

Effect of Environmental Tax on Carbon Dioxide Emission: A Mathematical Model

Shyam Sundar^{1*}, Ashish Kumar Mishra, Ram Naresh²

¹Department of Mathematics, PSIT, Kanpur-208020, India

²Department of Mathematics, H. B. Technological Institute, Kanpur-208002, India

*Corresponding author: ssmishra_75@yahoo.co.in

Abstract The objective of this work is to study effect of environmental tax on emitters to attain a pollution free environment. In this paper, an attempt has been made to study the effect of environmental tax to control carbon dioxide (a global warming gas) concentration in the atmosphere using a nonlinear mathematical model. It is assumed that the environmental tax is levied only when the concentration of carbon dioxide crosses a threshold level over which it is harmful to our environment. Analytically, it is shown that the concentration of carbon dioxide decreases as the rate of environmental tax increases. The variation of carbon dioxide concentration (with and without environmental tax) has also been shown in a plot to confirm analytical results. Further, carbon dioxide concentration can also be reduced by greenbelt (leafy trees) plantation. It is shown, analytically and numerically, that carbon dioxide concentration decreases as its rate of interaction (absorption) with leafy trees increases.

Keywords: *mathematical model, greenbelt, carbon dioxide, environmental tax*

Cite This Article: Shyam Sundar, Ashish Kumar Mishra, and Ram Naresh, "Effect of Environmental Tax on Carbon Dioxide Emission: A Mathematical Model." *American Journal of Applied Mathematics and Statistics*, vol. 4, no. 1 (2016): 16-23. doi: 10.12691/ajams-4-1-3.

1. Introduction

In present scenario, global warming is a serious threat to our planet due to emission of greenhouse gases into the atmosphere. Global warming gases are emitted into the atmosphere due to natural as well as anthropogenic sources. Carbon dioxide is one of the most crucial greenhouse gases responsible for global temperature change. Rapid growth of industrialization, urbanization and increasing demand of food are main driving force behind the increase in concentration of carbon dioxide in the atmosphere [27]. Though carbon dioxide increases plant growth significantly but it has harmful climatic effect to our environment too if it crosses a threshold level. Threshold concentration of carbon dioxide is that below which there are no visible harmful effects to our environment. The effect of global warming can be seen in the form of affecting agriculture and livelihood, infectious diseases, melting of glaciers, rise in sea levels and increase in average temperature of earth surface [2,7,15,17,19,24,26,31,33,37,38]. Since the climate of our environment is adversely affected by elevated concentration of carbon dioxide therefore it is crucial to comprehend and study the dynamics of carbon dioxide in the atmosphere.

Leafy trees in a greenbelt play a crucial role to regulate the concentration of carbon dioxide in the atmosphere. It acts like a natural sink and during photosynthesis gigatons of carbon dioxide per year from the atmosphere is taken by biomass of leafy trees and thus reducing the global atmospheric concentration of carbon dioxide [36]. In this way, leafy trees in a greenbelt can reduce the concentration

of carbon dioxide considerably in the atmosphere [4,11,14,22,23,25,30,39].

The main objective of this study is to control and stabilize the concentration of carbon dioxide in the atmosphere by taking into account the concept of environmental tax, as a control parameter, levied on emitters which may, in turn, reduce environmental damage and minimize harm to economic growth. It may also set the concentration of carbon dioxide at a level that would not be dangerous for our environment and enable us for economic development in a sustainable manner. According to Kyoto Protocol, Japan agreed to cut carbon dioxide and other greenhouse gases emission up to 5 percent below 1990 level. US decided to reduce up to 7 percent below the 1990 emission level. The worldwide reduction in carbon dioxide and other greenhouse gases emissions by participating countries in Kyoto Protocol has been targeted up to 5.2 percent below 1990 level [3,28]. Environmental tax imposed on emitters can in the form of,

1. Tax imposed directly on the emitters according to the amount of emissions and extent of environmental damage.
2. Tax imposed on individuals causing environmental damage.
3. Indirect taxes on production inputs or consumer goods that can make an undesirable change to our environment.
4. Tax on the type and extent of resources use.
5. Incentive charges levied on those who intentionally cause damage to the environment

In recent years, several investigations have been made to discuss the importance of tax policies and their

effective implementation on emitters without affecting overall energy consumptions to make our environment clean [5,6,8,9,16,18,21,29,32,35]. Some mathematical models have also been developed to study the environmental tax as one of the important mitigation options for the abatement of pollutant emission causing deterioration to our environment [1,12,13,34]. In this regard, He [12] proposed a mathematical model to study the effect of pollution tax on toxicant emitters in a polluted environment when their emission crosses the permissible limit (the limit up to which there is no harm to the population) and obtained some sufficient conditions for the persistence of population. Agarwal and Devi [1] have proposed and analyzed a nonlinear mathematical model to study the survival of biological species in a polluted environment by taking into account the effect of environmental tax on the pollutant emitters. They have shown that as the concentration level of pollutants in the environment increases the density of biological species decreases. They have also shown that the density of biological species can achieve a desired level if some pollution tax is imposed on the emitters to control the emission of pollutants in the environment. Sundar and Naresh [34] developed a mathematical model to study the effect of environmental tax on emitters for the survival of biological species in a polluted environment considering that the tax is imposed on the emitters only when the pollutants concentration goes beyond a permissible limit over which the population is exposed to the pollutants. In their analysis they have shown that beyond a certain level of pollutant emission environmental tax is one of the important ways to control pollutant concentration in the environment and for the survival of biological species.

