American Journal of Applied Mathematics and Statistics, 2014, Vol. 2, No. 1, 47-52
Available online at http://pubs.sciepub.com/ajams/2/1/8
© Science and Education Publishing
DOI:10.12691/ajams-2-1-8

Penalties for Misclassification of a Pure Diagonal Bilinear Process of Order Two as a Moving Average Process of Order Two

O. E. Okereke[1,*], I. S. Iwueze[2], C. O. Omekara[1]

[1]Department of Statistics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria
[2]Department of Statistics, Federal University of Technology, Owerri, Imo State, Nigeria
*Corresponding author: emmastat5000@yahoo.co.uk

Received July 26, 2013; Revised January 20, 2014; Accepted February 11, 2014

Abstract The penalty function based on misclassification of a pure diagonal bilinear process of order two as a moving average process of order two was derived in this study. Computation of penalties using the penalty function revealed that such misclassification increases the error variance. Regression analysis of the penalties on the parameters of the pure diagonal bilinear process suggested a second order polynomial regression model. A test of significance of each of the parameters of the fitted model showed that all the parameter estimates were statistically significant at 5% level of significance. The analysis of variance technique was also used to confirm the adequacy of the fitted model.

Keywords: *autocorrelation function, penalty function, pure diagonal bilinear process, moving average process, polynomial regression*

Cite This Article: O. E. Okereke, I. S. Iwueze, and C. O. Omekara, "Penalties for Misclassification of a Pure Diagonal Bilinear Process of Order Two as a Moving Average Process of Order Two." *American Journal of Applied Mathematics and Statistics*, vol. 2, no. 1 (2014): 47-52. doi: 10.12691/ajams-2-1-8.

1. Introduction

Much attention is usually given to the structures of the autocorrelation function (ACF) and partial autocorrelation function (PACF) during the identification stage of a time series model. The moving average model is a linear time series model with known structures of autocorrelation function and partial autocorrelation function [10]. Let $\mu_t, t \in Z$ be a sequence of independent and identically distributed random variables with zero mean and variance $\left(\sigma_1^{\,2}\right)$. Then $X_t, t \in Z$ is a non zero mean moving average process of order two (MA(2) process) if:

$$X_t = \beta_0 + \beta_1\mu_{t-1} + \beta_2\mu_{t-2} + \mu_t \quad (1.1)$$

The moving average process in (1.1) has the following first and second moments [3]:

$$E(X_t) = \beta_0 \quad (1.2)$$

$$R(k) = \begin{cases} \sigma_1^{\,2}(1+\beta_1^{\,2}+\beta_2^{\,2}), k=0 \\ \sigma_1^{\,2}\beta_1(1+\beta_2), k=\pm 1 \\ \beta_2\sigma_1^{\,2}, k=\pm 2 \\ 0, k \neq 0, \pm 1, \pm 2 \end{cases} \quad (1.3)$$

and

$$\rho_k = \begin{cases} 1, k=0 \\ \dfrac{\beta_1(1+\beta_2)}{(1+\beta_1^{\,2}+\beta_2^{\,2})}, k=\pm 1 \\ \dfrac{\beta_2}{(1+\beta_1^{\,2}+\beta_2^{\,2})}, k=\pm 2 \\ 0, otherwise \end{cases} \quad (1.4)$$

The autocorrelation function in (1.4) cuts off at lag two ([2,3]) .Other properties of the autocorrelation function are $-\frac{\sqrt{2}}{2} \le \rho_1 \le \frac{\sqrt{2}}{2}$ and $-\frac{1}{2} \le \rho_2 \le \frac{1}{2}$ [9].

