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Statistical Simulations of Galactic Planetary Nebulae

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Abstract In the present paper, two catalogues $J/A+A/327/736$ and $J/A+A/541/A98$ of VizieR database were used for the statistical simulations of galactic planetary nebulae. Each catalogue was utilized for certain purposes, the first catalogue for, the correlation coefficients between the entireties of the data, the determination of the position of the maximum (minimum) of each entry in the data together with the values of the other entries at this position. In addition, we deuced the histograms and descriptive, location, dispersion and shape statistics for the entries of the data. Finally, the best fit and its error analysis was established between LH &LR where LH is the logarithm of HI Zanstrs luminosity and LR is the logarithm of nebular radius. The second catalogue was used to display two-dimensional distributions of the interacting planetary nebulae, moreover IAU and other references used the equatorial coordinates of these nebulae to determine the equatorial coordinates of the north galactic pole (NGP). The results of this application are in good agreement with those given.

Keywords: *statistical simulations of planetary nebulae, two-dimensional distributions of the interacting planetary nebulae, equatorial coordinates of the north galactic pole*

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1. Introduction

A planetary nebula is created when a star blows off its outer layers after it has run out of fuel to burn. These outer layers of gas expand into space, forming a nebula, which is often the shape of a ring, bubble or other types ([4,5,11]). Only about 20% of planetary nebulae are spherically symmetric (for example, see Abell 39) [2] and likely produced by the old stars similar to the Sun [8]. Planetary nebulae play a very important role in galactic evolution. The early universe consisted almost entirely of hydrogen and helium, but stars create heavier elements via nuclear fusion. In recent studies ([3,9]), out of 200 billion stars, about 3500 planetary nebulae are now known to exist in our galaxy, more than double what it was a decade ago. They are found mostly near the plane of the Milky Way, with the greatest concentration near the galactic center [7].

The present paper, is devoted for the statistical simulations of galactic planetary nebulae, for such studies we used two catalogues and of VizieR database.

The first catalogue (its original reference is [14]) was utilized for establishing: (1) The correlation coefficients between the entireties of the data, (2) the position of the maximum (minimum) of each entry in the data together with the values of the other entries at this position, (3) the histograms of the entireties, (4) descriptive location, dispersion and shape statistics, (5) finally the best and its error analysis was developed between LH &LR where LH is the logarithm of HI Zanstrs luminosity and LR is logarithm of nebular radius.

The second catalogue (the original reference of the catalogue is [1]) was utilized for establishing the two dimensional distribution of the its nebulae and for determining the equatorial coordinates of the north galactic pole (NGP). The results of this application are in good agreements with those of IAU(1959), [15] and [13]. A good review to the history of the Galactic coordinate system is given in [6].

2. Statistical Analysis of Planetary Nebulae Properties

For the present analysis, we used catalogue: of VizieR database (the original reference of the catalogue is [14]). What concerning us among the entries of the catalogue are the rows {6, 7, 8, 9, 10, 12, 16, 17}, the explanation of each row together with its used abbreviation are listed in Table 1.

2.1. Correlations between the entries

A correlation coefficient is a statistical measure of the degree to which two variables are related to each other.

The linear correlation coefficient between (x_i, y_i), $i = 1, 2, \ldots, N$ is

$$r = \frac{N\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{\sqrt{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \times \sqrt{N\sum_{i=1}^{N} y_i^2 - \left(\sum_{i=1}^{N} y_i\right)^2}} \quad (1)$$

This coefficient is a dimensionless quantity; that is, it does not depend on the units employed. Its value is always between, such that, the closest the value of $|r|$ to one is the more correlation between the two variables will be. The sign of r only fells us whether y is increasing or decreasing when x increases.

We find for the correlation coefficients between the entries of Table 1 (N=49), the values listed in Table 2.

