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# Hermite-Hadamard and Simpson Type Inequalities for Differentiable Quasi-Geometrically Convex Functions

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**Abstract** In this paper, the authors define a new identity for differentiable functions. By using of this identity, authors obtain new estimates on generalization of Hadamard and Simpson type inequalities for quasi-geometrically convex functions.

***Keywords:*** *quasi-geometrically convex functions, hermite–hadamard type inequalities, simpson type inequality*

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## 1. Introduction

Let real function $f$ be defined on some nonempty interval $I$ of real line R. The function $f$ is said to be convex on $I$ if inequality

$$f(tx+(1-t)y)\leq tf(x)+(1-t)f(y)$$

holds for all $x,y\in I$ and $t\in[0,1]$.

Following inequalities are well known in the literature as Hermite-Hadamard inequality and Simpson inequality respectively:

**Theorem 1.** Let $f:I\subseteq\mathbb{R}\to\mathbb{R}$ be a convex function defined on the interval $I$ of real numbers and $a,b\in I$ with $a<b$. The following double inequality holds

$$f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)dx\leq\frac{f(a)+f(b)}{2}.$$

**Theorem 2.** Let $f:[a,b]\to\mathbb{R}$ be a four times continuously differentiable mapping on $(a,b)$ and $\left\|f^{(4)}\right\|_\infty=\sup_{x\in(a,b)}\left|f^{(4)}(x)\right|<\infty.$ Then the following inequality holds:

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\frac{a+b}{2}\right)\right]-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\leq\frac{1}{2880}\left\|f^{(4)}\right\|_\infty(b-a)^4.$$

In recent years, many athors have studied errors estimations for Hermite-Hadamard, Ostrowski and Simpson inequalities; for refinements, counterparts, generalization see [2,9,10].

The following definitions are well known in the literature.

**Definition 1** ([7,8]). A function $f:I\subseteq(0,\infty)\to\mathbb{R}$ is said to be GA-convex (geometric-arithmatically convex) if

$$f(x^t y^{1-t})\leq tf(x)+(1-t)f(y)$$

for all $x,y\in I$ and $t\in[0,1]$.

**Definition 2** ([7,8]). A function $f:I\subseteq(0,\infty)\to(0,\infty)$ is said to be GG-convex (called in [13] geometrically convex function) if

$$f(x^t y^{1-t})\leq f(x)^t f(y)^{(1-t)}$$

for all $x,y\in I$ and $t\in[0,1]$.

In [3], İşcan gave definition of quasi-geometrically convexity as follows:

**Definition 3.** A function $f:I\subseteq(0,\infty)\to\mathbb{R}$ is said to be quasi-geometrically convex on $I$ if

$$f\left(x^t y^{1-t}\right)\leq\sup\{f(x),f(y)\},$$

for any $x,y\in I$ and $t\in[0,1]$.

Clearly, any GA-convex and geometrically convex functions are quasi-geometrically convex functions. Furthermore, there exist quasi-geometrically convex functions which are neither GA-convex nor GG-convex [3].

For some recent results concerning Hermite-Hadamard type inequalities for GA-convex, GG-convex, quasi-geometrically convex functions we refer interestes reader to [1,3,4,5,6,11,12,14].

The goal of this article is to establish some new general integral inequalities of Hermite-Hadamard and Simpson type for quasi-geometrically convex functions by using a new integral identity.

## 2. Main Results

Let $f : I \subseteq (0,\infty) \to \mathbb{R}$ be a differentiable function on $I^\circ$, the interior of $I$, throughout this section we will take

$$I_f(\lambda,\mu,a,b) = (\lambda-\mu) f\left(\sqrt{ab}\right) + \mu f(a)$$
$$+(1-\lambda) f(b) - \frac{1}{\ln(b/a)}\int_a^b \frac{f(u)}{u}du$$

where $a,b \in I$ with $a<b$ and $\lambda,\mu \in \mathbb{R}$.

In order to prove our main results we need the following identity.