Keeping in view of the above, in this paper, we have developed a simple nonlinear mathematical model to study the introduction of environmental taxation, as a control parameter, for the mitigation of carbon dioxide emission in the atmosphere, thus reducing the effect of global warming.

2. Formulation of Mathematical Model

To describe the phenomenon of control of carbon dioxide concentration by planting leafy trees in a greenbelt and imposing environmental tax on emitters, we assume that C be the concentration of carbon dioxide in the atmosphere, B be the biomass density of leafy trees in a green belt and E be the environmental tax imposed on emitters. We have made the following assumptions to formulate the mathematical model,

1. The emission rate of carbon dioxide is constant (say Q) though it may be a function of time.
2. Carbon dioxide is used by leafy trees in a greenbelt during the process of photosynthesis in the presence of sunlight
3. Environmental tax is imposed on the emitters if the concentration of carbon dioxide crosses a threshold i.e. the concentration below which there is no impairment to our environment and therefore it is assumed to be proportional to the difference of carbon dioxide concentration and its threshold level.

In the first equation of the model, the constant δ_0 is the natural depletion rate coefficient of carbon dioxide. As we know that plants make their food in the presence of sunlight by taking carbon dioxide from the atmosphere during photosynthesis and therefore the abatement in carbon dioxide concentration in the atmosphere is assumed to be directly proportional to the biomass density of leafy trees as well as the concentration of carbon dioxide (i.e. δBC), δ being the rate of depletion of carbon dioxide due to presence of leafy trees in a greenbelt. Due to environmental tax, the diminution in carbon dioxide concentration is assumed to be at the rate μ . In the second equation of the model, the growth of leafy trees in a greenbelt is assumed to be governed by logistic equation where r and K are the intrinsic growth rate and the carrying capacity of leafy trees in a greenbelt respectively. The constant π is the rate at which carbon dioxide is used in photosynthesis process by leafy trees in their growth. In the third equation, ν is the tax rate coefficient and the constant ν_0 denotes the tax evasion due to some practical problems like pilferages, natural and administrative problems. The threshold concentration of carbon dioxide is denoted by C_0 .

In view of the above, we have developed a three dimensional mathematical model governing the phenomenon of reduction of carbon dioxide in the atmosphere, as follows.

$$\frac{dC}{dt} = Q - \delta_0 C - \delta BC - \mu E \tag{1}$$

$$\frac{dB}{dt} = rB \left(1 - \frac{B}{K} \right) + \pi \delta BC \tag{2}$$

$$\frac{dE}{dt} = \nu(C - C_0) - \nu_0 E \tag{3}$$

with $C(0) > C_0, B(0) \geq 0, E(0) \geq 0$

Remark: For the physical significance of the model system, it is required that the cumulative concentration of carbon dioxide must be greater than its threshold concentration (i.e $C > C_0$), and then the environmental tax will be imposed on the industrialists and would continue till $C \leq C_0$. If $C \leq C_0$, then there is no need to impose environmental tax. Further, if δ and μ are so large that

$\frac{dC}{dt}$ becomes negative, then this implies that the excess carbon dioxide is removed completely from the atmosphere.

To analyze the model system (1) – (3), we need the bounds of dependent variables. For this, we establish the region of attraction in the following lemma, Freedman and So [10].

Lemma 2.1. The set

$$\Omega = \left\{ (C, B, E) \in R_+^3 : 0 \leq C \leq \frac{Q}{\delta_0}, \right. \\ \left. 0 \leq B \leq \frac{K}{r} \left(r + \pi \delta \frac{Q}{\delta_0} \right), 0 \leq E \leq \frac{\nu}{\nu_0} \left(\frac{Q}{\delta_0} - C_0 \right) \right\}$$

is the region of attraction for all solutions of the model system (1) – (3) initiating in the interior of positive octant.

As discussed above, $C > C_0$ for all practical purposes and hence it is obvious that $\frac{Q}{\delta_0} > C_0$.

2.1. Equilibrium Analysis

The model system (1) – (3) has following two non-negative equilibria

- (i) $\bar{E}(\bar{C}, 0, \bar{E})$
- (ii) $E^*(C^*, B^*, E^*)$.

2.1.1. Existence of $\bar{E}(\bar{C}, 0, \bar{E})$

The positive solution of $\bar{E}(\bar{C}, 0, \bar{E})$ is given by the following algebraic equations,

$$Q - \delta_0 C - \mu E = 0 \quad (4)$$

$$\nu(C - C_0) - \nu_0 E = 0 \quad (5)$$

Using the value of E from (5) in (4), we get,

$$C = \frac{Q + \frac{\mu\nu}{\nu_0}}{\delta_0 + \frac{\mu\nu}{\nu_0}} = \bar{C} \quad (\text{say}) \quad (6)$$

Using this value in (5), we get,

$$E = \frac{\nu\delta_0}{\nu_0} \left\{ \frac{\frac{Q}{\delta_0} - C_0}{\delta_0 + \frac{\mu\nu}{\nu_0}} \right\} = \bar{E} \quad (\text{say}) \quad (7)$$

It may be noted here that the equilibrium $\bar{E}(\bar{C}, 0, \bar{E})$ exists provided $\frac{Q}{\delta_0} > C_0$. This condition implies that the tax would be imposed if the concentration of carbon dioxide (CO_2) is higher than that of its threshold concentration (C_0). But, it is obvious that $\frac{Q}{\delta_0} > C_0$ for the practical importance of the model system and hence $\bar{E}(\bar{C}, 0, \bar{E})$ exist without any condition.