Table 1. Explanations and the used abbreviations of the catalogue entries

Name	Explanation	Abbreviation
log[THI] phys. temperature	logarithm of HI Zanstrs temperature	LT
log[LHI] phys. luminosity	logarithm of HI Zanstrs luminosity	LH
Mv phys. mag.Abs	central star absolute visual magnitude	MV
log[Rad] phys. size.radius	logarithm of nebular radius	LR
log[SuBr] phto. flux.sb	logarithm of nebular surface brightness in Hβ	LS
DistS pos.distance	Shklovsky distance	Dists
Age 10^3 year time age	Derived evolutionary age	Age
Dist kpc pos.distance	Derived distance	Dist

Table 2. Correlation coefficients between the entries of Table 1.

	LH	MV	LR	LS	Dists	Age	Dist
LT	-0.353	0.698	0.218	-0.561	-0.355	0.082	-0.273
LH		-0.916	-0.978	0.970	0.744	-0.403	0.683
MV			0.844	-0.983	-0.722	0.348	-0.644
LR				-0.927	-0.725	0.444	-0.715
LS					0.749	0.444	-0.715
Dists						-0.303	0.778
Age							-0.235

2.2. The Maxima and Minima

Table 3 (Table 4) gives the position of the maximum (minimum) of each entry in the data together with the values of the other entries at this position.

Table 3. The position of the maximum of each entry V and the values of the other entries at this position

V	M	$(LT)_H$	$(LH)_H$	$(MV)_H$	$(LR)_H$	$(LS)_H$	$(Dists)_H$	$(Age)_H$	$(Dist)_H$
LT	45	5.24	2.65	6.18	-0.74	-7.92	2.8	23.5	1.9
LH	22	4.29	5.11	-6.4	-1.25	-1.88	23.4	4.1	4.1
MV	31	4.81	0.61	8.13	-0.12	-9.94	0.3	0.0	0.0
LR	31	4.81	0.61	8.13	-0.12	-9.94	0.3	0.0	0.0
LS	22	4.29	5.11	-6.4	-1.25	-1.88	23.4	4.1	4.1
Dists	14	4.58	4.58	-3.32	-1.38	-2.84	34.4	2.6	13.9
Age	29	4.67	2.13	3.4	-0.6	-7.09	8.5	41.1	8.5
Dist	14	4.58	4.58	-3.32	-1.38	-2.84	34.4	2.6	13.9

Table 4. The position of the minimum of each entry V and the values of the other entries at this position

V	m	(LT)m	(LH)m	(MV)m	(LR)m	(LS)m	(Dists)m	(Age)m	(Dist)m
LT	22	4.29	5.11	-6.4	-1.25	-1.88	23.4	4.1	4.1
LH	31	4.81	0.61	8.13	-0.12	-9.94	0.3	0.0	0.0
MV	22	4.29	5.11	-6.4	-1.25	-1.88	23.4	4.1	4.1
LR	6	4.54	4.74	-4.0	-1.41	-2.51	26.9	1.6	9.6
LS	31	4.81	0.61	8.13	-0.12	-9.94	0.3	0.0	0.0
Dists	31	4.81	0.61	8.13	-0.12	-9.94	0.3	0.0	0.0
Age	21	5.11	1.57	7.85	-0.41	-9.24	1.2	0.0	0.0
Dist	21	5.11	1.57	7.85	-0.41	-9.24	1.2	0.0	0.0

where M (m) of Table 3 (4) is used to denote the position of the maximum (minimum) of V.

