

A New Numerical Method for Solving First Order Differential Equations

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Abstract In this paper, we present a new numerical method for solving first order differential equations. The new numerical integration scheme was obtained which is particularly suited to solve oscillatory and exponential problems. We verify the reliability of the new scheme and the results obtained show that the scheme is computationally reliable, and competes favourably with other existing ones.

Keywords: numerical integration, oscillatory and exponential problems, algorithms

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1. Introduction

We shall consider the initial value problem of the form

$$y' = f(x, y), y(x_0) = y_0. \quad (1)$$

Which in time past, many scholars have derived different polynomial integrating functions to produce schemes capable of solving ordinary differential equations. Therefore we shall developed numerical scheme to solve problem (1) in tune with those developed by Fatunla [1,2], Ibijola [3], and Ogunrinde et.al [4].

2. Derivation of New Scheme

The mathematical formulation of physical phenomena in almost every sphere of human endeavor, be it engineering, control theory, biological, and economics often leads to one or a set of nth order differential equations in the form

Let us assume that the theoretical solution $y(x)$ to the initial value problem (1) can be locally represented in the interval $[x_n, x_{n+1}]$, $n \geq 0$ by the non-polynomial interpolating function;

$$F(x) = (\alpha_1 + \alpha_2)e^{-2x} + \alpha_3x^2 + \alpha_4x + \alpha_5 \quad (2)$$

Where $\alpha_1, \alpha_2, \alpha_3$ and α_4 are real undetermined coefficients and α_5 is a constant. We shall assume y_n is a numerical estimate to the theoretical solution $y(x)$ and $f_n = f(x_n, y_n)$.

We define mesh points as follows:

$$\begin{aligned} x_n &= a + nh, \\ n &= 0, 1, 2, \dots \end{aligned} \quad (3)$$

Imposing the following constraints on the interpolating function (2) in order to get the undetermined coefficients.

2.1. The Interpolating Function.

a. The interpolating function must coincide with the theoretical solution at

$$x = x_n \text{ and } x = x_{n+1}.$$

Hence we required that

$$\begin{aligned} F(x_n) & \\ &= (\alpha_1 + \alpha_2)e^{-2x_n} + \alpha_3x_n^2 + \alpha_4x_n + \alpha_5 \end{aligned} \quad (4)$$

$$\begin{aligned} F(x_{n+1}) &= (\alpha_1 + \alpha_2)e^{-2x_{n+1}} + \alpha_3x_{n+1}^2 \\ &+ \alpha_4x_{n+1} + \alpha_5 \end{aligned} \quad (5)$$

b. Secondly, the derivatives of the interpolating function are required to coincide with the differential equation as well as its first, second, and third derivatives with respect to x at $x = x_n$. We denote the i th total derivatives of $f(x, y)$ with respect to x with $f^{(i)}$ such that

$$F^1(x_n) = f_n \quad (6)$$

$$F^2(x_n) = f_n^1 \quad (7)$$

$$F^3(x_n) = f_n^2 \quad (8)$$

2.2. The Derivatives of the Interpolant

$$f_n = -2(\alpha_1 + \alpha_2)e^{-2x_n} + 2\alpha_3x_n + \alpha_4 \quad (9)$$

$$f_n^1 = 4(\alpha_1 + \alpha_2)e^{-2x_n} + 2\alpha_3 \quad (10)$$

$$f_n^2 = -8(\alpha_1 + \alpha_2)e^{-2x_n} \quad (11)$$

Solving for $\alpha_1 + \alpha_2$ from equation (11), we have

$$(\alpha_1 + \alpha_2) = -\frac{1}{8} f_n^2 e^{2x_n} \tag{12}$$

Substituting (12) into (10), we have

$$\alpha_3 = \frac{1}{2} (f_n^1 + \frac{1}{2} f_n^2) \tag{13}$$

Substituting (12) and (13) into (9), we have

$$\alpha_4 = (f_n - \frac{1}{4} f_n^2) - (f_n^1 + \frac{1}{2} f_n^2) x_n \tag{14}$$