**Lemma 1.** Let $f : I \subseteq (0,\infty) \to \mathbb{R}$ be a differentiable function on $I^\circ$ such that $f' \in L[a,b]$, where $a,b \in I$ with $a<b$. Then for all $\lambda,\mu \in \mathbb{R}$ we have:

$$I_f(\lambda,\mu,a,b) = \ln(b/a)\left\{\int_0^{1/2} (t-\mu) a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt \right. \tag{1}$$
$$\left. + \int_{1/2}^{1} (t-\lambda) a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt\right\}.$$

*Proof.* By integration by parts and changing the variable, we can state

$$\ln(b/a)\int_0^{1/2} (t-\mu) a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt$$
$$= \int_0^{1/2} (t-\mu) df\left(a^{1-t}b^t\right)$$
$$= (t-\mu) f\left(a^{1-t}b^t\right)\Big|_0^{1/2} - \int_0^{1/2} f\left(a^{1-t}b^t\right)dt$$
$$= \left(\frac{1}{2}-\mu\right) f\left(\sqrt{ab}\right) + \mu f(a) - \frac{1}{\ln(b/a)}\int_a^{\sqrt{ab}} \frac{f(u)}{u}du$$

and similarly we get

$$\ln(b/a)\int_{1/2}^{1} (t-\lambda) a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt$$
$$= \int_{1/2}^{1} (t-\lambda) df\left(a^{1-t}b^t\right)$$
$$= (t-\lambda) f\left(a^{1-t}b^t\right)\Big|_{1/2}^{1} - \int_{1/2}^{1} f\left(a^{1-t}b^t\right)dt$$
$$= (1-\lambda) f(b) - \left(\frac{1}{2}-\lambda\right) f\left(\sqrt{ab}\right) - \frac{1}{\ln(b/a)}\int_{\sqrt{ab}}^{b} \frac{f(u)}{u}du.$$

Adding the resulting identities we obtain the desired result.

**Theorem 3** Let $f : I \subset (0,\infty) \to \mathbb{R}$ be a differentiable function on $I^\circ$ such that $f' \in L[a,b]$, where $a,b \in I^\circ$ with $a<b$. If $|f'|^q$ is quasi-geometrically convex on $[a,b]$ for some fixed $q \ge 1$ and $0 \le \mu \le 1/2 \le \lambda \le 1$, then the following inequality holds

$$I_f(\lambda,\mu,a,b) \le \ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q, \left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}} \tag{2}$$
$$\left\{C_1^{1-1/q}(\mu)C_3^{1/q}(\mu,q,a,b) + C_2^{1-1/q}(\lambda)C_4^{1/q}(\lambda,q,a,b)\right\}$$

where

$$C_1(\mu) = \mu^2 - \frac{\mu}{2} + \frac{1}{8}, \tag{3}$$

$$C_2(\lambda) = \lambda^2 - \frac{3\lambda}{2} + \frac{5}{8},$$
$$C_3(\mu,q,a,b)$$
$$= \begin{cases} \dfrac{1}{2q\ln(b/a)}\Big[(1-2\mu)(ab)^{q/2} + 4\mu a^{(1-\mu)q} L\left(a^{q\mu},b^{q\mu}\right) \\ -a^{q/2}L\left(a^{q/2},b^{q/2}\right) - 2\mu a^q\Big], 0<\mu\le 1/2, \\ \dfrac{a^{q/2}}{2q\ln(b/a)}\left[b^{q/2} - L\left(a^{q/2},b^{q/2}\right)\right], \mu=0 \end{cases}$$
$$C_4(\lambda,q,a,b) =$$
$$\frac{1}{2q\ln(b/a)}\Big[2(1-\lambda)b^q - (2\lambda-1)(ab)^{q/2} - 2L\left(a^q,b^q\right)$$
$$+4\lambda a^{(1-\lambda)q} L\left(a^{q\lambda},b^{q\lambda}\right) - a^{q/2}L\left(a^{q/2},b^{q/2}\right)\Big],$$

and $L(a,b)$ is logarithmic mean defined by $L(a,b) = (b-a)/(\ln b - \ln a)$.

