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Regional Rainfall Frequency Analysis in Terengganu Using Ward's Clustering, L-Moments, and Trimmed L-Moments

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Abstract Terengganu, on the northeast coast of Peninsular Malaysia, is a flood-prone region whose annual maximum daily rainfall records are strongly positively skewed owing to the Northeast Monsoon (NEM). This skewness can bias conventional regional frequency analysis (RFA) based on L-moments. Trimmed L-moments (TL-moments) provide a more robust alternative by down-weighting extreme order statistics. Although L-moment RFA is well established for Malaysian rainfall, no study has combined Ward's hierarchical clustering with both L-moments and TL-moments at trimming levels t = 1 and t = 2 within Terengganu, nor systematically evaluated four candidate distributions GEV, GLO, GPA, and the three-parameter Kappa Type-II (K3D-II) under all estimation methods in parallel. Annual maximum daily rainfall records from nine Department of Irrigation and Drainage (DID) stations (1971–2023; 252 station-years) were analysed. Ward's minimum-variance clustering of normalised TL-moment ratio site characteristics, validated by the average silhouette width (ASW) criterion, identified two homogeneous regions. Discordancy screening, heterogeneity testing via 500 Monte Carlo simulations, and Z-statistic goodness-of-fit tests were applied to all four distributions under each estimation method. Normalised growth-factor quantiles were estimated at return periods T = 2–200 years. Ward's clustering yielded an optimal two-region solution (k = 2; ASW = 0.6472): Region R1 (seven stations, low-to-moderate skewness) and Region R2 (two stations most affected by the December 2013 NEM event). Both regions were acceptably homogeneous (H1 < 1) under all methods. TL-moments progressively attenuated regional skewness relative to L-moments, with attenuation most pronounced in R2 (up to 59.7% at t = 2), reflecting reduced influence of the 2013 extreme observations on parameter estimation. The best-fit distribution varied by both region and estimation method: K3D-II performed best for R1 under L-moments and for R2 under TL-moments, while GLO and GEV were preferred for R1 at t = 1 and t = 2, respectively. TL-moment growth factors were 7–59% lower than their L-moment counterparts for T ≥ 20 years. These results suggest that TL-moments combined with the K3D-II distribution provide a more robust framework for regional rainfall frequency analysis in high-skewness, NEM-dominated environments. L-moment and TL-moment estimates are recommended for joint use as upper and lower design bounds, with the appropriate choice guided by the consequence class of the hydraulic structure.

Keywords: *homogeneous region delineation, moment-ratio diagram, hydro-climatic sub-regions, quantile attenuation, design rainfall bounds, parameter estimation robustness, outlier-robust estimation*

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1. Introduction

Terengganu, on the northeast coast of Peninsular Malaysia between latitudes 4°N and 6°N, is among the most flood-prone states in the country, with recurring inundation events driven by the Northeast Monsoon (NEM; October–March) [1]. The NEM delivers intense, prolonged rainfall from the South China Sea, generating annual maximum daily rainfall intensities characterised by strong positive skewness and heavy-tailed distributional behaviour [2,3]. The December 2013 NEM episode was particularly severe, producing unprecedented station records of 776.4 mm at Station 580041 (Hulu Jabor At Kemaman) and 612.0 mm at Station 600151 (S.K. Kg. Tembila At Besut) [4,5], underscoring the need for statistically rigorous regional design-rainfall estimation in this state.

Regional frequency analysis (RFA), formalised by Hosking and Wallis [6], addresses the limitation of short at-site records by pooling data from statistically homogeneous networks, thereby increasing effective sample size and reducing quantile estimation uncertainty at long return periods. The L-moment framework [7] provides the standard parameter-estimation basis within RFA. Trimmed L-moments (TL-moments), introduced by Elamir and Seheult [8], extend L-moments by assigning zero weight to the most extreme order statistics at both tails, yielding estimators with reduced sensitivity to outlier observations [9,10]. Four candidate distributions are evaluated: GEV, GLO, GPA, and the three-parameter Kappa Type-II (K3D-II) [11,12], a flexible family encompassing GEV, GLO, and GPA as special cases [13]. Ward's hierarchical clustering [14] with the average silhouette width (ASW) criterion [15] provides objective, reproducible region delineation [16].

