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On A Class of New Type Generalized Difference Sequences Related to the P-Normed l^p Space Defined By Orlicz Functions

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Abstract The idea of difference sequence spaces were defined by Kizmaz [6] and generalized by Et and Colak [5]. Later Esi et al. [4] introduced the notion of the new difference operator Δ_m^n for fixed n,m∈N. In this article we introduce new type generalized difference sequence space $m\left(M,\Delta_m^n,\varphi,p\right)$ using by the Orlicz function. We give various properties and inclusion relations on this new type difference sequence space.

Keywords: *Orlicz function, difference sequence space, solid space, symmetric space*

1. Introduction

Throughout the article w, l_∞ and l^p denote the spaces all, bounded and p absolutely summable sequences, respectively. The zero sequence is denoted by $\Theta=(0,0,0,...)$. The sequence space $m(\varphi)$ was introduced by Sargent [11], who studied some of its properties and obtained its relationship with the space l^p . Later on it was investigated from sequence space point of view by Rath [9], Rath and Tripathy [10], Tripathy and Sen [15], Tripathy and Mahanta [14], Esi [2] and others.

An Orlicz function is a function M: $[0,\infty)\to[0,\infty)$, which is continuous, non-decreasing and convex with M(0)=0, M(x) > 0 for x > 0 and M(x) $\to\infty$ as $x\to\infty$.

An Orlicz function is said to satisfy Δ_2-condition for all values of u, if there exists a constant K > 0, such that M(2u) ≤ KM(u), u ≥ 0.

Remark. An Orlicz function satisfies the inequality $M(\lambda x)\le\lambda M(x)$ for all λ *with* $0<\lambda\le1$.

Lindenstrauss and Tzafriri [7] used the idea of Orlicz function to construct the sequence space

$$l_M=\left\{(x_k):\ \sum_k M\left(\frac{|x_k|}{\rho}\right)<\infty,\quad for\ some\ \rho>0\right\}.$$

The space l_M with the norm

$$\|x\|=\inf\left\{\rho>0:\ \sum_k M\left(\frac{|x_k|}{\rho}\right)\le1\right\}$$

becomes a Banach space which is called an Orlicz sequence space. The space l_M is closely related to the space l_p which is an Orlicz sequence space with

$$M(x)=x^p,1\le p<\infty.$$

In the later stage different Orlicz sequence spaces were introduced and studied by Tripathy and Mahanta [14] , Esi [1], Esi and Et [3], Parashar and Choudhary [8], and many others.

Kizmaz [6] defined the difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$ as follows:

$$Z(\Delta)=\{(x_k):(\Delta x_k)\in Z\},$$

for Z = ℓ_∞, c and c_o, where $\Delta x=(\Delta x_k)=(x_k-x_{k+1})$ for all $k\in N$.

The above spaces are Banach spaces, normed by

$$\|x\|_\Delta=|x_1|+\sup_k\|\Delta x_k\|$$

Later, the difference sequence spaces were generalized by Et and Çolak [5] as follows: Let $n\in N$ be fixed integer, then $X(\Delta^n)=\{(x_k):(\Delta^n x_k)\in X\}$ for $X=l_\infty, c\ and\ c_o$, where $\Delta^n x_k=\Delta^{n-1}x_k-\Delta^{n-1}x_{k+1}$ and so $\Delta^n x_k=\sum_{v=0}^{n}(-1)^v\binom{n}{v}x_{k+v}$.