2.1.2. Existence of $E^*(C^*, B^*, E^*)$

The positive solution of $E^*(C^*, B^*, E^*)$ is given by the following algebraic equations,

$$Q - \delta_0 C - \delta BC - \mu E = 0 \quad (8)$$

$$B = \frac{K}{r}(r + \pi\delta C) \quad (9)$$

$$E = \frac{\nu}{\nu_0}(C - C_0) \quad (10)$$

In view of (9) and (10), equation (8) can be written as

$$F(C) = Q - \delta_0 C - \delta \frac{K}{r}(r + \pi\delta C)C - \mu \frac{\nu}{\nu_0}(C - C_0) = 0 \quad (11)$$

From (11), we have,

$$F(0) = Q + \mu \frac{\nu}{\nu_0} C_0 > 0$$

$$F\left(\frac{Q}{\delta_0}\right) = -\delta \frac{K}{r} \left(r + \pi\delta \frac{Q}{\delta_0}\right) \frac{Q}{\delta_0} - \mu \frac{\nu}{\nu_0} \left(\frac{Q}{\delta_0} - C_0\right) < 0$$

$$F'(C) = -\delta_0 - \delta \frac{K}{r}(r + 2\pi\delta C) - \mu \frac{\nu}{\nu_0} < 0$$

This implies that, $F(C) = 0$ has a unique positive root (say C^*) in $0 < C \leq \frac{Q}{\delta_0}$ without any condition.

2.1.3. Variations of C with Different Parameters

Using (9) and (10) in equation (8) we have

$$\frac{\pi K}{r} \delta^2 C^2 + \left(\delta_0 + \delta K + \mu \frac{\nu}{\nu_0}\right) C - \left(Q + \mu \frac{\nu}{\nu_0} C_0\right) = 0 \quad (12)$$

Variation of C with δ

Differentiating (12) with respect to δ , we have

$$\frac{dC}{d\delta} = -\frac{\frac{2\pi K}{r} \delta C^2 + KC}{\frac{2\pi K}{r} \delta^2 C + \left(\delta_0 + \delta K + \mu \frac{\nu}{\nu_0}\right)} < 0$$

Thus $\frac{dC}{d\delta} < 0$, this implies that the concentration of carbon dioxide decreases as δ increases.

Variation of C with ν

Differentiating (12) with respect to ν , we have

$$\frac{dC}{d\nu} = -\frac{\frac{\mu}{\nu_0}(C - C_0)}{\frac{2\pi K}{r} \delta^2 C + \left(\delta_0 + \delta K + \mu \frac{\nu}{\nu_0}\right)} < 0$$

Therefore, $\frac{dC}{d\nu} < 0$. This implies that the concentration of carbon dioxide decreases as ν increases.

Remark: The above analysis implies that the equilibrium level of carbon dioxide decreases as the value of control parameters, discussed above, increases.

2.2. Stability Analysis

2.2.1. Local Stability of Equilibria

To establish the local stability behaviour of equilibria, we compute the following Jacobian matrix M for model system (1) – (3),

$$M = \begin{bmatrix} -(\delta_0 + \delta B) & -\delta C & -\mu \\ \pi\delta B & r\left(1 - \frac{2B}{K}\right) + \pi\delta C & 0 \\ \nu & 0 & -\nu_0 \end{bmatrix}$$

Let \bar{M} and M^* be the Jacobian matrices corresponding to $\bar{E}(\bar{C}, 0, \bar{E})$ and $E^*(C^*, B^*, E^*)$ respectively.

It can easily be found that the eigenvalues of Jacobian matrix M corresponding to $\bar{E}(\bar{C}, 0, \bar{E})$ are $-\delta_0$, $r + \pi\delta\bar{C}$ and $-\nu_0$. It means that the equilibrium $\bar{E}(\bar{C}, 0, \bar{E})$ is a saddle point which is locally stable in $C - E$ plane and unstable in B direction.

Now, we use Routh - Hurwitz criterion to study the local stability behaviour of the equilibrium $E^*(C^*, B^*, E^*)$. The eigenvalues of Jacobian matrix M corresponding to $E^*(C^*, B^*, E^*)$ are given by the following characteristic equation,

$$x^3 + a_1x^2 + a_2x + a_3 = 0 \tag{13}$$

where,

$$a_1 = (\delta_0 + \delta B^*) + \frac{r}{K} B^* + \nu_0$$

$$a_2 = (\delta_0 + \delta B^*) \frac{r}{K} B^* + \frac{r}{K} B^* \nu_0 + \nu_0(\delta_0 + \delta B^*) + \pi\delta^2 B^* C^* + \mu\nu$$

$$a_3 = (\delta_0 + \delta B^*) \frac{r}{K} B^* \nu_0 + \pi\delta^2 \nu_0 B^* C^* + \mu\nu \frac{r}{K} B^*$$