*Proof.* Since $|f'|^q$ is quasi-geometrically convex on $[a,b]$, for all $t \in [0,1]$

$$\left|f'\left(a^{1-t}b^t\right)\right|^q \le \sup\left\{\left|f'(a)\right|^q, \left|f'(b)\right|^q\right\}.$$

Hence, using Lemma 1 and power mean inequality we get

$$I_f(\lambda,\mu,a,b) \le \ln(b/a)$$
$$\times\left\{\left(\int_0^{1/2}|t-\mu|dt\right)^{1-\frac{1}{q}}\left(\int_0^{1/2} |t-\mu|\left(a^{1-t}b^t\right)^q \sup\left\{\left|f'(a)\right|^q, \left|f'(b)\right|^q\right\}dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\int_{1/2}^{1}|t-\lambda|dt\right)^{1-\frac{1}{q}}\left(\int_{1/2}^{1} |t-\lambda|\left(a^{1-t}b^t\right)^q \sup\left\{\left|f'(a)\right|^q, \left|f'(b)\right|^q\right\}dt\right)^{\frac{1}{q}}\right\}$$
$$\le \ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q, \left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$
$$\times\left\{\left(\int_0^{1/2}|t-\mu|dt\right)^{1-\frac{1}{q}}\left(\int_0^{1/2} |t-\mu|\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\int_{1/2}^{1}|t-\lambda|dt\right)^{1-\frac{1}{q}}\left(\int_{1/2}^{1} |t-\lambda|\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right\},$$

where

$$\int_0^{1/2}|t-\mu|\,dt = C_1(\mu) = \mu^2-\frac{\mu}{2}+\frac{1}{8},$$

$$\int_{1/2}^{1}|t-\lambda|\,dt = C_2(\lambda) = \lambda^2-\frac{3\lambda}{2}+\frac{5}{8},$$

$$\int_0^{1/2}|t-\mu|\left(a^{1-t}b^t\right)^q dt = C_3(\mu,q,a,b),$$

$$\int_{1/2}^{1}|t-\lambda|\left(a^{1-t}b^t\right)^q dt = C_4(\lambda,q,a,b),$$

which completes the proof.

**Corollary 1** Under the assumptions of Theorem 3 with $\lambda=\mu=1/2$, the inequality (2) reduced to the following inequality

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\left(\frac{1}{8}\right)^{1-1/q}\ln(b/a)\left(\sup\left\{\begin{array}{l}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{array}\right\}\right)^{\frac{1}{q}}\{C_3^{1/q}(1/2,q,a,b)$$

$$\le\left(\frac{1}{8}\right)^{1-1/q}\ln(b/a)\left(\sup\left\{\begin{array}{l}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{array}\right\}\right)^{\frac{1}{q}}$$

$$\times\left\{C_3^{1/q}(0,q,a,b)+C_4^{1/q}(1,q,a,b)\right\}+C_4^{1/q}(1/2,q,a,b)\big\}.$$

**Corollary 2** Under the assumptions of Theorem 3 with $\mu=0$ and $\lambda=1$, the inequality (2) reduced to the following inequality

$$\left|f\left(\sqrt{ab}\right)-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\left(\frac{1}{8}\right)^{1-1/q}\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left\{C_3^{1/q}(0,q,a,b)+C_4^{1/q}(1,q,a,b)\right\}.$$

**Corollary 3** Under the assumptions of Theorem 3 with $\mu=1/6$ and $\lambda=5/6$, the inequality (2) reduced to the following inequality

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\sqrt{ab}\right)\right]-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\left(\frac{5}{72}\right)^{1-1/q}\ln(b/a)\left(\sup\left\{\begin{array}{l}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{array}\right\}\right)^{\frac{1}{q}}$$

$$\times\left\{\begin{array}{l}C_3^{1/q}(1/6,q,a,b)\\ +C_4^{1/q}(5/6,q,a,b)\end{array}\right\}$$