A non linear time series model which competes with the moving average process in (1.1) in terms of autocorrelation function structure is the pure diagonal bilinear time series process of order two (PDB(2) process) defined by [4]:

$$Y_t = \theta_1 X_{t-1}e_{t-1} + \theta_2 X_{t-2}e_{t-2} + e_t \quad (1.5)$$

where $e_t, t \in Z$ is a sequence of independent and identically distributed random variables with zero mean and constant variance $\left(\sigma_2^{\,2}\right)$, θ_1 and θ_2 are real constants. If $\lambda_1 = \theta_1\sigma_2$ and $\lambda_2 = \theta_2\sigma_2$, then the first and second moments of the model in (1.5) are as follows [8]:

$$E(Y_t) = \left(\theta_1 + \theta_2\right)\sigma_2^{\,2} \quad (1.6)$$

$$R(k)=\begin{cases}\dfrac{\sigma_2{}^2\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)}{1-\lambda_1{}^2-\lambda_2{}^2}, k=0\\ \dfrac{\sigma_2{}^2\left(\begin{array}{l}\lambda_1{}^2-\lambda_1{}^4\\+\lambda_1{}^2\lambda_2{}^2+3\lambda_1\lambda_2\end{array}\right)}{1-\lambda_1{}^2-\lambda_2{}^2}, k=\pm 1\\ \sigma_2{}^2(\lambda_1\lambda_2+\lambda_2{}^2), k=\pm 2\\ 0, k\neq 0,\pm 1,\pm 2\end{cases} \quad (1.7)$$

$$\rho_k=\begin{cases}1, k=0\\ \dfrac{\left(\lambda_1{}^2-\lambda_1{}^4+\lambda_1{}^2\lambda_2{}^2+3\lambda_1\lambda_2\right)}{\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)}, k=\pm 1\\ \dfrac{(\lambda_1\lambda_2+\lambda_2{}^2)(1-\lambda_1{}^2-\lambda_2{}^2)}{\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)}, k=\pm 2\\ 0, k\neq 0,\pm 1,\pm 2\end{cases} \quad (1.8)$$

It is quite obvious that the ACFs in (1.4) and the one in (1.8) all cut off after lag two. This is indicative of the fact that a moving average process of order two and a pure diagonal bilinear time series process of order two have similar autocorrelation structures. As a result, there is a possibility of misclassifying a pure diagonal bilinear process of order two as a moving average process of order two. The ease with which linear models are fitted and the practice of approximating nonlinear models by linear models can also cause misspecification of the nonlinear pure diagonal bilinear process of order two.

From the foregoing, it is imperative to investigate the statistical implication of the aforementioned model misclassification. In this regard, we will focus on the penalty function associated with misclassification of a PDB(2) process as an MA(2) process.

2. Relationship between the Parameters of the Pure Diagonal Bilinear Process of Order Two and Moving Average Process of Order Two

Having observed that the moving average process of order two and pure diagonal bilinear process of order two have similar autocorrelation structures, it is worthwhile to derive the relationship between the parameters of the two models. These relationships will help us to obtain the penalty function for misclassifying the nonlinear model as the competing linear model. The method of moments which involves equating the first and second moments of the pure diagonal bilinear model to the corresponding moments of the non zero moving average process of order two shall be used for this purpose.

Equating means, we have

$$\beta_0=\sigma_2{}^2(\theta_1+\theta_2) \quad (2.1)$$

Equating variances, we obtain

$$\sigma_1{}^2\left(1+\beta_1{}^2+\beta_2{}^2\right)=\frac{\sigma_2{}^2\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)}{1-\lambda_1{}^2-\lambda_2{}^2} \quad (2.2)$$

Equating first order autocorrelations leads to:

$$\frac{\beta_1(1+\beta_2)}{(1+\beta_1{}^2+\beta_2{}^2)}=\frac{\lambda_1{}^2-\lambda_1{}^4+\lambda_1{}^2\lambda_2{}^2+3\lambda_1\lambda_2}{\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)} \quad (2.3)$$

Equating second order autocorrelations gives

$$\frac{\beta_2}{(1+\beta_1{}^2+\beta_2{}^2)}=\frac{\left(\lambda_1\lambda_2+\lambda_2{}^2\right)\left(1-\lambda_1{}^2-\lambda_2{}^2\right)}{\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)} \quad (2.4)$$

Dividing (2.4) by (2.3), we have

$$\frac{\beta_2}{\beta_1(1+\beta_2)}=\frac{\left(\lambda_1\lambda_2+\lambda_2{}^2\right)\left(1-\lambda_1{}^2-\lambda_2{}^2\right)}{\lambda_1{}^2-\lambda_1{}^4+3\lambda_1\lambda_2+\lambda_1{}^2\lambda_2{}^2} \quad (2.5)$$