It can easily be seen that $a_1, a_2 > 0$ and $a_3 > 0$. Further,

$$a_1 a_2 - a_3 = (\delta_0 + \delta B^*) \left(\frac{r}{K} B^* \nu_0 + \nu_0(\delta_0 + \delta B^*) + \pi\delta^2 B^* C^* + \mu\nu \right) + \frac{r}{K} B^* \left((\delta_0 + \delta B^*) \frac{r}{K} B^* + \frac{r}{K} B^* \nu_0 + \nu_0(\delta_0 + \delta B^*) + \pi\delta^2 B^* C^* \right) + \nu_0 \left((\delta_0 + \delta B^*) \frac{r}{K} B^* + \frac{r}{K} B^* \nu_0 + \nu_0(\delta_0 + \delta B^*) + \mu\nu \right) > 0.$$

Since $a_1, a_2, a_3 > 0$ and $a_1 a_2 - a_3 > 0$, therefore, by Routh-Hurwitz criterion, eigenvalues of the Jacobian matrix corresponding to $E^*(C^*, B^*, E^*)$ are either negative or have negative real part without any condition. Thus, we have the following local stability theorem,

Theorem 2.2.1. The interior equilibrium $E^*(C^*, B^*, E^*)$ is locally asymptotically stable without any condition.

This theorem implies that if the initial state of dependent variables of the model system (1) - (3) is close to $E^*(C^*, B^*, E^*)$, the solution trajectories approach to the equilibrium $E^*(C^*, B^*, E^*)$ as $t \rightarrow \infty$. Thus, if the initial states of dependent variables C, B and E is near the equilibrium C^*, B^* and E^* , respectively, the concentration of carbon dioxide will finally get stabilized without any condition.

2.2.2. Nonlinear Stability of Equilibrium $E^*(C^*, B^*, E^*)$

In the following, the nonlinear stability behaviour of an interior equilibrium $E^*(C^*, B^*, E^*)$ inside the region of

attraction has been studied by using Liapunov's second method (LaSalle and Lefschetz [20]).

To establish the nonlinear stability behaviour of $E^*(C^*, B^*, E^*)$, we consider the following positive definite function,

$$V = \frac{1}{2} m_1 (C - C^*)^2 + m_2 \left(B - B^* - B^* \log \frac{B}{B^*} \right) + \frac{1}{2} m_3 (E - E^*)^2 \tag{14}$$

where m_i ($i=1,2,3$) are positive constants to be chosen appropriately.

Differentiating (14) with respect to 't' along the model system (1) - (3), we get,

$$\begin{aligned} \frac{dV}{dt} = & -m_1 \delta B (C - C^*)^2 - m_1 \delta_0 (C - C^*)^2 \\ & - m_2 \frac{r}{K} (B - B^*)^2 - m_3 \nu_0 (E - E^*)^2 \\ & + (-m_1 \delta C^* + m_2 \pi \delta) (C - C^*) (B - B^*) \\ & + (-m_1 \mu + m_3 \nu) (C - C^*) (E - E^*). \end{aligned}$$

Now choosing $m_1 = 1$, $m_2 = \frac{C^*}{\pi}$ and $m_3 = \frac{\mu}{\nu}$, $\frac{dV}{dt}$

will be negative definite inside the region of attraction Ω without any condition and hence the theorem.

Now, we state the nonlinear stability result of the interior equilibrium $E^*(C^*, B^*, E^*)$ in the form of the following theorem,

Theorem 2.2.2. The interior equilibrium $E^*(C^*, B^*, E^*)$ is nonlinearly asymptotically stable within the region of attraction Ω without any condition.

This theorem implies that for every initial start inside the region of attraction Ω solution always approaches to the equilibrium $E^*(C^*, B^*, E^*)$.

3. Numerical Simulation and Discussion

In this section, to study the local and nonlinear stability behaviour and feasibility of the model system (1) - (3) numerically, we have performed some numerical simulations, using MAPLE 7, by choosing the following set of parameters.

$$\begin{aligned} Q = 30, \delta_0 = 0.0002, \delta = 0.00008, \\ \mu = 0.01, K = 1500, r = 0.1, \pi = 0.75 \\ \nu = 0.3, \nu_0 = 0.25, C_0 = 100. \end{aligned}$$

The equilibrium values of $E^*(C^*, B^*, E^*)$ corresponding to above data are given as,

$$C^* = 211.616669, B^* = 1690.455003, E^* = 133.940000$$

The eigenvalues of Jacobian matrix corresponding to equilibrium $E^*(C^*, B^*, E^*)$ for the model system (1) - (3) are $-0.1389 + 0.0387i$, $-0.1389 - 0.0387i$ and -0.2202 . Since all the eigenvalues of Jacobian matrix corresponding to

$E^*(C^*, B^*, E^*)$ are either negative or have negative real part, therefore the interior equilibrium $E^*(C^*, B^*, E^*)$ is locally asymptotically stable. The nonlinear stability behaviour of interior equilibrium E^* in $C-B$, $C-E$ and $E-B$ planes has been shown in Figure 1, Figure 2 and Figure 3 respectively. From these figures, it is speculated that the solution trajectories started at any point within the region of attraction approach to equilibrium E^* .