**Theorem 4** Let $f:\ I\subset(0,\infty)\to\mathbb{R}$ be a differentiable function on $I^\circ$ such that $f'\in L[a,b]$, where $a,b\in I^\circ$ with $a<b$. If $|f'|^q$ is quasi-geometrically convex on $[a,b]$ for some fixed $q>1$ and $0\le\mu\le1/2\le\lambda\le1$, then the following inequality holds.

$$I_f\left(\lambda,\mu,a,b\right)\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}\tag{4}$$

$$\times\left\{C_5^{1/p}(p,\mu)C_7^{1/q}(q,a,b)+C_6^{1/p}(p,\lambda)C_8^{1/q}(q,a,b)\right\}$$

where

$$C_5(p,\mu)=\frac{1}{p+1}\left[\mu^{p+1}+(\frac{1}{2}-\mu)^{p+1}\right],$$

$$C_6(p,\lambda)=\frac{1}{p+1}\left[(\lambda-\frac{1}{2})^{p+1}+(1-\lambda)^{p+1}\right],$$

$$C_7(q,a,b)=\frac{1}{2}a^{q/2}L\left(a^{q/2},b^{q/2}\right),$$

$$C_8(q,a,b)=L\left(a^q,b^q\right)-C_7(q,a,b)$$

and $\frac{1}{p}+\frac{1}{q}=1$.

*Proof.* Since $|f'|^q$ is quasi-geometrically convex on $[a,b]$ and using Lemma 1 and Hölder inequality, we get

$$I_f\left(\lambda,\mu,a,b\right)\le\ln(b/a)$$

$$\times\left\{\left(\int_0^{1/2}|t-\mu|^p\,dt\right)^{\frac{1}{p}}\left(\int_0^{1/2}\left(a^{1-t}b^t\right)^q\sup\left\{\begin{array}{l}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{array}\right\}dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_{1/2}^{1}|t-\lambda|^p\,dt\right)^{\frac{1}{p}}\left(\int_{1/2}^{1}\left(a^{1-t}b^t\right)^q\sup\left\{\begin{array}{l}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{array}\right\}dt\right)^{\frac{1}{q}}\right\}$$

$$\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left\{\left(\int_0^{1/2}|t-\mu|^p\,dt\right)^{\frac{1}{p}}\left(\int_0^{1/2}\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_{1/2}^{1}|t-\lambda|^p\,dt\right)^{\frac{1}{p}}\left(\int_{1/2}^{1}\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right\},$$

here it is seen by simple computation that

$$\int_0^{1/2}|t-\mu|^p\,dt=\frac{1}{p+1}\left[\mu^{p+1}+(\frac{1}{2}-\mu)^{p+1}\right],$$

$$\int_{1/2}^{1}|t-\lambda|^p\,dt=\frac{1}{p+1}\left[(\lambda-\frac{1}{2})^{p+1}+(1-\lambda)^{p+1}\right],$$

$$\int_0^{1/2}\left(a^{1-t}b^t\right)^q dt=\frac{a^{q/2}}{2}L\left(a^{q/2},b^{q/2}\right)$$

$$and\int_{1/2}^{1}\left(a^{1-t}b^t\right)^q dt=L\left(a^q,b^q\right)-\frac{a^{q/2}}{2}L\left(a^{q/2},b^{q/2}\right).$$

Hence, the proof is completed.

**Corollary 4** *Under the assumptions of Theorem 4 with* $\lambda=\mu=1/2$, *the inequality (4) reduced to the following inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left(\frac{1}{2^{p+1}(p+1)}\right)^{1/p}\left\{C_7^{1/q}(q,a,b)+C_8^{1/q}(q,a,b)\right\}.$$

**Corollary 5** *Under the assumptions of Theorem 4 with* $\mu=0$ *and* $\lambda=1$, *the inequality (4) reduced to the following inequality.*

$$\left|f\left(\sqrt{ab}\right)-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left(\frac{1}{2^{p+1}(p+1)}\right)^{1/p}\left\{C_7^{1/q}(q,a,b)+C_8^{1/q}(q,a,b)\right\}.$$

**Corollary 6** *Under the assumptions of Theorem 4 with* $\mu=1/6$ *and* $\lambda=5/6$, *the inequality (4) reduced to the following inequality*