From (2.5), we obtain

$$\beta_1=\frac{\left(\lambda_1{}^2-\lambda_1{}^4+3\lambda_1\lambda_2+\lambda_1{}^2\lambda_2{}^2\right)\beta_2}{\left(\begin{array}{l}\lambda_2{}^2-\lambda_2{}^4+\lambda_1\lambda_2\\-\lambda_1{}^2\lambda_2{}^2-\lambda_1{}^3\lambda_2-\lambda_1\lambda_2{}^3\end{array}\right)(1+\beta_2)} \quad (2.6)$$

Substituting (2.6) into (2.3), we obtain

$$\beta_2{}^4+A\beta_2{}^3+B\beta_2{}^2+A\beta_2+1=0 \quad (2.7)$$

where

$$A=\frac{\left[\begin{array}{l}2\left(\lambda_2{}^2-\lambda_2{}^4+\lambda_1\lambda_2-\lambda_1{}^2\lambda_2{}^2-\lambda_1{}^3\lambda_2-\lambda_1\lambda_2{}^3\right)\\-\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)\end{array}\right]}{\lambda_2{}^2-\lambda_2{}^4+\lambda_1\lambda_2-\lambda_1{}^2\lambda_2{}^2-\lambda_1{}^3\lambda_2-\lambda_1\lambda_2{}^3} \quad (2.8)$$

and

$$B=\frac{\left[\begin{array}{l}\left(\begin{array}{l}\lambda_1{}^2-\lambda_1{}^4\\+3\lambda_1\lambda_2+\lambda_1{}^2\lambda_2{}^2\end{array}\right)^2+2\left(\begin{array}{l}\lambda_2{}^2-\lambda_2{}^4+\lambda_1\lambda_2\\-\lambda_1{}^2\lambda_2{}^2-\lambda_1{}^3\lambda_2-\lambda_1\lambda_2{}^3\end{array}\right)^2\\-\left(\begin{array}{l}2\left(\lambda_2{}^2-\lambda_2{}^4+\lambda_1\lambda_2-\lambda_1{}^2\lambda_2{}^2-\lambda_1{}^3\lambda_2-\lambda_1\lambda_2{}^3\right)\\\times\left(\begin{array}{l}1+\lambda_1{}^2+\lambda_2{}^2+\lambda_1{}^4+2\lambda_1{}^2\lambda_2{}^2\\+2\lambda_1{}^3\lambda_2+2\lambda_1\lambda_2{}^3+\lambda_2{}^4\end{array}\right)\end{array}\right)\end{array}\right]}{\lambda_2{}^2-\lambda_2{}^4+\lambda_1\lambda_2-\lambda_1{}^2\lambda_2{}^2-\lambda_1{}^3\lambda_2-\lambda_1\lambda_2{}^3} \quad (2.9)$$

One way of solving (2.7) involves modifying it first to obtain

$$\begin{aligned}&\beta_2{}^4+A\beta_2{}^3+B\beta_2{}^2+A\beta_2+1+(a\beta_2+b)^2\\&=(\beta_2{}^2+\frac{A}{2}\beta_2+k)^2\end{aligned} \quad (2.10)$$

Here, a, b and k are constants such that

$$a^2 + B = 2k + \frac{A^2}{4} \quad (2.11)$$

$$2ab + A = kA \quad (2.12)$$

$$b^2 + 1 = k^2 \quad (2.13)$$

The value of k satisfying (2.11), (2.12) and (2.13) is obtained by solving the equation

$$k^3 - \frac{1}{2}Bk^2 + \frac{1}{4}\left(A^2 - 4\right)k + \frac{1}{8}\left(4B - A^2\right) = 0 \quad (2.14)$$

Substituting $k = Z + \frac{B}{6}$ into (2.14), we obtain

$$Z^3 + \frac{(3A^2 - B^2 - 12)}{12}Z + \frac{\begin{pmatrix} 9A^2B - 2B^3 \\ +72B - 27A^2 \end{pmatrix}}{216} = 0 \quad (2.15)$$