They showed that the above spaces are Banach spaces, normed by

$$\|(x_k)\|_\Delta=\sum_{i=1}^{n}|x_i|+\sup_k\|\Delta^n x_k\|$$

After then, the notion new type of difference sequence spaces were further generalized Esi and et.al. [4] as follows:

Let $m, n \in N$ be fixed integers, then

$$X(\Delta_m^n) = \{(x_k) : (\Delta_m^n x_k) \in X\}$$

for $X = l_\infty, c \text{ and } c_o$, where $\Delta_m^n x_k = \Delta_m^{n-1} x_k - \Delta_m^{n-1} x_{k+m}$ and $\Delta_m^0 x_k = x_k$ for all $k \in N$. The new type generalized difference has the following binomial representation:

$$\Delta_m^n x_k = \sum_{v=0}^{n} (-1)^v \binom{n}{v} x_{k+mv}$$

They showed that the above spaces are Banach spaces, normed by

$$\|(x_k)\|_{\Delta_m^n} = \sum_{i=1}^{r} |x_i| + \sup_k \|\Delta_m^n x_k\|$$

where, $r = mn$ for $m, n \ge 1$; $r = n \text{ for } m = 0$ and $r = m \text{ for } n = 0$.

2. Definitions and Background

Throughout the article $\wp_s$ denotes the set of all subsets of N, the set of natural numbers, those do not contain more than s elements. Further (φ_s) will denote a non-decreasing sequence of positive real numbers such that $n\varphi_{n+1} \le (n+1)\varphi_n$ for all $n \in N$. The class of all the sequences (φ_s) satisfying this property is denoted by Φ.

The space $m(\varphi)$ introduced and studied by Sargent [11] is defined as follows:

$$m(\varphi) = \left\{(x_k) : \|x\|_{m(\varphi)} = \sup_{s\ge1, \sigma\in\wp_s} \frac{1}{\varphi_s} \sum_{k\in\sigma} |x_k| < \infty\right\}$$

Recently Tripathy and Mahanta [13] defined and studied the following sequence space: Let M be an Orlicz function, then

$$m(M, \Delta, \varphi) = \left\{(x_k) : \sup_{s\ge1, \sigma\in\wp_s} \frac{1}{\varphi_s} \sum_{k\in\sigma} \left[M\left(\frac{|\Delta x_k|}{\rho}\right)\right] < \infty, \quad \text{for some } \rho > 0\right\}.$$

The purpose of this paper is to introduce and study a class of new type generalized difference sequences related to the space $l^p(\Delta)$ using by Orlicz function.

In this article we introduce the following sequence space: Let M be an Orlicz function and p= (p_k) be bounded sequence of strictly positive real numbers and $m, n \ge 0$ be fixed integers, then

$$m(M, \Delta_m^n, \varphi, p) = \left\{(x_k) : \sup_{s\ge1, \sigma\in\wp_s} \frac{1}{\varphi_s} \sum_{k\in\sigma} \left[M\left(\frac{|\Delta_m^n x_k|}{\rho}\right)\right]^{p_k} < \infty, \quad \text{for some } \rho > 0\right\}.$$

Taking $p_k = 1$ for all k and m=n=1 i.e., considering only first difference we have the following difference sequence space which were defined and studied by Tripathy and Mahanta [13]

$$m(M, \Delta, \varphi) = \left\{(x_k) : \sup_{s\ge1, \sigma\in\wp_s} \frac{1}{\varphi_s} \sum_{k\in\sigma} M\left(\frac{|\Delta x_k|}{\rho}\right) < \infty, \quad \text{for some } \rho > 0\right\}.$$

Taking $p_k = 1$ for all k, M(x)=x and m=n=1 i.e., considering only first difference we have the following difference sequence space which were defined and studied by Tripathy [12]

$$m(\Delta, \varphi) = \left\{(x_k) : \sup_{s\ge1, \sigma\in\wp_s} \frac{1}{\varphi_s} \sum_{k\in\sigma} |\Delta x_k| < \infty\right\}.$$

Taking $p_k = 1$ for all k, M(x)=x and n=1, we have the following difference sequence space which were defined and studied by Esi [2]

$$m(\Delta_m, \varphi) = \left\{(x_k) : \sup_{s\ge1, \sigma\in\wp_s} \frac{1}{\varphi_s} \sum_{k\in\sigma} |\Delta_m x_k| < \infty\right\}.$$

The space $l^p(\Delta)$ for $0 < p < 1$ is defined by Rath [9] as follows:

$$l^p(\Delta) = \left\{(x_k) : \sum_{k=1}^{\infty} |\Delta x_k|^p < \infty\right\}.$$

Let $x = (x_k)$ be a sequence, then $S(X)$ denotes the set of all permutations of the elements of (x_k) *i.e.*, $S(X) = \{(x_{\pi(k)}) : \pi(k)$ is a permutation on $N\}$. A sequence space E is said to be *symmetric* if $S(X) \subset E$ for all $x \in E$.