2.3. The Histograms

The following figures show the histograms of the entries

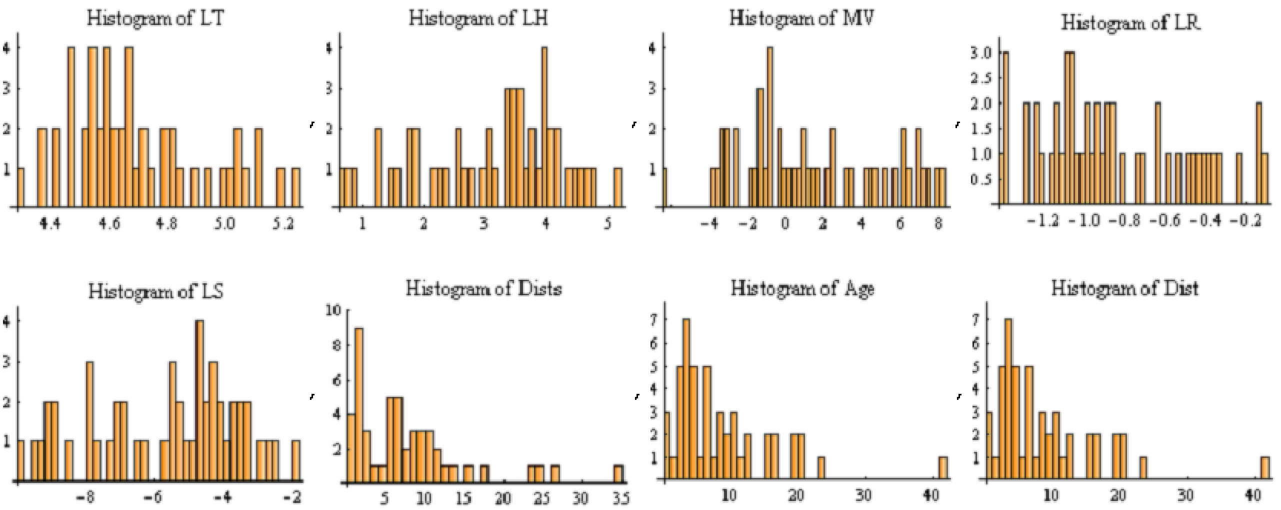

Figure 1. Histograms of the entries

2.4. Descriptive Statistics

The following are some descriptive statistics of the entries

2.4.1. Basic Descriptive Statistics

Table 5. Basic Descriptive statistics of entries

V	Mean	Median	Variance
LT	4.69184	4.64	0.0547278
LH	3.06633	3.36	1.31614
MV	1.25224	0.52	13.9265
LR	-0.867143	-0.95	0.130879
LS	-5.69265	-5.23	4.57117
Dists	7.73878	6.3	53.8012
Age	8.69592	6.5	59.7346
Dist	5.32041	5.6	13.7937

2.4.2. Location Statistics

Table 6. Location statistics of entries

V	Root Mean Square
LT	4.69755
LH	3.26981
MV	3.90005
LR	0.93816
LS	6.07323
Dists	10.6109
Age	11.5816
Dist	6.46676

2.4.3. Location Statistics

Table 7. Dispersion statistics of entries

V	Vr. M	S.E.S.M	C. Var	M. D	Med. D	S. Ra
LT	0.00111689	0.03342	0.049861	0.189188	0.15	0.95
LH	0.0268601	0.16389	0.374139	0.950879	0.71	4.5
MV	0.284213	0.533117	2.9801	3.13236	2.63	14.53
LR	0.002671	0.0516818	-0.4172	0.298717	0.24	1.29
LS	0.0932892	0.305433	-0.375577	1.81998	1.67	8.06
Dists	1.09798	1.04785	0.947815	5.24182	3.9	34.1
Age	1.21907	1.10412	0.888786	5.72153	3.6	41.1
Dist	0.281505	0.53057	0.698065	3.08855	3.4	13.9

where, $Vr.M. \equiv$ variance of mean, $S.E.S.M. \equiv$ standard error of sample mean, $C.Var. \equiv$ coefficient of variation, $M.D. \equiv$ mean deviation, $Med.D. \equiv$ median deviation and $S.Ra. \equiv$ sample range.