Following the methods of [5] and [1], the real solution of (2.15) is found to be

$$Z = \sqrt[3]{Z_1 + \sqrt{Z_1^2 + Z_2}} + \sqrt[3]{Z_1 - \sqrt{Z_1^2 + Z_2}} \quad (2.16)$$

where

$$Z_1 = \frac{-\left(9A^2B - 2B^3 + 72B - 27A^2\right)}{432} \quad (2.17)$$

and

$$Z_2 = \frac{\left(3A^2 - B^2 - 12\right)^3}{46656} \quad (2.18)$$

Hence,

$$k = \sqrt[3]{Z_1 + \sqrt{Z_1^2 + Z_2}} + \sqrt[3]{Z_1 - \sqrt{Z_1^2 + Z_2}} + \frac{B}{6} \quad (2.19)$$

Comparing (2.7) and (2.10), we have

$$\left(\beta_2^2 + \frac{A}{2}\beta_2 + k\right)^2 = \left(a\beta_2 + b\right)^2 \quad (2.20)$$

It can be deduced from (2.20) that

$$\beta_2^2 + \frac{A}{2}\beta_2 + k = a\beta_2 + b \quad (2.21)$$

or

$$\beta_2^2 + \frac{A}{2}\beta_2 + k = -a\beta_2 - b \quad (2.22)$$

Solving (2.21), we obtain

$$\beta_{21} = \frac{-\left(\frac{A-2a}{2}\right) - \sqrt{\left(\frac{A-2a}{2}\right)^2 - 4\left(k - \sqrt{k^2 - 1}\right)}}{2} \quad (2.23)$$

or

$$\beta_{22} = \frac{-\left(\frac{A-2a}{2}\right) + \sqrt{\left(\frac{A-2a}{2}\right)^2 - 4\left(k - \sqrt{k^2 - 1}\right)}}{2} \quad (2.24)$$

From (2.22), we have

$$\beta_{23} = \frac{-\left(\frac{A+2a}{2}\right) - \sqrt{\left(\frac{A+2a}{2}\right)^2 - 4\left(k + \sqrt{k^2 - 1}\right)}}{2} \quad (2.25)$$

or

$$\beta_{24} = \frac{-\left(\frac{A+2a}{2}\right) + \sqrt{\left(\frac{A+2a}{2}\right)^2 - 4\left(k + \sqrt{k^2 - 1}\right)}}{2} \quad (2.26)$$

When $\beta_2 = \beta_{21}$, we have

$$\beta_{11} = \frac{\left(\lambda_1^2 - \lambda_1^4 + 3\lambda_1\lambda_2 + \lambda_1^2\lambda_2^2\right)\beta_{21}}{\begin{pmatrix} \lambda_2^2 - \lambda_2^4 + \lambda_1\lambda_2 \\ -\lambda_1^2\lambda_2^2 - \lambda_1^3\lambda_2 - \lambda_1\lambda_2^3 \end{pmatrix}(1 + \beta_{21})} \quad (2.27)$$

For $\beta_2 = \beta_{22}$, we obtain

$$\beta_{12} = \frac{\left(\lambda_1^2 - \lambda_1^4 + 3\lambda_1\lambda_2 + \lambda_1^2\lambda_2^2\right)\beta_{22}}{\begin{pmatrix} \lambda_2^2 - \lambda_2^4 + \lambda_1\lambda_2 \\ -\lambda_1^2\lambda_2^2 - \lambda_1^3\lambda_2 - \lambda_1\lambda_2^3 \end{pmatrix}(1 + \beta_{22})} \quad (2.28)$$

Using $\beta_2 = \beta_{23}$, we have

$$\beta_{13} = \frac{\left(\lambda_1^2 - \lambda_1^4 + 3\lambda_1\lambda_2 + \lambda_1^2\lambda_2^2\right)\beta_{23}}{\begin{pmatrix} \lambda_2^2 - \lambda_2^4 + \lambda_1\lambda_2 \\ -\lambda_1^2\lambda_2^2 - \lambda_1^3\lambda_2 - \lambda_1\lambda_2^3 \end{pmatrix}(1 + \beta_{23})} \quad (2.29)$$