A sequence space E is said to be *monotone*, if it contains the canonical pre-images of its step spaces.

The following inequality will be used throughout the paper

$$|x_k + y_k|^{p_k} \le C\left(|x_k|^{p_k} + |y_k|^{p_k}\right)$$

where x_k and y_k are complex numbers, $C = \max\left(1, 2^{H-1}\right)$ and $H = \sup_k p_k < \infty$.

3. Main Results

In this section we prove some results involving the sequence space $m(M, \Delta_m^n, \varphi, p)$.

Theorem 1. Let p= (p_k) be bounded sequence of strictly positive real numbers.Then the space $m(M, \Delta_m^n, \varphi, p)$. is a linear space over the complex field **C**.

Proof: Let (x_k), $(y_k) \in m(M, \Delta_m^n, \varphi, p)$. and $\alpha, \beta \in \mathbf{C}$. Then there exists positive numbers ρ_1 and ρ_2 such that

$$\sup_{s\ge1, \sigma\in\wp_s} \frac{1}{\varphi_s} \sum_{k\in\sigma} \left[M\left(\frac{|\Delta_m^n x_k|}{\rho_1}\right)\right]^{p_k} < \infty$$

and

$$\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n y_k\right|}{\rho_2}\right)\right]^{p_k}<\infty$$

Let $\rho_3=\max\left(2\left|\alpha\right|\rho_1,2\left|\beta\right|\rho_2\right)$. Since M is non-decreasing and convex

$$\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(\alpha x_k+\beta y_k\right)\right|}{\rho_3}\right)\right]^{p_k}$$

$$\leq\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(\alpha x_k\right)\right|}{\rho_3}+\frac{\left|\Delta_m^n\left(\beta y_k\right)\right|}{\rho_3}\right)\right]^{p_k}$$

$$\leq C\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(x_k\right)\right|}{\rho_1}\right)\right]^{p_k}+C\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(y_k\right)\right|}{\rho_2}\right)\right]^{p_k}$$

$\Rightarrow$

$$\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(\alpha x_k+\beta y_k\right)\right|}{\rho_3}\right)\right]^{p_k}$$

$$\leq C\left\{\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(x_k\right)\right|}{\rho_1}\right)\right]^{p_k}+\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(y_k\right)\right|}{\rho_2}\right)\right]^{p_k}\right\}$$

$<\infty$

Hence $\alpha\left(x_k\right)+\beta\left(y_k\right)\in m\left(M,\Delta^m,\varphi,p\right)$.

Theorem 2. Let p= $\left(p_k\right)$ be bounded sequence of strictly positive real numbers and $H=\max\left(1,\sup_k p_k\right)$. Then $m\left(M,\Delta_m^n,\varphi,p\right)$. is a linear topological space paranormed by

$$g(x)=\left(\sum_{i=1}^{r}\left|\Delta_m^n x_i\right|^{p_i}\right)^{1/H}+$$
$$\inf\left\{\rho^{p_n/H}:\left(\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k}\right)^{1/H}\leq 1,\ n,m=1,2,3,\ldots\right\}$$

where $r=mn$ for $m\geq 1,\ n\geq 1$; $r=n$ for m=0 and r=m for n=0.

Proof: Clearly $g\left(x\right)=g\left(-x\right)$. Next $\left(x_k\right)=\Theta$ implies $\Delta_m^n x_k=0$ and such as $M\left(0\right)=0$, therefore $g\left(\Theta\right)$=0. It can be easily shown that $g\left(x\right)=0\Rightarrow\left(x_k\right)=\Theta$.

