015M09
Included observations: 208
Convergence achieved after 8 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-0.138728	0.073611	-1.884608	0.0609
MA(1)	-0.751699	0.055116	-13.63839	0.0000
SMA(12)	-0.106877	0.072784	-1.468420	0.1435
SIGMASQ	0.050962	0.002754	18.50354	0.0000

R-squared	0.470518	Mean dependent var	-0.000962
Adjusted R-squared	0.462732	S.D. dependent var	0.310989
S.E. of regression	0.227950	Akaike info criterion	-0.094588
Sum squared resid	10.60012	Schwarz criterion	-0.030404
Log likelihood	13.83713	Hannan-Quinn criter.	-0.068635
Durbin-Watson stat	2.001667		

Inverted AR Roots	-.14			
Inverted MA Roots	.83	.75	.72+.41i	.72-.41i
	.41+.72i	.41-.72i	.00+.83i	-.00-.83i
	-.41-.72i	-.41+.72i	-.72-.41i	-.72+.41i
	-.83			

Results on Table 6 show that the coefficient on AR(1) and SMA(12) parameters are not statistically significant on 5% level of significance.

Table 7. Estimation Model SARIMA(0,2,1)(0,2,1)$_{12}$

Dependent Variable: DDUNE
Method: ARMA Maximum Likelihood (BFGS)
Date: 03/07/16 Time: 12:32
Sample: 1998M06 2015M09
Included observations: 208
Convergence achieved after 8 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.792147	0.038170	-20.75309	0.0000
SMA(12)	-0.134387	0.071066	-1.891025	0.0600
SIGMASQ	0.051621	0.002788	18.51265	0.0000

R-squared	0.463676	Mean dependent var	-0.000962
Adjusted R-squared	0.458443	S.D. dependent var	0.310989
S.E. of regression	0.228858	Akaike info criterion	-0.091231
Sum squared resid	10.73711	Schwarz criterion	-0.043093
Log likelihood	12.48803	Hannan-Quinn criter.	-0.071767
Durbin-Watson stat	2.187099		

Inverted MA Roots	.85	.79	.73+.42i	.73-.42i
	.42-.73i	.42+.73i	.00+.85i	-.00-.85i
	-.42-.73i	-.42+.73i	-.73+.42i	-.73-.42i
	-.85			

Results on Table 7 show that coefficient on SMA(12) parameter is not statistically significant on 5% level of significance.

Table 8. Estimation Model SARIMA(0,2,1)(1,2,1)$_{12}$

Dependent Variable: DDUNE
Method: ARMA Maximum Likelihood (BFGS)
Date: 03/07/16 Time: 12:29
Sample: 1998M06 2015M09
Included observations: 208
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(12)	0.749178	0.172405	4.345459	0.0000
MA(1)	-0.798245	0.039135	-20.39742	0.0000
SMA(12)	-0.922079	0.170068	-5.421812	0.0000
SIGMASQ	0.048878	0.003670	13.31699	0.0000

R-squared	0.492169	Mean dependent var	-0.000962
Adjusted R-squared	0.484701	S.D. dependent var	0.310989
S.E. of regression	0.223241	Akaike info criterion	-0.116559
Sum squared resid	10.16667	Schwarz criterion	-0.052375
Log likelihood	16.12212	Hannan-Quinn criter.	-0.090606
Durbin-Watson stat	2.154698		

Inverted AR Roots	.98	.85+.49i	.85-.49i	.49+.85i
	.49-.85i	-.00-.98i	-.00+.98i	-.49-.85i
	-.49+.85i	-.85+.49i	-.85-.49i	-.98
Inverted MA Roots	.99	.86+.50i	.86-.50i	.80
	.50+.86i	.50-.86i	.00+.99i	-.00-.99i
	-.50+.86i	-.50-.86i	-.86+.50i	-.86-.50i
	-.99			

The results of the above table show that all coefficients are statistically significant thus that model can be used for forecasting.