Since $F(x_{n+1}) = y(x_{n+1})$ and $F(x_n) = y(x_n)$
Implies that

$$y(x_{n+1}) = y_{n+1} \text{ and } y(x_n) = y_n \tag{15}$$

$$F(x_{n+1}) - F(x_n) = y_{n+1} - y_n$$

Then we shall have

$$y_{n+1} - y_n = (\alpha_1 + \alpha_2) [e^{-2x_{n+1}} - e^{-2x_n}] + \alpha_3 [x_{n+1}^2 - x_n^2] + \alpha_4 [x_{n+1} - x_n] \tag{16}$$

Recall that

$$x_n = a + nh, x_{n+1} = a + (n+1)h \text{ with } n = 0, 1, 2, \dots \tag{17}$$

Substitute (12), (13) and (14) into (16), we have

$$y_{n+1} = y_n - \frac{1}{8} f_n^2 (e^{-2h} - 1) + \frac{1}{2} (f_n^1 + \frac{1}{2} f_n^2) h^2 + (f_n - \frac{1}{4} f_n^2) h \tag{18}$$

Hence (18) is the new schemes for solution of the first order differential equation.

3. The Implementation of the Scheme

Example 1

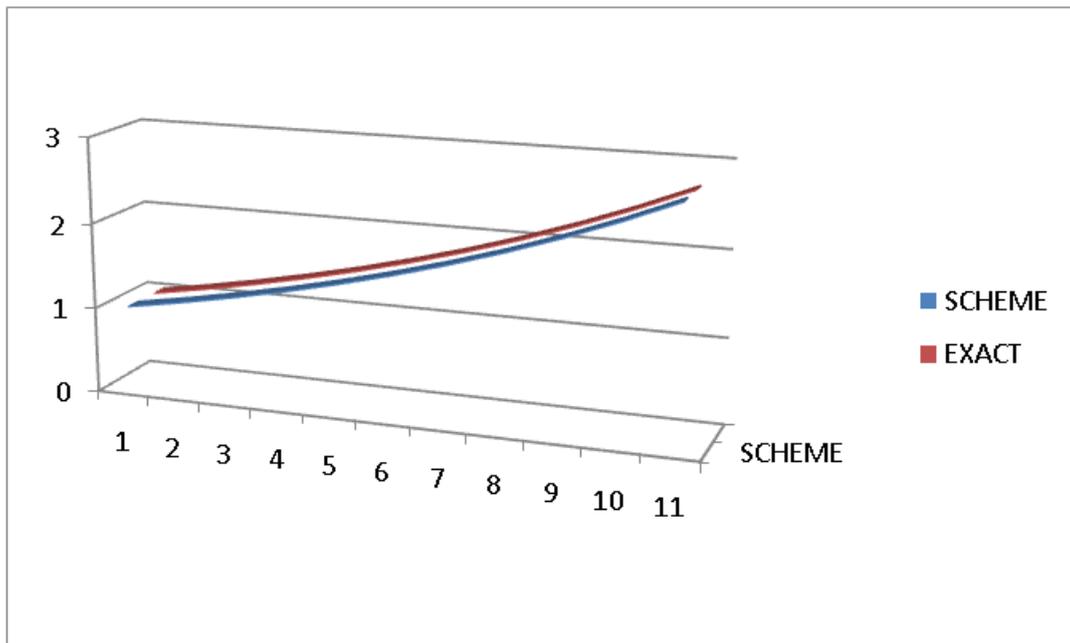
Table 3.1

Using scheme (18) to solve the initial value problem

$y' = y, y(0) = 1, \text{ in the interval } 0 \leq x \leq 1, \text{ The theoretical solution } y(x) = e^x, h = 0.1$