2.4.4. Shape Statistics

Table 8. Shape statistics of entries

V	Skewness	Pearson Skewness1
LT	0.616807	1.43417
LH	-0.478641	-2.36309
MV	0.256835	0.717274
LR	0.486573	1.59927
LS	-0.356673	-0.901746
Dists	1.67154	2.75617
Age	1.82368	1.97802
Dist	0.253014	4.29759

2.5. Best Fit between LH &LR and Its Error Analysis

1. We established for the best fit between $LH(\equiv y)$ & LR($\equiv$ x) the analytical representations:

$$y = Q + \left(\frac{A + Exp[a_1 + b_1 x]}{B + Exp[a_2 + b_2 x]} \right)^2 \quad (2)$$

where the coefficients and their probable errors are:

$$a_1 = -3.62204 \pm 2.45439$$
$$b_1 = 1.71915 \pm 0.493344$$
$$a_2 = 2.96017 \pm 0.767171$$
$$b_2 = 0.392709 \pm 0.181332$$
$$A = -93.9761 \pm 6.52625$$
$$B = 57.6549 \pm 10.6897$$
$$Q = -1.41011 \pm 0.0198879$$

2. The estimated variance = 0.0020664 .
3. Residuals at some points

Table 9. Residuals at some points of the entries

Point	Residual
1	-0.0318278
6	0.0000403678
11	-0.0374801
16	-0.0465094
21	-0.00348401
26	0.0554333
31	-0.0154322
36	-0.000721013
41	-0.0320737

4. The mean prediction confidence interval for some values of x are shown in the following table:

Table 10. The mean prediction confidence interval

x	Predicted	Standard Error	Confidence Interval
-1.01	-0.978172	0.011099	(-1.0007, -0.95564)
-1.3	-1.32601	0.016672	(-1.35985, -1.29216)
-1.09	-1.04349	0.0105586	(-1.06493, -1.02206)
-0.6	-0.583292	0.0139538	(-0.61162, -0.554964)
-0.82	-0.772599	0.014092	(-0.801208, -0.743991)
-0.71	-0.709279	0.0138444	(-0.737385, -0.681173)
-1.14	-1.13975	0.0129654	(-1.16607, -1.11343)

3. Galactic Pole Using Planetary Nebulae

For the present analysis, we used catalogue: $J/A+A/541/A98$ of VizieR database (the original reference of the catalogue is [1]), hereafter will be referred to as Paper I). What concerning us among the entries of the catalogue, are the galactic coordinates and the equatorial coordinates (right ascension & declination).

The sample used in Paper I is 117 planetary nebulae, the two dimensional distribution of the sample is shown in Figure 2.

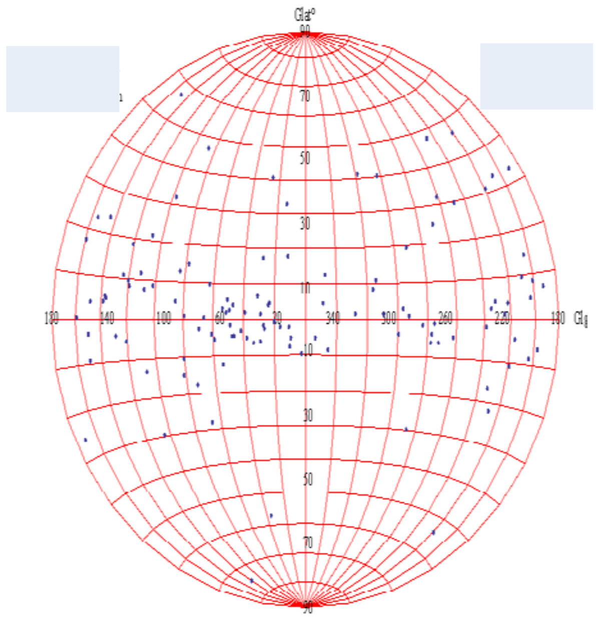

Figure 2. Two-dimensional distribution of the sample

Figure 2 confirms the result of Paper I in that, the majority of nebulae of the sample are located close to the galactic plane.