4.5. Diagnostic Checking of the Model SARIMA(0,2,1)(1,2,1)$_{12}$

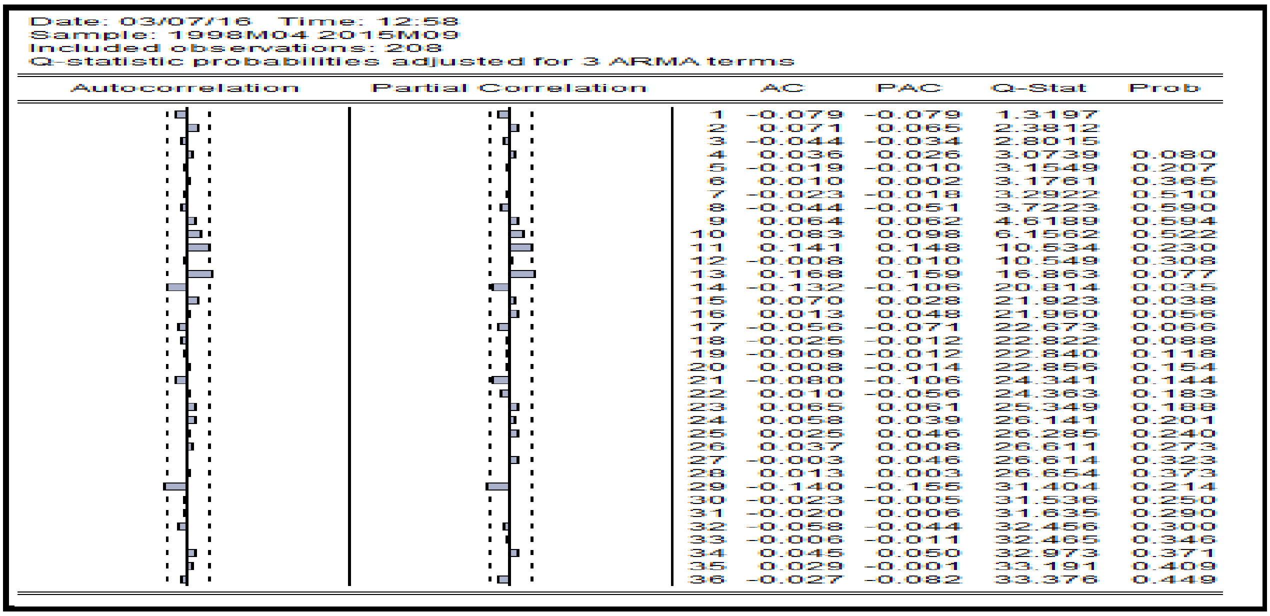
On the Figure 13 and Figure 14, the residuals test for the autocorrelation with conditional heteroscedasticity (ARCH model) is provided.

From the results of Figure 13 and Figure 14, we can see that autocorrelation and partial autocorrelation coefficients are non statistical significant in all lags as all Q-statistics have large probability-values. So, we can regard that residuals are not autocorrelated and don't form ARCH models. Thus, SARIMA (0,2,1)(1,2,1)$_{12}$ model can be used for forecasting.

4.6. Forecasting.

For forecasting SARIMA(0,2,1)(1,2,1)$_{12}$ model we use both dynamic and static forecasting procedure. The dynamic procedure calculates forecasts for periods after the first period in the sample by using the previously forecasted values from lagged dependent variable and ARMA terms. This procedure is called n-step ahead forecast. Static procedure uses actual and non forecasted values of dependent variable. This procedure is called one step- ahead forecast.