S/N	Xn	Scheme	Exact	Error
[0]	[0.00]	[1.0000000000000000]	[1.0000000000000000]	[0.0000000000000000]
[1]	[0.10]	[1.105158655865252]	[1.105170918075648]	[1.226221039551945e-005]
[2]	[0.20]	[1.221389206321850]	[1.221402758160170]	[1.355183832019158e-005]
[3]	[0.30]	[1.349843830478405]	[1.349858807576003]	[1.497709759790133e-005]
[4]	[0.40]	[1.491808145388568]	[1.491824697641270]	[1.655225270247307e-005]
[5]	[0.50]	[1.648702977631813]	[1.648721270700128]	[1.829306831546695e-005]
[6]	[0.60]	[1.822098583423405]	[1.822118800390509]	[2.021696710463594e-005]
[7]	[0.70]	[2.013730364266381]	[2.013752707470477]	[2.234320409577606e-005]
[8]	[0.80]	[2.225516235433084]	[2.225540928492468]	[2.469305938346267e-005]
[9]	[0.90]	[2.459575821105841]	[2.459603111156950]	[2.729005110868599e-005]
[10]	[1.00]	[2.718251668288208]	[2.718281828459046]	[3.016017083767864e-005]

Hence the graph of Table 3.1



Example 2.

Table 3.2

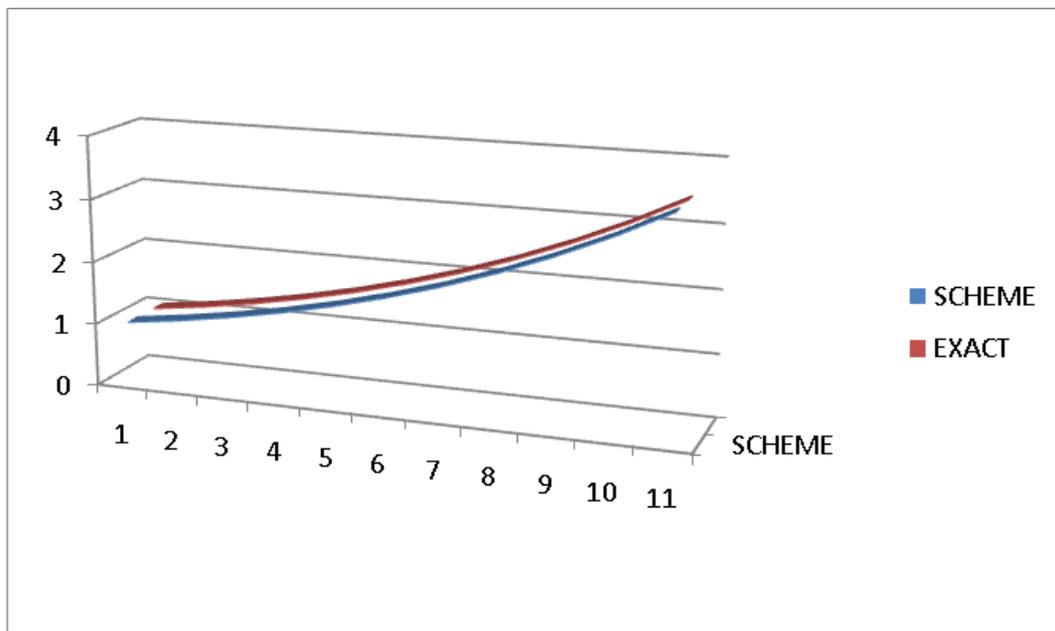
Using scheme (18) to solve the initial value problem

$y' = x^2 + y, y(0) = 1, \text{ in the interval } 0 \leq x \leq 1$

The theoretical solution $y(x) = -2 - 2x - x^2 + 3e^x, h = 0.1$

S/N	Xn	Scheme	Exact	Error
[0]	[0.00]	[1.0000000000000000]	[1.0000000000000000]	[0.0000000000000000]
[1]	[0.10]	[1.110317311730505]	[1.110341836151295]	[2.452442079081685e-005]
[2]	[0.20]	[1.242778412643699]	[1.242805516320340]	[2.710367664016111e-005]
[3]	[0.30]	[1.399687660956810]	[1.399717615152007]	[2.995419519646880e-005]
[4]	[0.40]	[1.583616290777136]	[1.583649395282541]	[3.310450540472409e-005]
[5]	[0.50]	[1.797405955263626]	[1.797442541400256]	[3.658613663071186e-005]
[6]	[0.60]	[2.044197166846809]	[2.044237600781018]	[4.043393420927188e-005]
[7]	[0.70]	[2.327460728532762]	[2.327505414940953]	[4.468640819110803e-005]
[8]	[0.80]	[2.651032470866169]	[2.651081856984936]	[4.938611876692534e-005]
[9]	[0.90]	[3.019151642211683]	[3.019206222313899]	[5.458010221648380e-005]
[10]	[1.00]	[3.436503336576414]	[3.436563656918091]	[6.032034167668954e-005]