For the determination of the equatorial coordinates of the galactic pole, (α_p, δ_p) we obviously shall except to get the best results choosing objects, which exhibit strong concentration towards the galactic plane. So the data of the sample of Paper I is suitable for such determination

3.1. Analytical Developments

It was shown that [12], the determination of (α_p, δ_p) is a typical constrained minimization problem with:

1. the objective function is the sum of the N (say) squared distances of the selected objects from the plane of galactic plane.

$$f_1(\ell, m, n) = \sum_{i=1}^{N} D_i^2 = \sum_{i=1}^{N} \left(\left(\ell\, x_i + m\, y_i \right) n\, z_i \right)^2, \qquad (3)$$

where, on the assumption that, all objects having the equatorial coordinates $\alpha_i, \delta_i; i = 1, 2, \cdots, N$ are all at unit desistance from the Sun, so

$$x_i = \cos\delta_i \cos\alpha_i,\ y_i = \cos\delta_i \sin\alpha_i,\ z_i = \sin\delta_i, \qquad (4)$$

and ℓ, m, n are direction cosines of the perpendicular to the plane drawn from the origin.

2. The constrained of the problem is:

$$f_2(\ell, m, n) = \ell^2 + m^2 + n^2 - 1 = 0 \qquad (5)$$

Now, the solution of the constrained minimization problem is found by Lagrange's method so, the objective function becomes:

$$F(\ell, m, n) = f_1(\ell, m, n) - \lambda\, f_2(\ell, m, n) \qquad (6)$$

where λ is Lagrange's multiplier to be determined.

The necessary conditions for the minimum are:

$$\frac{\partial F}{\partial \ell} = 0\ ,\ \frac{\partial F}{\partial m} = 0\ ,\frac{\partial F}{\partial n} = 0\ , \qquad (7)$$

$$\mathbf{Ax} = \lambda \mathbf{x}, \qquad (8)$$

where, the elements of the symmetric matrix A are given by:

$$a_{11} = \sum_{i=1}^{N} x_i^2, a_{12} = \sum_{i=1}^{N} x_i y_i, a_{13} = \sum_{i=1}^{N} x_i z_i, \qquad (9)$$

$$a_{22} = \sum_{i=1}^{N} y_i^2, a_{23} = \sum_{i=1}^{N} y_i z_i, a_{33} = \sum_{i=1}^{N} z_i^2, \qquad (10)$$

It is well known that [10], the symmetric least square matrix A has all its eigen values $\lambda_i\ ;\ i = 1, 2, 3$ real and distinct.

Now after solving the eigen value problem of Equation (8) for $(\lambda_i, \ell_i, m_i, n_i), i = 1, 2, 3$ select from this set, the values (λ, ℓ, m, n) that gives the global minimum of F, then the required values of (α_p, δ_p) could be determined from:

$$\alpha_p = \tan^{-1}\left(\frac{m}{\ell}\right),\ \delta_p = \sin^{-1}(n)\,. \qquad (11)$$

3.2. Numerical Developments

Applying the above analytic developments for the data of Paper I, result as the average of five values corresponding to galactic latitudes $b = 1^o, 2^o, 3^o, 4^o, 5^o$ of the members yields $\alpha_p = 192,949^o$ & $\delta_p = 26.9684^o$.

In concluding the present paper, two catalogues $J / A + A / 327 / 736$ and $J / A + A / 541 / A98$ of VizieR database were used for the statistical simulations of galactic planetary nebulae. Each catalogue was utilized for certain purposes, the first catalogue for, the correlation coefficients between the entireties of the data, the determination of the position of the maximum (minimum) of each entry in the data together with the values of the other entries at this position. In addition, the histograms and descriptive, location, dispersion and shape statistics for the entries of the data are also developed. Finally, the best fit and its error analysis was established between LH &LR where LH is the logarithm of HI Zanstrs luminosity and LR is the logarithm of nebular radius.

The second catalogue was used to display two-dimensional distributions of the interacting planetary nebulae, moreover IAU and other references used the equatorial coordinates of these nebulae to determine the equatorial coordinates of the north galactic pole (NGP). The results of this application are in good agreement with those given.

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