Next, let $\rho_1>0$ and $\rho_2>0$ be such that

$$\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho_1}\right)\right]^{p_k}\leq 1$$

and

$$\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n y_k\right|}{\rho_2}\right)\right]^{p_k}\prec\infty$$

Let $\rho=\rho_1+\rho_2$. Then we have

$$\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(x_k+y_k\right)\right|}{\rho}\right)\right]^{p_k}$$

$$\leq\left(\frac{\rho_1}{\rho_1+\rho_2}\right)^H\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho_1}\right)\right]^{p_k}$$

$$+\left(\frac{\rho_1}{\rho_1+\rho_2}\right)^H\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n y_k\right|}{\rho_2}\right)\right]^{p_k}$$

≤ 1

Since the $\rho's$ are non-negative, we have

$$g(x+y)=\left(\sum_{i=1}^{r}\left|\Delta_m^n\left(x_i+y_i\right)\right|^{p_i}\right)^{1/H}+$$
$$\inf\left\{\rho^{p_n/H}:\left(\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(x_k+y_k\right)\right|}{\rho}\right)\right]^{p_k}\right)^{1/H}\leq 1,\ n,m=1,2,3,\ldots\right\}$$

$$\leq\left(\sum_{i=1}^{r}\left|\Delta_m^n\left(x_i\right)\right|^{p_i}\right)^{1/H}+\left(\sum_{i=1}^{r}\left|\Delta_m^n\left(y_i\right)\right|^{p_i}\right)^{1/H}$$

$$+\inf\left\{\rho_1^{p_n/H}:\left(\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(x_k\right)\right|}{\rho_1}\right)\right]^{p_k}\right)^{1/H}\leq 1,\ n,m=1,2,3,\ldots\right\}$$

$$+\inf\left\{\rho_2^{p_n/H}:\left(\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(y_k\right)\right|}{\rho_2}\right)\right]^{p_k}\right)^{1/H}\leq 1,\ n,m=1,2,3,\ldots\right\}$$

$=g\left(x+y\right)$

Next, for $\lambda\in C$, without loss of generality, let $\lambda\neq 0$, then

$$g(\lambda x)=\left(\sum_{i=1}^{r}\left|\Delta_m^n\left(\lambda x_i\right)\right|^{p_i}\right)^{1/H}+$$
$$\inf\left\{\rho^{p_n/H}:\left(\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(\lambda x_k\right)\right|}{\rho}\right)\right]^{p_k}\right)^{1/H}\leq 1,\ n,m=1,2,3,\ldots\right\}$$

$$=\left(\sum_{i=1}^{r}\left|\Delta_m^n\left(\lambda x_i\right)\right|^{p_i}\right)^{1/H}+$$
$$\inf\left\{\left(\left|\lambda\right|r\right)^{p_n/H}:\left(\sup_{s\geq 1,\sigma\in\wp_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n\left(\lambda x_k\right)\right|}{r}\right)\right]^{p_k}\right)^{1/H}\leq 1,\ n,m=1,2,3,\ldots\right\}$$

where $r = \frac{\rho}{|\lambda|}$

$$\Rightarrow g(\lambda x) = \max(1, |\lambda|)\left(\sum_{i=1}^{r} \left|\Delta_m^n(x_i)\right|^{p_i}\right)^{1/H} +$$

$$\max(1, |\lambda|)\inf\left\{r^{p_n/H} : \left(\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n(\lambda x_k)\right|}{r}\right)\right]^{p_k}\right)^{1/H} \leq 1, n, m = 1,2,3,...\right\}$$

$$= \max(1, |\lambda|)\, g(x)$$

So, the continuity of the scalar multiplication follows from the above inequality.

Theorem 3. $m(M, \Delta_m^n, \varphi, p). \subseteq m(M, \Delta_m^n, \Psi, p)$ if and only if $\sup_{s\geq 1} \frac{\varphi_s}{\Psi_s} < \infty$.