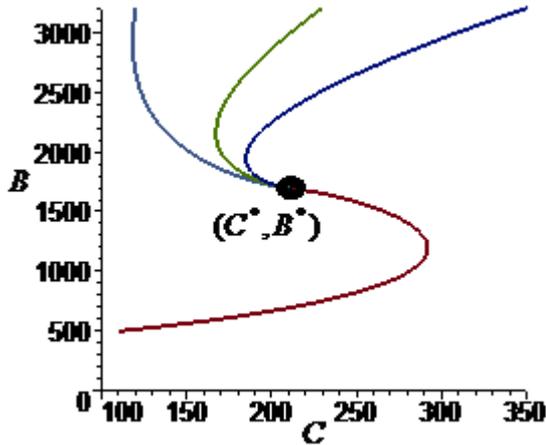


Figure 1. Nonlinear stability in $C-B$ plane

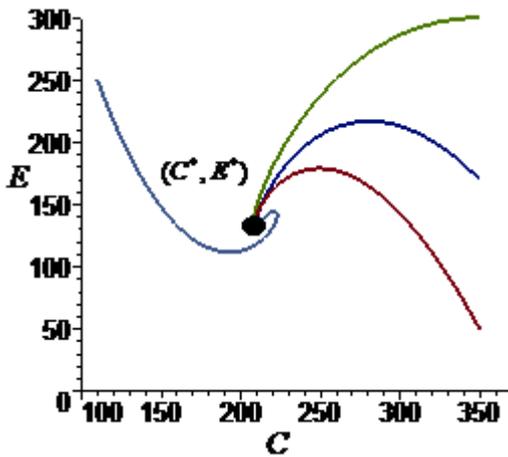


Figure 2. Nonlinear stability in $C-E$ plane

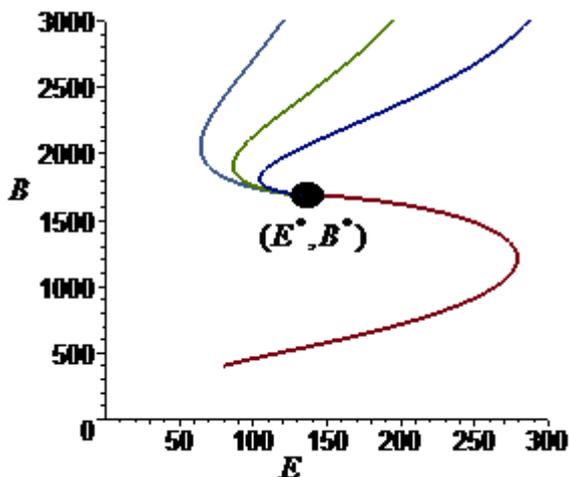


Figure 3. Nonlinear stability in $E-B$ plane

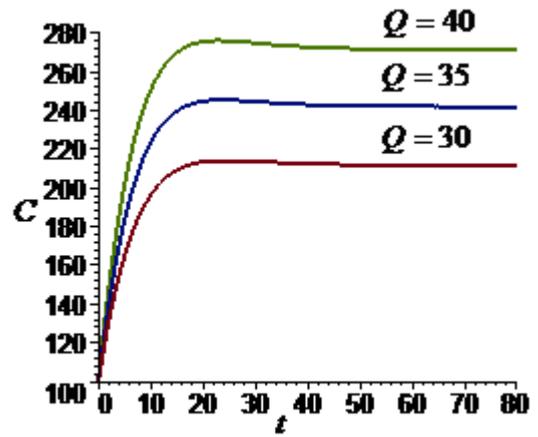


Figure 4. Variation of concentration C with time ' t ' for different values of Q

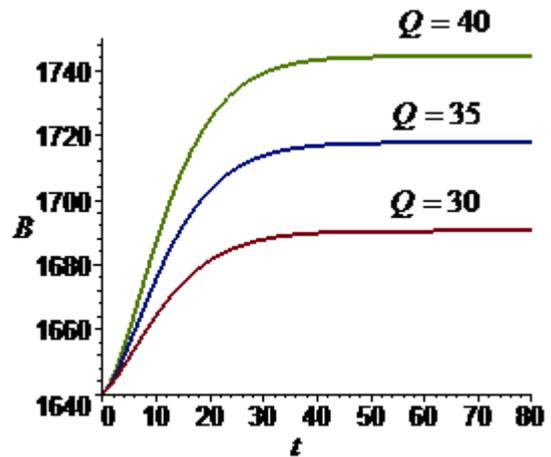


Figure 5. Variation of plant biomass density B with time ' t ' for different values of Q

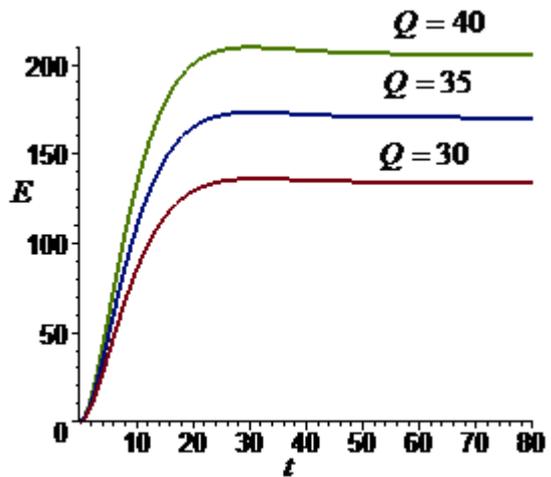


Figure 6. Variation of environmental tax E with time ' t ' for different values of Q