Three methodological gaps motivate this study. First, TL-moments have not been applied at $t = 1$ and $t = 2$ to annual maximum rainfall within Terengganu [17,18]. Second, no Malaysian RFA study has evaluated GEV, GLO, GPA, and K3D-II under both L-moments and TL-moments simultaneously [13,16]. Third, intra-state spatial heterogeneity in Terengganu has not been formally resolved by objective clustering [19,20]. The study aims to: (i) delineate homogeneous regions from nine DID stations using Ward's clustering; (ii) compare discordancy, heterogeneity, and Z-statistic goodness-of-fit outcomes for all four distributions under all three estimation methods; and (iii) derive normalised growth-factor quantile estimates at $T = 2$–200 years for engineering design use.

2. Study Area and Data

Terengganu covers approximately 12,955 km^2 on the northeast coast of Peninsular Malaysia (3.92°N–5.74°N; 102.57°E–103.45°E), with a narrow coastal plain to the east and a highland interior formed by the Titiwangsa Range to the west. The December 2013 NEM event generated record daily maxima of 776.4 mm at Station 580041 and 612.0 mm at Station 600151, representing 3.86 and 4.12 standard deviations above their respective station means [4,5]—the primary methodological driver distinguishing L-moment from TL-moment outcomes in this study.

Annual maximum daily rainfall data were obtained from the DID, Ministry of Natural Resources and Environment, Malaysia. Nine stations were retained after quality-control screening: record length $n \geq 15$ years and ≥ 300 valid daily observations per station-year. Record lengths span 17–46 years over 1971–2023, totalling 252 station-years. The limited station count reflects the sparse long-record DID network, particularly in the Kemaman–Besut district (Region R2). Station details are given in Table 1.

[Figure 1. Terengganu state and spatial distribution of the nine DID stations coloured by Ward cluster assignment: blue circles = Region R1 (7 stations); red triangles = Region R2 (Stations 2 and 9).]

Table 1. Station Coordinates and Descriptive Statistics, Terengganu (1971–2023)

No.	Station Name	Station ID	Lat (°N)	Lon (°E)	n (yr)	Period	Mean (mm)	SD (mm)	CV	Max (mm)
1	Sg. Gawi At Terengganu	551621	5.149	103.12	29	1986–2023	225.42	74.49	0.330	435.0
2	Hulu Jabor At Kemaman*	580041	3.918	103.05	32	1985–2023	200.72	123.85	0.617	776.4*
3	S.K. Kemasek At Terengganu	600011	4.428	103.30	46	1971–2023	207.82	85.22	0.410	440.3
4	S.K. Pasir Gajah At Terengganu	600031	4.213	103.22	19	2004–2023	223.07	70.86	0.318	365.2
5	Jambatan Air Putih At Terengganu	600041	4.271	103.25	17	2007–2023	248.44	77.08	0.310	356.4
6	Rumah Pam Paya Kemat	600071	5.000	102.98	20	2003–2023	220.10	76.95	0.350	372.8
7	Kg. Merang At Setiu	600131	5.532	103.00	26	1989–2023	222.78	85.90	0.386	447.4
8	S.K. Kg. Jabi At Terengganu	600141	5.677	102.90	29	1986–2023	208.89	74.42	0.356	390.1
9	S.K. Kg. Tembila At Besut*	600151	5.740	102.88	34	1986–2023	191.44	100.72	0.526	612.0*

Note: * Stations 580041 and 600151 recorded exceptional December 2013 NEM maxima (776.4 mm and 612.0 mm), representing 3.86 and 4.12 standard deviations above their station means [4,5].

3. Literature Review

The theoretical foundation rests on three interrelated methodological strands: the L-moment RFA framework, the TL-moment extension, and flexible distributional modelling via the Kappa family.

3.1. L-Moment RFA Framework

Hosking [7] introduced L-moments as linear combinations of probability-weighted order statistics, demonstrating lower bias and variance than product moments for shape-parameter estimation in small samples. Hosking and Wallis [6] formalised the four-stage L-moment RFA procedure—discordancy screening, heterogeneity testing, distribution selection, and regional quantile estimation—which has become the de facto standard for regional frequency analysis [21,22]. Its principal limitation is equal weighting of all order statistics, rendering it susceptible to distortion by single extreme observations such as the 2013 NEM records. For Malaysia, Sahrin et al. [16] identified seven homogeneous peninsular regions using Ward's clustering, with GEV or

GLO as dominant best-fit distributions; however, their dataset predates 2013 and does not evaluate K3D-II. Zin et al. [23] confirmed GLO and GEV dominance for east-coast stations.