With $\beta_2 = \beta_{24}$, we obtain

$$\beta_{14} = \frac{\left(\lambda_1^2 - \lambda_1^4 + 3\lambda_1\lambda_2 + \lambda_1^2\lambda_2^2\right)\beta_{24}}{\begin{pmatrix} \lambda_2^2 - \lambda_2^4 + \lambda_1\lambda_2 \\ -\lambda_1^2\lambda_2^2 - \lambda_1^3\lambda_2 - \lambda_1\lambda_2^3 \end{pmatrix}(1 + \beta_{24})} \quad (2.30)$$

Simulation concerning (2.23), (2.24), (2.25), (2.26) and their corresponding values of β_1 namely β_{11}, β_{12}, β_{13} and β_{14} respectively shows that only β_{11} and β_{21} take on values satisfying the invertibility condition of a moving average process of order two.

3. Penalty Function for Misclassification of a PDB(2) Process as an MA(2) Process

Penalty function based on model misclassification in time series analysis is defined by [6] as a function of error standard deviations. Let σ_2 be the standard deviation of the errors associated with a PDB(2) process. Suppose σ_1 represents the standard deviation of the errors corresponding to an MA(2) process. Then the penalty function for the misclassification of PDB(2) as an MA(2) is given as

$$P = \frac{\sigma_1 - \sigma_2}{\sigma_2} \quad (3.1)$$

We can write (3.1) as

$$P = \sqrt{\frac{\sigma_1^2}{\sigma_2^2}} - 1 \tag{3.2}$$

Using (2.2), we obtain

$$\frac{\sigma_1^2}{\sigma_2^2} = \frac{\left(\begin{array}{l} 1+\lambda_1^2+\lambda_2^2+\lambda_1^4+2\lambda_1^2\lambda_2^2 \\ +2\lambda_1^3\lambda_2+2\lambda_1\lambda_2^3+\lambda_2^4 \end{array}\right)}{\left(1-\lambda_1^2-\lambda_2^2\right)\left(1+\beta_1^2+\beta_2^2\right)} \tag{3.3}$$

Substituting (3.3) into (3.2) leads to

$$P = \sqrt{\frac{\left(\begin{array}{l} 1+\lambda_1^2+\lambda_2^2+\lambda_1^4+2\lambda_1^2\lambda_2^2 \\ +2\lambda_1^3\lambda_2+2\lambda_1\lambda_2^3+\lambda_2^4 \end{array}\right)}{\left(1-\lambda_1^2-\lambda_2^2\right)\left(1+\beta_1^2+\beta_2^2\right)}} - 1 \tag{3.4}$$

β_1 and β_2 in (3.3) are as defined in (2.27) and (2.23) respectively. Table 1 contains the penalties (P) corresponding to various values of λ_1, λ_2, β_1 and β_2.

Considering the complete table containing 2129 sets of values, we can see that the penalty function for misclassification of a PDB(2) process as an MA(2) process (P) takes on positive values for all values of λ_1, λ_2, β_1 and β_2. The positive value of the penalty for misclassification of a PDB(2) process as an MA(2) process shows that this misclassification leads to increase in variance of the errors. This finding agrees with the results obtained by [6] with regard to misclassification of a PDB(1) process as an MA(1) process.

For predictive purposes, we have to find the relationship between P and λ_1 and λ_2. First, we plot P against each of λ_1 and λ_2. Figure 1 shows the plot of P against λ_1.

Table 1. Penalties for various Values of Parameters of MA(2) Process and PDB (2) Process