Proof: Let $\sup_{s\geq 1} \frac{\varphi_s}{\Psi_s} < \infty$ and $(x_k) \in m(M, \Delta_m^n, \varphi, p)$. Then

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k} < \infty,$$

for some $\rho > 0$.

So,

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\Psi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k}$$

$$\leq \left\{\sup_{s\geq 1}\frac{\varphi_s}{\Psi_s}\right\}\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k}$$

$$< \infty.$$

Therefore $(x_k) \in m(M, \Delta^m, \Psi, p)$

Conversely , let $m(M, \Delta_m^n, \varphi, p). \subseteq m(M, \Delta_m^n, \Psi, p)$. Suppose that $\sup_{s\geq 1} \frac{\varphi_s}{\Psi_s} = \infty$ .Then there exists a sequence of natural numbers (s_i) such that $\lim_{i\to\infty} \frac{\varphi_{s_i}}{\Psi_{s_i}} = \infty$. Let $(x_k) \in m(M, \Delta_m^n, \varphi, p)$. Then there exists $\rho > 0$ such that

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k} < \infty.$$

Now we have

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\Psi_s}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k}$$

$$\geq \left\{\sup_{i\geq 1}\frac{\varphi_{s_i}}{\Psi_{s_i}}\right\}\sup_{i\geq 1, \sigma\in\wp_s} \frac{1}{\varphi_{s_i}}\sum_{k\in\sigma}\left[M\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k} = \infty.$$

Therefore $(x_k) \notin m(M, \Delta_m^n, \Psi, p)$. As such we arrive at a contradiction. Hence $\sup_{s\geq 1} \frac{\varphi_s}{\Psi_s} < \infty$.

The following result is a consequence of Theorem 3.

Corollary 4: Let M be an Orlicz function. Then $m(M, \Delta_m^n, \varphi, p) = m(M, \Delta_m^n, \Psi, p)$ if and only if $\sup_{s\geq 1} \frac{\varphi_s}{\Psi_s} < \infty$ and $\sup_{s\geq 1} \frac{\Psi_s}{\varphi_s} < \infty$ for all s=1,2,3,... .

Theorem 5: Let p= (p_k) be bounded sequence of strictly positive real numbers and let M *and* M_1 be Orlicz functions satisfying Δ_2-condition. Then $m(M, \Delta_m^n, \varphi, p) \subseteq m(MoM_1, \Delta_m^n, \varphi, p)$

Proof: Let $(x_k) \in m(M_1, \Delta_m^n, \varphi, p)$. Then we have

$$\sup_{s\geq 1, \sigma\in\wp_s} \frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M_1\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)\right]^{p_k} < \infty,$$

for some $\rho > 0$.

Let $0 < \varepsilon < 1$ and choose δ with $0 < \delta < 1$ such that $M(t) < \varepsilon$ for $0 \leq t \leq \delta$. Let $y_k = M_1\left(\frac{\left|\Delta_m^n x_k\right|}{\rho}\right)$ for all m and n and for any $\sigma \in P_s$, let

$$\sum_{k\in\sigma}\left[M\ (y_k)\right]^{p_k} = \sum_1\left[M\ (y_k)\right]^{p_k} + \sum_2\left[M\ (y_k)\right]^{p_k}$$

where the first summation is over $y_k \leq \delta$ and the second is over $y_k > \delta$. For the first summation above, we can write

$$\sum_1\left[M(y_k)\right]^{p_k} \leq [M(1)]^H \sum_1\left[(y_k)\right]^{p_k} \leq [M(2)]^H \sum_1\left[(y_k)\right]^{p_k} \quad (1)$$

(by using Remark)

For the second summation, we will make following procedure. For $y_k > \delta$, we have

$$y_k < 1 + \frac{y_k}{\delta}$$

Since M is non-decreasing and convex, it follows that

$$M(y_k) < M\left(1 + \frac{y_k}{\delta}\right) \leq \frac{1}{2}M(2) + \frac{1}{2}M\left(2\frac{y_k}{\delta}\right)$$