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\sqrt{ab}\right)\right]-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\frac{\ln(b/a)}{2}\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left(\frac{1+2^{p+1}}{6^{p+1}(p+1)}\right)^{1/p}\left\{C_7^{1/q}(q,a,b)+C_8^{1/q}(q,a,b)\right\}.$$

**Theorem 5** *Let* $f:\ I\subset(0,\infty)\to\mathbb{R}$ *be a differentiable function on* $I^\circ$ *such that* $f'\in L[a,b]$, *where* $a,b\in I^\circ$ *with* $a<b$. *If* $|f'|^q$ *is quasi-geometrically convex on* $[a,b]$ *for some fixed* $q>1$ *and* $0\le\mu\le 1/2\le\lambda\le 1$, *then the following inequality holds*

$$I_f\left(\lambda,\mu,a,b\right)$$

$$\le\ln(b/a)\left(\sup\left\{\begin{matrix}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{matrix}\right\}\right)^{\frac{1}{q}} \tag{5}$$

$$\times\left\{C_7^{1/p}(p,a,b)C_5^{1/q}(q,\mu)+C_8^{1/p}(p,a,b)C_6^{1/q}(q,\lambda)\right\}$$

where $C_5,C_6,C_7,C_8$ are defined as in Theorem 4 and $\frac{1}{p}+\frac{1}{q}=1$.

*Proof.* Since $|f'|^q$ is quasi-geometrically convex on $[a,b]$ and using Lemma 1 and Hölder inequality, we get

$$I_f\left(\lambda,\mu,a,b\right)\le\ln(b/a)$$

$$\times\left\{\left(\int_0^{1/2}\left(a^{1-t}b^t\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^{1/2}|t-\mu|^q\sup\left\{\begin{matrix}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{matrix}\right\}dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_{1/2}^{1}\left(a^{1-t}b^t\right)^p dt\right)^{\frac{1}{p}}\left(\int_{1/2}^{1}|t-\lambda|^q\sup\left\{\begin{matrix}\left|f'(a)\right|^q,\\ \left|f'(b)\right|^q\end{matrix}\right\}dt\right)^{\frac{1}{q}}\right\}$$

$$\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\left\{\left(\int_0^{1/2}\left(a^{1-t}b^t\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^{1/2}|t-\mu|^q dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_{1/2}^{1}\left(a^{1-t}b^t\right)^p dt\right)^{\frac{1}{p}}\left(\int_{1/2}^{1}|t-\lambda|^q dt\right)^{\frac{1}{q}}\right\},$$

$$\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\left\{C_7^{1/p}(p,a,b)C_5^{1/q}(q,\mu)+C_8^{1/p}(p,a,b)C_6^{1/q}(q,\lambda)\right\}.$$

Hence, the proof is completed.

**Corollary 7** *Under the assumptions of Theorem 5 with* $\lambda=\mu=1/2$, *the inequality (5) reduced to the following inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left(\frac{1}{2^{q+1}(q+1)}\right)^{1/q}\left\{C_7^{1/p}(p,a,b)+C_8^{1/p}(p,a,b)\right\}.$$

**Corollary 8** *Under the assumptions of Theorem 5 with* $\mu=0$ *and* $\lambda=1$, *the inequality (5) reduced to the following inequality*

$$\left|f\left(\sqrt{ab}\right)-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$

$$\le\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left(\frac{1}{2^{q+1}(q+1)}\right)^{1/q}\left\{C_7^{1/p}(p,a,b)+C_8^{1/p}(p,a,b)\right\}.$$

**Corollary 9** *Under the assumptions of Theorem 5 with* $\mu = 1/6$ *and* $\lambda = 5/6$, *the inequality (5) reduced to the following inequality*

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\sqrt{ab}\right)\right]-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du\right|$$
$$\le \frac{\ln(b/a)}{2}\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$
$$\times\left(\frac{1+2^{q+1}}{6^{q+1}(q+1)}\right)^{1/q}\left\{C_7^{1/p}(p,a,b)+C_8^{1/p}(p,a,b)\right\}.$$

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