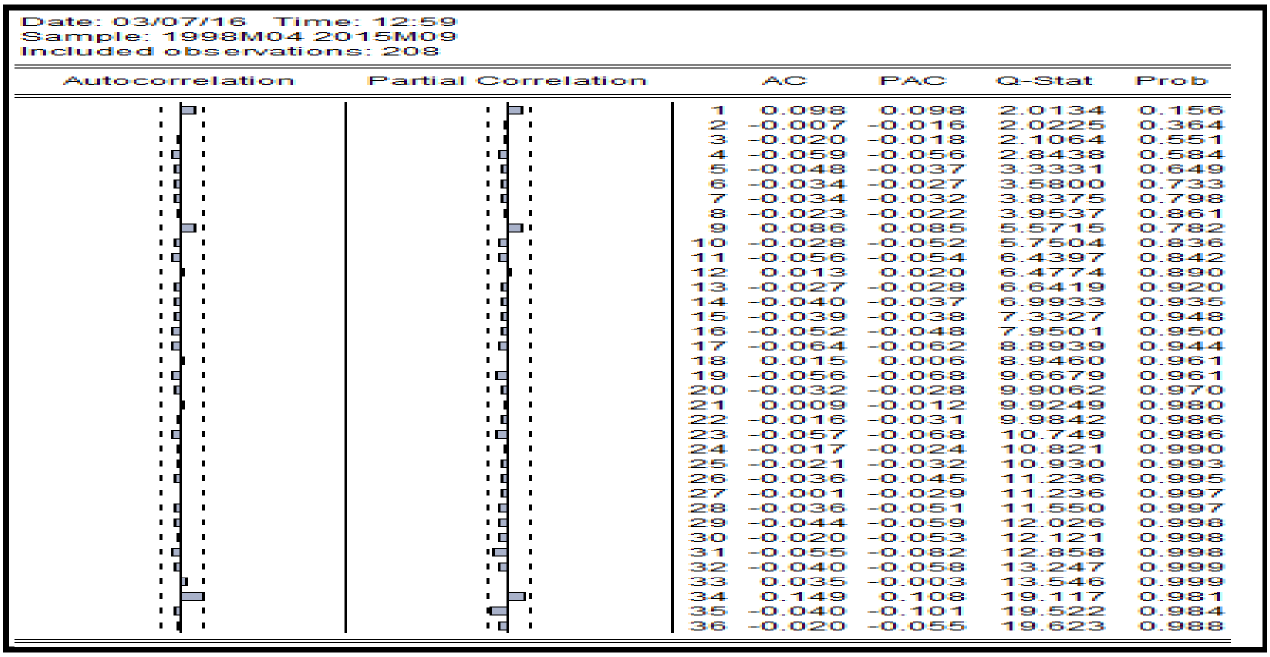
In Figure 15 and Figure 16 we represent the criteria for the evaluation of the forecasts in and out sample forecast at level form of the dependent variable, using dynamic and static forecast respectively.



Date: 03/07/16 Time: 12:58
Sample: 1998M04 2015M09
Included observations: 208
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.079	-0.079	1.3197	
		2	0.071	0.065	2.3812	
		3	-0.044	-0.034	2.8015	
		4	0.036	0.026	3.0739	0.080
		5	-0.019	-0.010	3.1549	0.207
		6	0.010	0.002	3.1761	0.365
		7	-0.023	-0.018	3.2922	0.510
		8	-0.044	-0.051	3.7223	0.590
		9	0.064	0.062	4.6189	0.594
		10	0.083	0.098	6.1562	0.522
		11	0.141	0.148	10.534	0.230
		12	-0.008	0.010	10.549	0.308
		13	0.168	0.159	16.863	0.077
		14	-0.132	-0.106	20.814	0.035
		15	0.070	0.028	21.923	0.038
		16	0.013	0.048	21.960	0.056
		17	-0.056	-0.071	22.673	0.066
		18	-0.025	-0.012	22.822	0.088
		19	-0.009	-0.012	22.840	0.118
		20	0.008	-0.014	22.856	0.154
		21	-0.080	-0.106	24.341	0.144
		22	0.010	-0.056	24.363	0.183
		23	0.065	0.061	25.349	0.188
		24	0.058	0.039	26.141	0.201
		25	0.025	0.046	26.285	0.240
		26	0.037	0.008	26.611	0.273
		27	-0.003	0.046	26.614	0.323
		28	0.013	0.003	26.654	0.373
		29	-0.140	-0.155	31.404	0.214
		30	-0.023	-0.005	31.536	0.250
		31	-0.020	0.006	31.635	0.290
		32	-0.058	-0.044	32.456	0.300
		33	-0.006	-0.011	32.465	0.346
		34	0.045	0.050	32.973	0.371
		35	0.029	-0.001	33.191	0.409
		36	-0.027	-0.082	33.376	0.449

Figure 13. Diagnostic residuals' autocorrelation test of SARIMA(0,2,1)(1,2,1)$_{12}$ model