Since M satisfies Δ_2 condition, we can write

$$M(y_k) \leq \frac{K}{2}M(2)\left(\frac{y_k}{\delta}\right) + \frac{K}{2}M(2)\left(\frac{y_k}{\delta}\right) = KM(2)\left(\frac{y_k}{\delta}\right)$$

Hence

$$\sum_2\left[M\ (y_k)\right]^{p_k} \leq \max\left(1, \left[\frac{K}{\delta}M(2)\right]^H\right)\sum_2\left[(y_k)\right]^{p_k} \quad (2)$$

By (1) and (2), we have $(x_k) \in m\left(MoM_1, \Delta_m^n, \varphi, p\right)$.

Taking $M_1(x) = x$ in Theorem 5, we have the following result.

Corollary 6: Let p= (p_k) be bounded sequence of strictly positive real numbers and let M be an Orlicz function satisfying Δ_2-condition. Then

$$m\left(\Delta_m^n, \varphi, p\right) \subseteq m\left(M, \Delta_m^n, \varphi, p\right)$$

From Theorem 3 and Corollary 6, we have

Corollary 7: Let p= (p_k) be bounded sequence of strictly positive real numbers and let M be an Orlicz function satisfying Δ_2-condition. Then

$$m\left(\Delta_m^n, \varphi, p\right) \subseteq m\left(M, \Delta_m^n, \Psi, p\right)$$

if and only if $\sup_{s\geq 1} \dfrac{\varphi_s}{\Psi_s} < \infty$.

Corollary 8: The space $m\left(M, \Delta_m^n, \varphi, p\right)$ is not solid and symmetric in general.

Proof: To show this space is not solid and symmetric in general, consider the following examples, respectively.

Example 1. Let m=n=1, $\varphi_k = 1$, $p_k = 1$ and $x_k = 1$ for all k $\in N$. Consider $\lambda = (\lambda_k) = \left((-1)^k\right)$ for all k $\in N$ and M(x)=x. Then $(x_k) \in m\left(M, \Delta_m^n, \varphi, p\right)$ but $(\lambda_k x_k) \notin m\left(M, \Delta_m^n, \varphi, p\right)$. Hence the space is not solid in general.

Example 2. Let m=n=1, $\varphi_k = k^{-1}$, $p_k = 1$ and $x_k = 1$ for all k $\in N$ and M(x)=x. Then the sequence (x_k) define $x_k = k$ for all k $\in N$ is in $m\left(M, \Delta_m^n, \varphi, p\right)$. Consider the sequence (y_k), the rearrangement of $x = (x_k)$ define as follows

$$(y_k) = (x_1, x_2, x_4, x_3, x_9, x_5, x_{16}, x_6, x_{27}, x_7, x_{36}, x_8, x_{49}, x_{10}, x_{64}, x_{11}, ...)$$

Then $(y_k) \notin m\left(M, \Delta_m^n, \varphi, p\right)$. Hence the space is not symmetric in general.

Finally, in this section, we consider that $p = (p_k)$ and $q = (q_k)$ are any bounded sequences of strictly positive real numbers. We are able to prove below results only under additional conditions.

Corollary 9: a) If $0 < \inf_k p_k \leq p_k \leq 1$ for all k, then

$$m\left(M, \Delta_m^n, \varphi, p\right) \subseteq m\left(M, \Delta_m^n, \varphi\right)$$

b) If $1 \leq p_k \leq \sup_k p_k = H < \infty$ for all k, then

$$m\left(M, \Delta_m^n, \varphi\right) \subseteq m\left(M, \Delta_m^n, \varphi, p\right)$$

c) Let $0 < p_k \leq q_k$ for all k and $\left(\dfrac{q_k}{p_k}\right)$ be bounded, then

$$m\left(M, \Delta_m^n, \varphi, q\right) \subseteq m\left(M, \Delta_m^n, \varphi, p\right)$$

Proof: Using the same technique as in Theorem 4 in [1], it is easy to prove the Corollary 9.

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