In Figure 4, Figure 5 and Figure 6 the variation of carbon dioxide concentration (C), plant biomass density (B) and environmental tax (E) is shown with respect to time for different values of Q , the emission rate of carbon dioxide in the atmosphere. From these figures, it is seen that with increase in the emission of carbon dioxide in the atmosphere, its concentration increases and also the plant biomass density increases. This increase in the level of carbon dioxide above its threshold results in increasing the

environmental tax (Figure 6). In Figure 7, the variation of carbon dioxide concentration with time is depicted for different values of ν , the tax rate coefficient. It is seen that the concentration of carbon dioxide decreases with increase in tax rate coefficient. This suggests that if the environmental tax is levied on the emitters, the level of carbon dioxide can be kept under control.

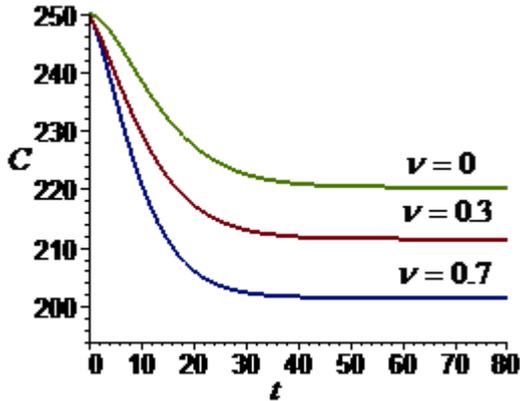


Figure 7. Variation of concentration C with time ' t ' for different values of ν

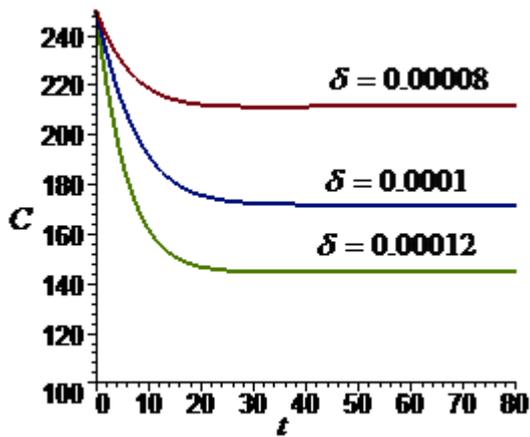


Figure 8. Variation of concentration C with time ' t ' for different values of δ

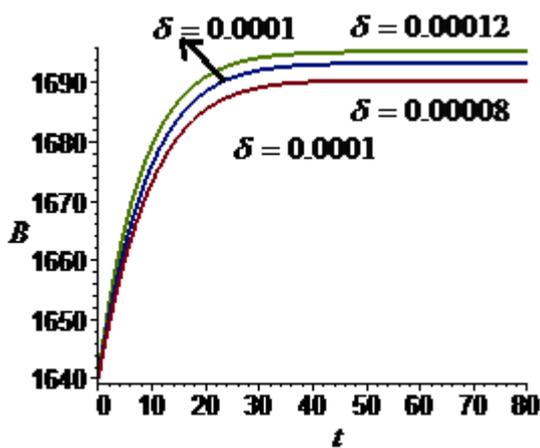


Figure 9. Variation of plant biomass density B with time ' t ' for different values of δ

In Figure 8, Figure 9 and Figure 10 we have shown the variation in carbon dioxide concentration in the atmosphere, plant biomass density and the environmental tax with time for different values of δ , the depletion rate

coefficient due to presence of biomass. It is observed that the concentration of carbon dioxide decreases in the atmosphere with increase in its depletion rate due to interaction with plant biomass. This decrease in carbon dioxide concentration in the atmosphere leads to lowering the environmental tax (Figure 10). The plant biomass density, however, increases due to higher depletion of carbon dioxide from the atmosphere for being used by plants for photosynthesis (Figure 9).

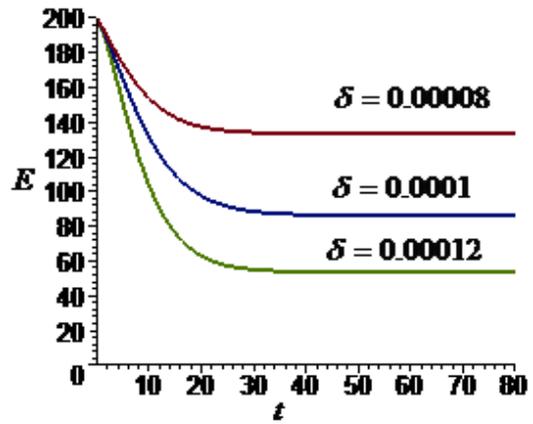


Figure 10. Variation of environmental tax E with time ' t ' for different values of δ