S/NO	λ_1	λ_2	θ_1	θ_2	P	$\hat{P}$	$P-\hat{P}$
1	-0.26	-0.42	0.40	0.12	0.24	0.2615	-0.0215
2	-0.26	-0.41	0.39	0.12	0.23	0.2519	-0.0219
3	-0.26	-0.40	0.37	0.12	0.22	0.2426	-0.0226
4	-0.26	-0.39	0.35	0.11	0.22	0.2334	-0.0134
5	-0.26	-0.38	0.34	0.11	0.21	0.2245	-0.0145
6	-0.26	-0.37	0.32	0.10	0.20	0.2159	-0.0159
7	-0.25	-0.43	0.40	0.13	0.24	0.2664	-0.0264
8	-0.25	-0.42	0.39	0.13	0.23	0.2565	-0.0265
9	-0.25	-0.41	0.37	0.12	0.22	0.2469	-0.0269
10	-0.25	-0.40	0.36	0.12	0.22	0.2375	-0.0175
11	-0.25	-0.39	0.34	0.11	0.21	0.2284	-0.0184
12	-0.25	-0.38	0.33	0.11	0.20	0.2195	-0.0195
13	-0.25	-0.37	0.31	0.10	0.20	0.2108	-0.0108
14	-0.25	-0.36	0.30	0.10	0.19	0.2024	-0.0124
15	-0.25	-0.35	0.28	0.10	0.19	0.1942	-0.0042
16	-0.25	-0.34	0.27	0.09	0.18	0.1862	-0.0062
17	-0.25	-0.33	0.25	0.09	0.18	0.1785	0.0015
18	-0.24	-0.43	0.39	0.13	0.24	0.2615	-0.0215
19	-0.24	-0.42	0.37	0.13	0.23	0.2517	-0.0217
20	-0.24	-0.41	0.36	0.12	0.22	0.2421	-0.0221
21	-0.24	-0.40	0.34	0.12	0.21	0.2327	-0.0227
22	-0.24	-0.39	0.33	0.11	0.21	0.2236	-0.0136
23	-0.24	-0.38	0.32	0.11	0.20	0.2147	-0.0147
24	-0.24	-0.37	0.30	0.11	0.19	0.2060	-0.0160
25	-0.24	-0.36	0.29	0.10	0.19	0.1976	-0.0076
26	-0.24	-0.35	0.27	0.10	0.18	0.1894	-0.0094
27	-0.24	-0.34	0.26	0.09	0.18	0.1814	-0.0014
28	-0.24	-0.33	0.24	0.09	0.17	0.1737	-0.0037
29	-0.24	-0.32	0.23	0.08	0.17	0.1662	0.0038
30	-0.24	-0.31	0.22	0.08	0.16	0.1589	0.0011
31	-0.24	-0.30	0.20	0.07	0.16	0.1519	0.0081
32	-0.24	-0.29	0.19	0.07	0.15	0.1451	0.0049
33	-0.23	-0.44	0.39	0.13	0.24	0.2670	-0.0270
34	-0.23	-0.43	0.37	0.13	0.23	0.2569	-0.0269
35	-0.23	-0.42	0.36	0.13	0.23	0.2471	-0.0171
36	-0.23	-0.41	0.35	0.12	0.22	0.2374	-0.0174
37	-0.23	-0.40	0.33	0.12	0.21	0.2281	-0.0181
38	-0.23	-0.39	0.32	0.11	0.20	0.2189	-0.0189
39	-0.23	-0.38	0.30	0.11	0.20	0.2100	-0.0100
40	-0.23	-0.37	0.29	0.10	0.19	0.2014	-0.0114
41	-0.23	-0.36	0.28	0.10	0.18	0.1929	-0.0129
42	-0.23	-0.35	0.26	0.10	0.18	0.1847	-0.0047
43	-0.23	-0.34	0.25	0.09	0.17	0.1768	-0.0068
44	-0.23	-0.33	0.23	0.09	0.17	0.1691	0.0009
45	-0.23	-0.32	0.22	0.08	0.16	0.1616	-0.0016
46	-0.23	-0.31	0.21	0.08	0.16	0.1543	0.0057
47	-0.23	-0.30	0.20	0.07	0.15	0.1473	0.0027