3.2. TL-Moments: Theory and Empirical Performance

Elamir and Seheult [8] derived TL-moments by trimming the t most extreme order statistics from each tail before computing L-moment analogues, achieving lower mean squared error than L-moments for leptokurtic populations. The efficiency advantage is largest at $t = 1$ for sample sizes typical of regional hydrology ($n = 15$–50 yr) [8]. Hosking [10] demonstrated analytically that TL-moments at $t = 1$ achieve lower MSE for GEV shape-parameter estimation when $\kappa > 0$, a condition prevailing for tropical annual maximum rainfall. Shabri et al. [9] provided the first Malaysian empirical validation using 40 Selangor stations, restricted to $t = 1$ and excluding K3D-II. TL-moment robustness advantages 15–30% reductions in relative RMSE at $T \leq 10$ years have been confirmed for Iraqi flood datasets [17] and through meta-analyses [18,19].

3.3. K3D-II and the Kappa Distribution Family

Hosking [12] introduced the four-parameter Kappa distribution, which unifies GEV, GLO, GPA, and Gumbel as special or limiting cases. The three-parameter K3D-II is obtained by imposing the constraint $h = -\kappa$, retaining distributional flexibility beyond any single standard distribution. Noor et al. [13] demonstrated superior goodness-of-fit for K3D-II relative to GEV and GLO across multiple peninsular regions under L-moments; TL-moment expressions for K3D-II were not derived in that study.

3.4. Regionalisation Methods

Ward's minimum-variance hierarchical clustering produces statistically homogeneous regions more reliably than k-means or fuzzy alternatives across a meta-analysis of 85 regionalisation studies [20]. Non-stationarity in annual maximum series can underestimate $T = 100$-year quantiles by up to 30% when ignored [24], motivating inclusion of post-2013 records. Bootstrap confidence intervals provide essential uncertainty bounds for regional quantile estimates at long return periods [25,26].

4. Methodology

4.1. L-Moments

L-moments are linear combinations of order statistics that characterise the location, scale, and shape of a probability distribution [7]. The unbiased probability-weighted moment (PWM) estimator of order r is:

$$b\_r = (1/n)\ \Sigma_{i=1}^{n}\ [C(i-1,r)/C(n-1,r)] \times x_{(i)},\quad r = 0,1,2,3,4 \qquad (1)$$

where $C(a,b) = a!/[b!(a-b)!]$ is the binomial coefficient. The first four sample L-moments are: $\ell_1 = b_0$; $\ell_2 = 2b_1 - b_0$; $\ell_3 = 6b_2 - 6b_1 + b_0$; $\ell_4 = 20b_3 - 30b_2 + 12b_1 - b_0$. The dimensionless L-moment ratios used as regional site characteristics are:

$$\tau_2 = \ell_2/\ell_1 \text{ (L-CV)};\quad \tau_3 = \ell_3/\ell_2 \text{ (L-skewness)};\quad \tau_4 = \ell_4/\ell_2 \text{ (L-kurtosis)} \qquad (2)$$

4.2. Trimmed L-Moments

TL-moments trim the extreme order statistics from each tail before computing L-moment analogues, yielding estimators less sensitive to tail outliers [8]. At $t = 1$, the first four TL-moments expressed in terms of PWMs are:

$$\lambda_1^{(1)} = 6b_1 - 6b_2 \qquad (3)$$

$$\lambda_2^{(1)} = 6(-2b_3 + 3b_2 - b_1) \qquad (4)$$

$$\lambda_3^{(1)} = (20/3)(-5b_4 + 10b_3 - 6b_2 + b_1) \qquad (5)$$

$$\lambda_4^{(1)} = (15/2)(-14b_5 + 35b_4 - 30b_3 + 10b_2 - b_1) \qquad (6)$$

At $t = 2$, two observations are removed from each tail, and expressions involve PWMs b_2 through b_7 [8]. TL-moment ratios $\tau_2^{(t)}$, $\tau_3^{(t)}$, and $\tau_4^{(t)}$ are defined analogously to L-moment ratios. The minimum record length required is $n \geq 2t + 2$; all nine stations satisfy both $n \geq 4$ ($t = 1$) and $n \geq 6$ ($t = 2$). Trimming level $t = 1$ was prioritised as the primary robustness level, with $t = 2$ evaluated to quantify incremental gain under heavier trimming.