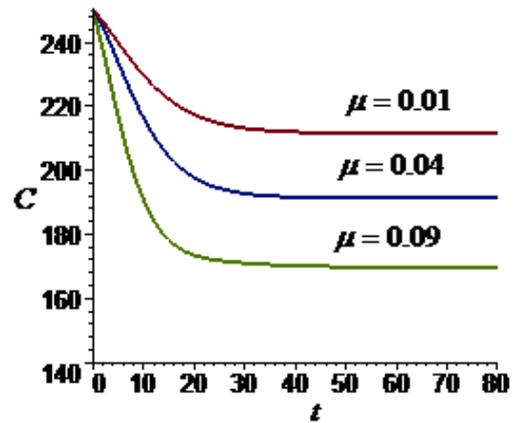


Figure 11. Variation of concentration C with time ' t ' for different values of μ

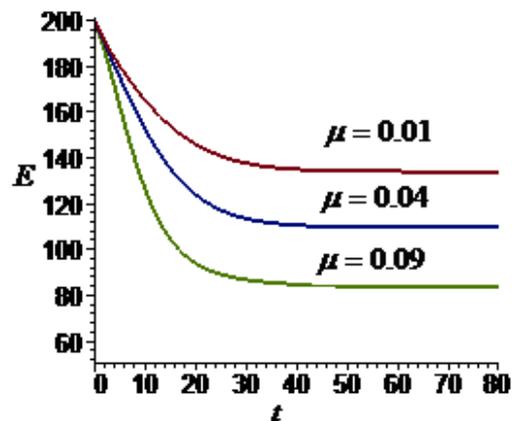


Figure 12. Variation of environmental tax E with time ' t ' for different values of μ

As the rate of diminution of carbon dioxide increases, the concentration of carbon dioxide decreases in the

atmosphere and as a consequence the burden of environmental tax decreases (Figure 11 and Figure 12 respectively). From the analysis, it is hypothesized that if the environmental tax is levied on the emitters of carbon dioxide above a threshold level, it may have a positive impact on emitters to keep the increasing emission of carbon dioxide under control.

4. Conclusion

Our main objective, in this paper, is to introduce environmental tax, as a control parameter, to be imposed on emitters in different forms for the sustainable development of environment and to stabilize the carbon dioxide concentration in the atmosphere to reduce global warming. It is assumed that the environmental tax is imposed on the emitters only when the emission of carbon dioxide crosses a permissible level, as discussed above. We have modeled the phenomenon considering three nonlinearly interacting dependent variables namely, concentration of carbon dioxide, biomass density of leafy trees and the environmental tax imposed on emitters. The existence and stability of non-negative equilibria for the proposed model system have been carried out. Equilibrium E^* is locally as well as nonlinearly stable without any condition. It is shown that analytically and numerically that when environmental tax is imposed on emitters, the concentration level of carbon dioxide in the atmosphere decreases. Thus, environmental tax is one of the important mitigation options to reduce global warming and to make our environment clean.