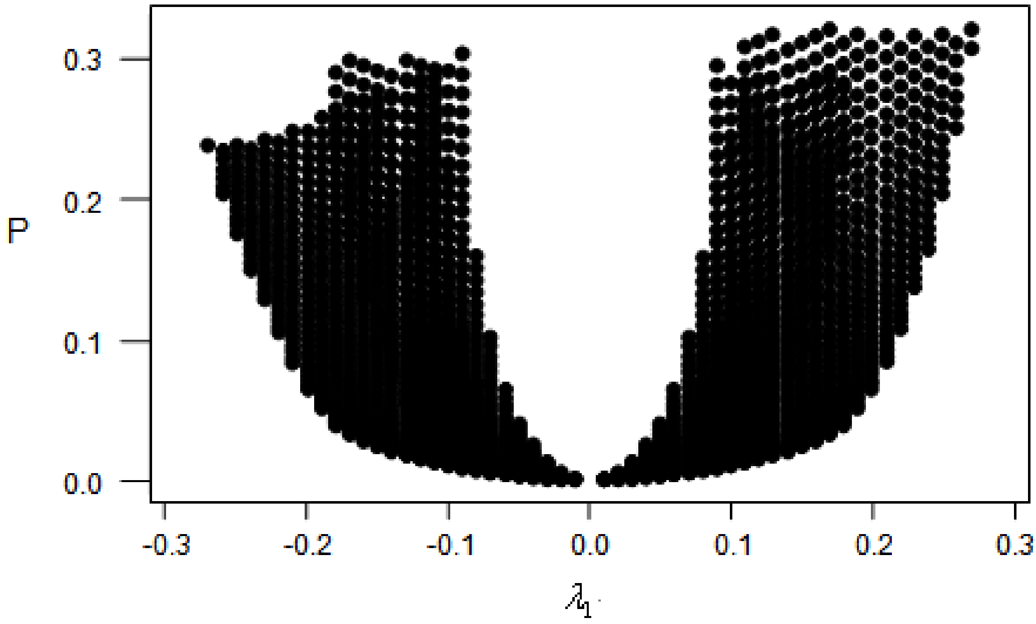

Figure 1. Plot of P against λ_1

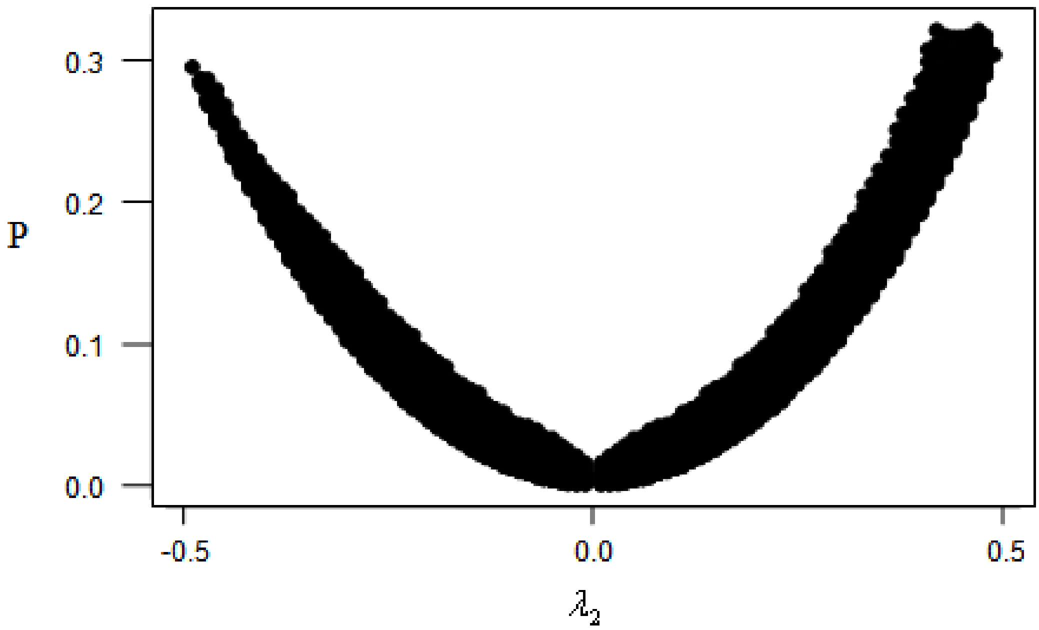

Figure 2. Plot of P against λ_2

It can easily be seen from Figure 1 that there is a curvilinear relationship between P and λ_1. In Figure 2, we have the plot of P against λ_2.

Figure 2 also reveals that there is a curvilinear relationship between P and λ_2.