Hence the graph of Table 3.2



Example 3

Table 3.3

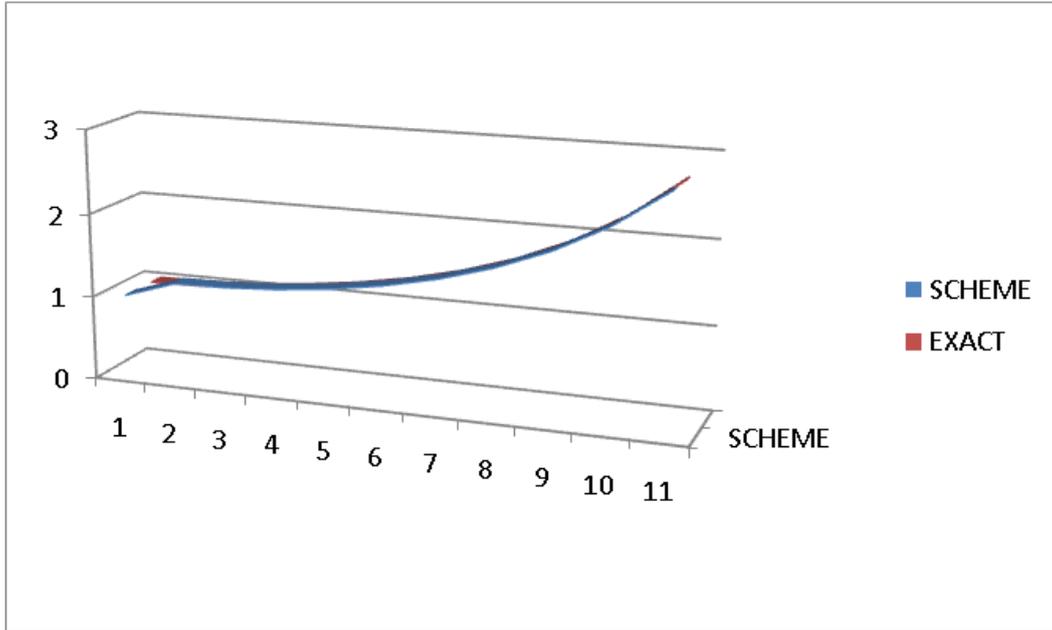
Using scheme (18) to solve the initial value problem

$y' = 2xy, y(0) = 1, \text{ in the interval } 0 \leq x \leq 1$

The theoretical solution $y(x) = e^{x^2}, h = 0.1$

S/N	Xn	Scheme	Exact	Error
[0]	[0.00]	[1.0000000000000000]	[1.0000000000000000]	[0.0000000000000000]
[1]	[0.10]	[1.2000000000000000]	[1.010050167084168]	[0.189949832915832]
[2]	[0.20]	[1.212263492537484]	[1.040810774192388]	[0.171452718345096]
[3]	[0.30]	[1.249816146017192]	[1.094174283705210]	[0.155641862311982]
[4]	[0.40]	[1.315016151145149]	[1.173510870991810]	[0.141505280153339]
[5]	[0.50]	[1.412063606381248]	[1.284025416687741]	[0.128038189693506]
[6]	[0.60]	[1.547454345272219]	[1.433329414560340]	[0.114124930711879]
[7]	[0.70]	[1.730708227029939]	[1.632316219955379]	[0.098392007074560]
[8]	[0.80]	[1.975486794533931]	[1.896480879304952]	[0.079005915228979]
[9]	[0.90]	[2.301284446910632]	[2.247907986676472]	[0.053376460234161]
[10]	[1.00]	[2.735985639959573]	[2.718281828459046]	[0.017703811500527]