References

- [1] Agarwal, M. and Devi, S., The effect of environmental tax on the survival of biological species in a polluted environment: a mathematical model, *Nonlinear Analysis: Modeling and Control*, 15(3), 271-286, 2010.
- [2] Arman, H., Uncertainties, risks and challenges relating to CO₂ emissions and its possible impact on climate change in the United Arab Emirates, *Int. J. of Global Warming*, 8(1), 1-17, 2015.
- [3] Anderson, J.W., The Kyoto Protocol on Climate Change: Background, unresolved issues and next steps 1997, <http://www.rff.org/files/sharepoint/WorkImages/Download/RFF-RPT-kyotoprot.pdf>, Accessed 28 Dec 2015.
- [4] Battipaglia, G., Saurer, M., Cherubini, P., Calfapietra, C., McCarthy, H.R., Norby, R.J. and Cotrufo, M.F., Elevated CO₂ increases tree-level intrinsic water use efficiency: insights from carbon and oxygen isotope analyses in tree rings across three forest FACE sites, *New Phytologist*, 197, 544-554, 2013.
- [5] Bovenberg, A.L. and Heijdra, B.J., Environmental tax policy and intergenerational distribution, *Journal of Public Economics*, 67, 1-24, 1998.
- [6] Braathen, N.A. and Greene, J., Taxation, Innovation and the Environment. OECD, www.oecd.org/env/taxes/innovation, 2011, Accessed 28 Dec 2015.
- [7] Casper, J.K., *Greenhouse gases: Worldwide Impacts (Global Warming)*, Facts on File Inc., New York, 2010.
- [8] Chen, H-J. and Li, M-C., Environmental tax policy, habit formation and nonlinear dynamics, *Nonlinear Analysis: Real World Applications*, 12, 246-253, 2011.
- [9] Eskeland, G.S. and Lindstad, H., Environmental Taxation in the Transport Sector, *GGKP Third annual conference*, Venice, Italy, January 29-30, 2015.
- [10] Freedman, H.I. and So, J.W.H., Global stability and persistence of simple food chains, *Math. Bios.*, 7, 69-86, 1985.
- [11] Gorte, R.W., Carbon Sequestration in Forests, Congressional Research Service 2009, <https://www.fas.org/sgp/crs/misc/RL31432.pdf>, Accessed 28 Dec 2015.
- [12] He, J-W., Effect of Pollution Tax on Population in a Polluted Environment, *Applied Mathematical Sciences*, 4(60), 2993-2998, 2010.
- [13] He, J-W., Effect of pollution tax on population in a polluted environment, *Mathematical Methods in the Applied Sciences*, 34, 1440-1444, 2011.
- [14] Houghton, R.A., Byers, B. and Nassikas, A.A., A role for tropical forests in stabilizing atmospheric CO₂, *Nature Climate Change*, 5, 1022-1023, 2015.
- [15] IPCC., Implications of Proposed CO₂ Emissions Limitations, Technical paper 4, ISBN: 92-9169-103-8, 1997.
- [16] Kanabar, D., Tax trends in emerging India: A survey. KPMG, kpmg.com/in, 2011.
- [17] Khasnis, A.A. and Nettleman, M.D., Global warming and infectious disease, *Arch. Med. Res.*, 36(6), 689-696, 2005.
- [18] Karp, L. and Rezai, A., The political economy of environment policy with overlapping generations, *International Economic review*, 55(3), 711-733, 2014.
- [19] Kurane, I., The effect of global warming on infectious diseases, *Osong Public Health Res. Perspect.*, 1, 4-9, 2010.
- [20] Lasalle, J.P. and Lefschetz, S., *Stability by Liapunov's Direct Method with Applications*, Academic Press, New York, 1961.
- [21] Liu, A.A., Tax Evasion and Optimal Environmental Taxes, Resources for the Future. Washington DC, USA, 2012.
- [22] Madhu, M. and Hatfield, J.L., Dynamics of plant root growth under increased atmospheric carbon dioxide, *Agronomy Journal*, 105(3), 657-669, 2013.
- [23] Malhi, Y. and Grace, J., Tropical forests and atmospheric carbon dioxide, *Trends Ecol. Evol.*, 15, 332-337, 2000.
- [24] Marland, G., Boden, T.A. and Andres, R.J., Global, regional, and national CO₂ emissions. In: *Trends: A Compendium of Data on Global Change*. Oak Ridge, Tenn.: Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, 2007 http://cdiac.ornl.gov/trends/emis/tre_glob.htm.
- [25] McPherson, E.G., Atmospheric carbon dioxide reduction by Sacramento's urban forest, *Journal of Arboriculture*, 24(4), 215-223, 1998.
- [26] Ninan, K.N. and Bedamatta, S., Climate change, agriculture, poverty and livelihoods: A status report, The Institute for Social and Economic Change, Bangalore, 2012.
- [27] NOAA., Atmospheric CO₂ concentrations (ppm) since March 1958, 2014, Accessed 28 Dec 2015.
- [28] Norregaard, J. and Reppelin-Hill, V., Controlling pollution using taxes and tradable permits, International Monetary Fund, <http://www.imf.org/external/pubs/ft/issues/issues25/2000>, Accessed 28 Dec 2015.
- [29] Ono, T., Environmental tax policy in a model of growth cycles, *Economic Theory*, 22, 141-168, 2003.
- [30] Poorter, H., Pot, C.S. and Lambers, H., The effect of an elevated atmospheric CO₂ concentration on growth, photosynthesis and respiration of *Plantago major*, *Physiologia Plantarum*, 73(4), 553-559, 1988.
- [31] Rautela, P. and Karki, B., Impact of climate change on life and livelihood of indigenous people of higher himalaya in Uttarakhand, India, *American Journal of Environmental Protection*, 3(4), 112-124, 2015.
- [32] Rezai, A., Foley, D.K. and Taylor, L., Global warming and economic externalities, *Economic Theory*, 49(2), 329-51, 2012.
- [33] Robinson, A.B., Robinson, N.E. and Soon, W., Environmental effects of increased atmospheric carbon dioxide, *Journal of American Physicians and Surgeons*, 12(3), 79-90, 2007.
- [34] Sterner, T., Gasoline Taxes: A useful instrument for climate policy, *Energy Policy*, 35, 3194-3202, 2007.
- [35] Sundar, S. and Naresh, R., Modeling and analysis of the survival of a biological species in a polluted environment: Effect of environmental tax, *Computational Ecology and Software*, 5(2), 201-221, 2015.
- [36] Symons, E., Proops, J. and Gay, P., Carbon Taxes, consumer demand and carbon dioxide emissions: A simulation analysis for the UK, *Fiscal Studies*, 15(2), 19-43, 1994.
- [37] Tennakone, K., Stability of the biomass-carbon dioxide equilibrium in the atmosphere: mathematical model, *Appl. Math. Comput.*, 35, 125-130, 1990

- [38] Usman, U., Yelwa, S.A., Gulumbe, S.U. and Danbaba, A., An assessment of the changing climate in Northern Nigeria using Cokriging, *American Journal of Applied Mathematics and Statistics*, 1(5), 90-98, 2013.
- [39] WHO, Global health risks: mortality and burden of disease attributable to selected major risks. http://www.who.int/healthinfo/global_burden_disease/GlobalHealthRisks_report_full.pdf, 2009, Accessed 11 Dec 2015
- [40] Yazaki, K., Maruyama, Y., Mori, S., Koike, T. and Funada, R., Effects of elevated carbon dioxide concentration on wood structure and formation in trees, *Plant responses to air pollution and global change*, (Edited by K. Omasa, I. Nouchi, and L J. De Kok: Springer-Verlag, Tokyo), 89-97, 2005.