Combining the information in Figure 1 and Figure 2, the regression model in (3.5) is suggested for the relationship between P and λ_1 and λ_2.

$$P = \phi_0 + \phi_1\lambda_1 + \phi_2\lambda_1^2 + \phi_3\lambda_2 + \phi_4\lambda_2^2 + \nu \qquad (3.5)$$

where ϕ_0, ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are the parameters of the regression equation (3.5) and ν is the associated error term. The least squares estimation of (3.5) based on the admissible values of . λ_1 and λ_2 leads to the predictive equation [7].

$$\hat{P} = -0.0034 + 0.0224\lambda_1 + 1.0320\lambda_1^2 + 0.0152\lambda_2 + 1.1757\lambda_2^2 \qquad (3.6)$$

Table 2 contains the summary of the test for significance of each of the parameters of the model in (3.6).

Table 2. Test for Significance of the Parameters of the Regression Model for Penalty for Misclassication of a PDB(2) Process as an MA(2) Process

Predictor	Coef	StDev	T	P
Constant	-0.0034	0.0003	-13.34	0.000
λ_1	0.0224	0.0011	21.25	0.000
λ_1^2	1.0320	0.0102	101.20	0.000
λ_2	0.0152	0.0005	29.26	0.000
λ_2^2	1.1757	0.0023	521.79	0.000

S = 0.0064, R-Sq = 99.4%, R-Sq(adj) = 99.4%

Each of the parameters appears to be significant at $\alpha = 5\%$ level of significance since the corresponding p value is less than 0.05. Next, we test for significance of the overall regression using the analysis of variance technique as shown in Table 3.

Table 3. ANOVA Table for Testing for Significance of the Fitted Penalty Model

Source	DF	SS	MS	F	P
Regression	4	15.5117	3.8779	93994.23	0.000
Error	2124	0.0876	0.0000		
Total	2128	15.5993			

The p value of 0.00 in the Table 3 implies that the fitted regression model is suitable for describing the relationship between P and λ_1 and λ_2.

4. Conclusion

In this study, we determined the effect of misclassifying a pure diagonal bilinear process of order two as a moving average process of order two. A penalty function was defined and was used to compute penalties for misclassification of the pure diagonal bilinear process of order two as the moving average process of order two based on various sets of values of the parameters of the two processes. The computed penalties assumed positive values. This indicated increase in error variance due to misclassification of pure diagonal bilinear process of order two as a moving average process of order two. A quadratic regression model was found suitable for predicting the penalties based on the parameters of the pure diagonal bilinear process of order two.

References

[1] Bessels, S. (2006). One step beyond the solvable equation. www.staff.science.uu.nc/…/Afstudeerscriptie_Sander_Bessels.pdf (This site was visited in June, 2013).

[2] Box, G. E. P., Jenkins, G. M. and Reinsel, G. C.(1994). Time Series Analysis: Forecasting and Control. 3rd ed., Prentice-Hall, Englewood Cliffs, N. J.

[3] Chatfield, C. (1995). The Analysis of Time Series. An Introduction. 5th ed. Chapmann and Hall, London.

[4] Granger, C. W. J. and Andersen, A. (1978). An Introduction to Bilinear Time Series Models. Vanderhoeck and Ruprecht, Gottingen.

[5] Hahn, K. (2005). Solving cubic and quartic polynomials. www.Karlscalculus.org/pdf/cubicquartic.pdf.

[6] Iwueze, I. S. and Ohakwe, J. (2009). Penalties for misclassification of first order and linear moving average time series processes. Interstat Journal of Statistics, No3, http//interstatjournals.net/Year/2009/articles/0906003.pdf.

[7] Okereke, O. E. (2013). Characterization of moments of pure diagonal bilinear process of order two and moving average process of order two, Unpublished Ph. D Dissertation.

[8] Okereke, O. E and Iwueze, I. S. (2013). Region of comparison for the second order moving average and pure diagonal bilinear processes. International Journal of Applied Mathematics and Statistical Sciences, 2(2): 17-25.

[9] Okereke, O. E, Iwueze, I. S and Johnson, O. (2013). Extrema for autocorrelation coefficients of moving average processes. Far East Journal of Theoritical Statistics, 42(2): 137-150.

[10] Wei, W. W. S. (2006). Time Series Analysis, Univariate and Multivariate Methods. 2nd ed. Pearson Addision Wesley, New York.