Hence the graph of Table 3.3



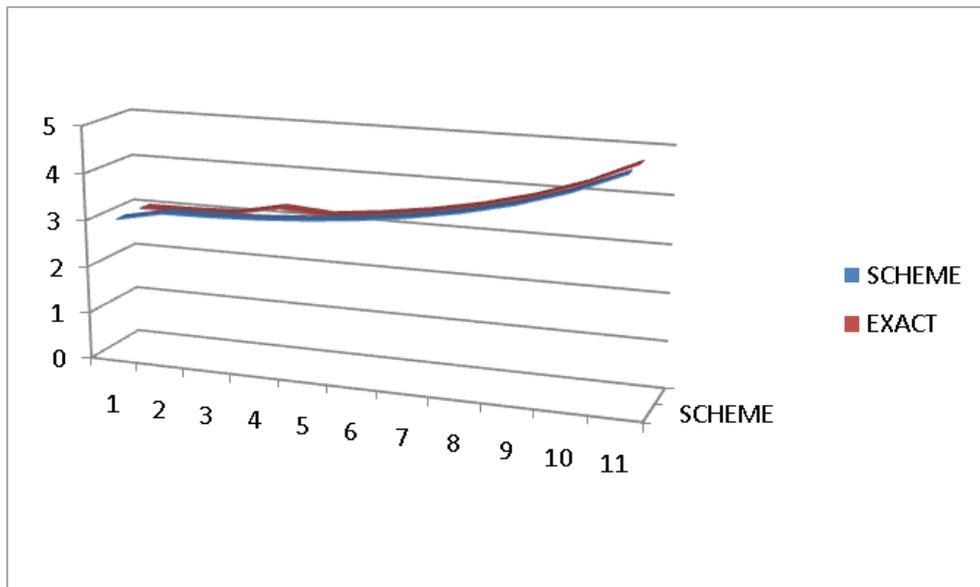
Example 4

Table 3.4

Using scheme (18) to solve the initial value problem $y' = 2xy + 4x, y(0) = 1$, in the interval $0 \leq x \leq 1$,
 The theoretical solution $y(x) = 3e^{x^2} - 2, h = 0.1$

S/N	Xn	Scheme	Exact	Error
[0]	[0.00]	[3.0000000000000000]	[3.0000000000000000]	[0.0000000000000000]
[1]	[0.10]	[3.2000000000000000]	[3.010050167084168]	[0.189949832915832]
[2]	[0.20]	[3.212263492537484]	[3.040810774192388]	[0.171452718345096]
[3]	[0.30]	[3.249816146017192]	[3.094174283705210]	[0.155641862311982]
[4]	[0.40]	[3.315016151145149]	[3.173510870991811]	[0.141505280153338]
[5]	[0.50]	[3.412063606381248]	[3.284025416687741]	[0.128038189693506]
[6]	[0.60]	[3.547454345272219]	[3.433329414560340]	[0.114124930711878]
[7]	[0.70]	[3.730708227029940]	[3.632316219955379]	[0.098392007074561]
[8]	[0.80]	[3.975486794533931]	[3.896480879304952]	[0.079005915228979]
[9]	[0.90]	[4.301284446910633]	[4.247907986676472]	[0.053376460234161]
[10]	[1.00]	[4.735985639959573]	[4.718281828459046]	[0.017703811500527]

Hence the graph of Table 3.4



4. Summary and Conclusion

The following can be deduced from this numerical experiment as summary and conclusion

(i) The results obtained from the implementation of the scheme revealed that the approximation/numerical solution and the exact/theoretical solution obtained leads to error that is miniature in size.

(ii) The smaller the step-size “h” the more accurate is the numerical methods.

(iii) It can be clearly seen from the results that the scheme can be used to solve any standard initial value problems as the scheme converges easily.

(iv) Since the error is not blown off, it showed the reasonability of the stability of the scheme.

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