Date: 03/07/16 Time: 12:59
Sample: 1998M04 2015M09
Included observations: 208

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.098	0.098	2.0134	0.156
		2	-0.007	-0.016	2.0225	0.364
		3	-0.020	-0.018	2.1064	0.551
		4	-0.059	-0.056	2.8438	0.584
		5	-0.048	-0.037	3.3331	0.649
		6	-0.034	-0.027	3.5800	0.733
		7	-0.034	-0.032	3.8375	0.798
		8	-0.023	-0.022	3.9537	0.861
		9	0.086	0.085	5.5715	0.782
		10	-0.028	-0.052	5.7504	0.836
		11	-0.056	-0.054	6.4397	0.842
		12	0.013	0.020	6.4774	0.890
		13	-0.027	-0.028	6.6419	0.920
		14	-0.040	-0.037	6.9933	0.935
		15	-0.039	-0.038	7.3327	0.948
		16	-0.052	-0.048	7.9501	0.950
		17	-0.064	-0.062	8.8939	0.944
		18	0.015	0.006	8.9460	0.961
		19	-0.056	-0.068	9.6679	0.961
		20	-0.032	-0.028	9.9062	0.970
		21	0.009	-0.012	9.9249	0.980
		22	-0.016	-0.031	9.9842	0.986
		23	-0.057	-0.068	10.749	0.986
		24	-0.017	-0.024	10.821	0.990
		25	-0.021	-0.032	10.930	0.993
		26	-0.036	-0.045	11.236	0.995
		27	-0.001	-0.029	11.236	0.997
		28	-0.036	-0.051	11.550	0.997
		29	-0.044	-0.059	12.026	0.998
		30	-0.020	-0.053	12.121	0.998
		31	-0.055	-0.082	12.858	0.998
		32	-0.040	-0.058	13.247	0.999
		33	0.035	-0.003	13.546	0.999
		34	0.149	0.108	19.117	0.981
		35	-0.040	-0.101	19.522	0.984
		36	-0.020	-0.055	19.623	0.988

Figure 14. Diagnostic test for residuals' conditional autocorrelation of SARIMA(0,2,1)(1,2,1)$_{12}$ model

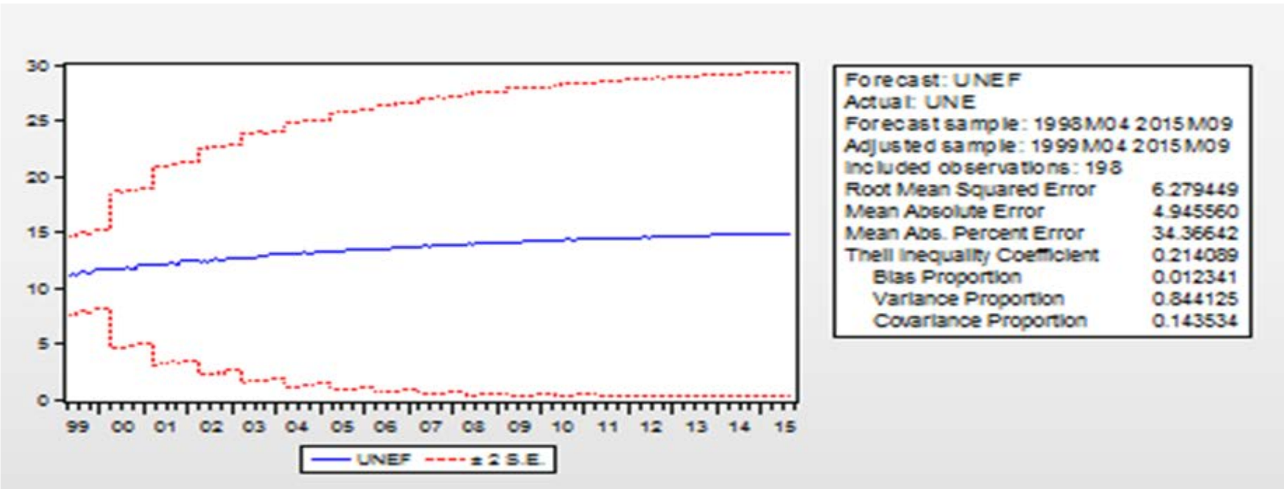


Figure 15. Dynamic Forecast of Unemployment

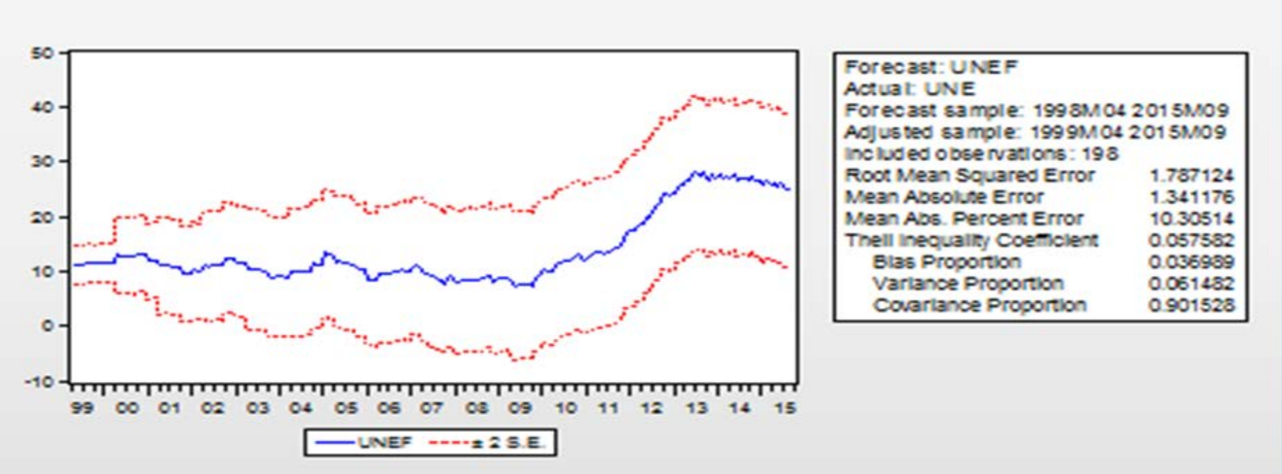


Figure 16. Static Forecast of Unemployment

From Figure 15 and Figure 16 we notice that the indices of root mean squared error and mean absolute error get smaller values on Figure 16 than that of Figure 15. We conclude that static procedure gives better forecasting ability on the examined model. Furthermore, Theil's Inequality index on the static procedure is smaller compared to the dynamic procedure and is close to zero. This indication on Theil's index shows the "goodness of fit" for the model. Moreover, the proportion of bias (it measures how far is the mean of simulated series from the mean of the actual series), and the proportion of variance (it measures how far is the variance of simulated series from the variance of actual series), has very small value on the static procedure. These indices prove that the forecasting ability of static procedure show more accurate forecasting. On the contrary, the proportion of covariance which measures the rest of non-systematic forecast error, is larger on the static procedure as it was expected.

Table 9 displays the comparison between the actual, forecasted and the residual starting from 2013:10 to 2015:09.

Table 9. Comparison between the actual, forecast values from 2013:10 to 2015:09

obs	Actual	Fitted	Residual	Residual Plot
2013M10	27.7000	26.5513	1.14871	
2013M11	27.7000	27.5143	0.18570	
2013M12	27.5000	27.0614	0.43862	
2014M01	27.2000	27.7758	-0.57583	
2014M02	27.2000	26.8291	0.37086	
2014M03	27.0000	27.1981	-0.19809	
2014M04	27.1000	27.0444	0.05563	
2014M05	27.0000	27.6775	-0.67749	
2014M06	26.7000	26.3221	0.37793	
2014M07	26.3000	26.8701	-0.57014	
2014M08	26.2000	26.7254	-0.52542	
2014M09	26.1000	26.6665	-0.56655	
2014M10	26.1000	27.4396	-1.33964	
2014M11	25.9000	26.6721	-0.77208	
2014M12	25.9000	26.6933	-0.79327	
2015M01	25.9000	25.6142	0.28585	
2015M02	25.8000	26.3687	-0.56866	
2015M03	25.9000	25.7348	0.16518	
2015M04	25.2000	26.0272	-0.82719	
2015M05	24.9000	25.3526	-0.45259	
2015M06	25.0000	25.9233	-0.92327	
2015M07	24.9000	24.8068	0.09325	
2015M08	24.7000	24.7522	-0.05218	
2015M09	24.6000	24.6292	-0.02919	

From Table 9 we can see that SARIMA$(0,2,1)(1,2,1)_{12}$ model has the best predictive power. The forecasted value of unemployment, deriving from the suggested model, refers to September 2015 and is 24.62%. This value is very close to the actual which is 24.6%. Therefore, SARIMA$(0,2,1)(1,2,1)_{12}$ model that we suggest has a good and precise forecasting for unemployment in Greece.

5. Conclusion

Unemployment plagues many countries so it is important to capture the trend of this series. The use of ARIMA models is a highly flexible tool in order to forecast unemployment rate if there is no government's intervention which will change this trend. The main goal of this paper is to find the most suitable model with a forecasting ability in order to forecast unemployment in Greece. Using Box-Jenkins methodology, we determined the form of SARIMA model and estimated this model with the non-linear optimization method of Maximum Likelihood, using numerical optimization Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. For the forecasting power of the model, both the dynamic and static procedure together with the criteria forecasting measures, were used. The results of the forecast showed that the forecasted value of unemployment is close to the actual value. This result showed that model's suitability can be used to forecast unemployment in Greece for the following years.

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