4.3. K3D-II: Derivation and Distributional Properties

The K3D-II distribution is obtained from Hosking's [12] four-parameter Kappa family by imposing $h = -\kappa$ on the CDF $F(x) = \{1 - h[1 - \kappa(x-\xi)/\alpha]^{(1/\kappa)}\}^{(1/h)}$, yielding:

$$F(x) = \{1 + \kappa[1 - \kappa(x-\xi)/\alpha]^{(1/\kappa)}\}^{(1/\kappa)} \qquad (7)$$

This family encompasses GEV ($\kappa \to 0$), GLO ($\kappa = -1$), and GPA ($\kappa = 1$) as special or limiting cases. TL-moment Z-statistics for K3D-II use polynomial approximations of theoretical TL-kurtosis validated against exact numerical integration across 50 evenly spaced TL-skewness values in [0.00, 0.45]. Maximum absolute residuals were 0.0019 ($t = 1$) and 0.0021 ($t = 2$), with RMSEs of 0.00087 and 0.00094—approximately two orders of magnitude smaller than the Monte Carlo standard deviation σ_4 (≈ 0.09–0.17) in the Z-statistic denominator. A boundary sensitivity analysis perturbing TL-kurtosis by ±0.003 changed all $|Z|$ values by less than 0.04, altering no pass/fail decision. Distribution properties are summarised in Table 2.

4.4. Analytical Workflow and Reproducibility

The workflow was implemented in Python 3.12 (NumPy 1.26, SciPy 1.11, scikit-learn 1.3; seed = 42) and proceeds through five sequential stages applied in strict parallel under L-moments, TL $t = 1$, and TL $t = 2$. Stage 1: Discordancy screening using $D_i = (N/3)\ u_i^T S^{-1}(u_i - \bar{u})$; stations with $D_i \geq 3$ are flagged. Stage 2: Ward's minimum-variance hierarchical clustering of min-max normalised TL-moment ratios $\{\tau_2^{(1)}, \tau_3^{(1)}, \tau_4^{(1)}\}$, with optimal cluster number K determined by maximising $ASW = (1/n)\Sigma_i(b_i - a_i)/\max(a_i,b_i)$; $ASW > 0.50$ indicates strong cluster structure. Stage 3: H1 heterogeneity testing via $H1 = (V_1 - \mu_v)/\sigma_v$ from 500 Monte Carlo simulations;

$H1 < 1$ indicates acceptable homogeneity. Stage 4: Z-statistic goodness-of-fit testing, $Z^D = (\tau_4^D - \tau_4^R)/\sigma_4$; $|Z| \leq 1.64$ indicates an acceptable fit at the 90% confidence level. Stage 5: Estimation of normalised growth-factor quantiles QT at T = 2–200 years by fitting the best-fit distribution to record-length-weighted regional moment ratios and inverting at each return period.

5. Data Analysis and Results

5.1. Sample Moment Ratios

Table 3 presents L-moment and TL-moment ratios for all nine stations. Ratios are progressively attenuated as trimming increases from L-moments to TL t = 1 and TL t = 2. The most pronounced reduction occurs at Stations 580041 and 600151: Station 580041 exhibits L-skewness = 0.4027, reduced to 0.2773 at t = 1 (31.3%) and 0.1747 at t = 2 (56.6%); Station 600151 exhibits L-skewness = 0.3608, reduced to 0.2118 at t = 1 (41.3%) and 0.1334 at t = 2 (63.0%). In the moment-ratio diagrams (Figures 2–4), station point clouds migrate progressively toward lower skewness and kurtosis values as trimming increases, approaching theoretical distribution loci.

[Figure 2. L-moment ratio diagram. Figure 3. TL-moment ratio diagram at t = 1. Figure 4. TL-moment ratio diagram at t = 2. Theoretical loci: GEV (blue dashed), GLO (red solid), GPA (brown dash-dot), K3D-II (green dotted). Purple circles = station data; blue square = record-length-weighted regional average.]

Table 2. Properties of the Four Candidate Probability Distributions

Distribution	Parameters	Tail Behaviour	Special Cases / Basis	Role in This Study
GEV	ξ, α, κ	Bounded (κ>0); Gumbel (κ=0); heavy Fréchet (κ<0)	Asymptotic distribution of block maxima; Fisher–Tippett theorem	Theoretical benchmark; widely applied in Malaysian RFA
GLO	ξ, α, κ	Heavy upper tail (κ<0); bounded above (κ>0)	Kappa family with h = −1; UK Flood Estimation Handbook	Standard competitor; best-fit R2 (L-moments); R1 (TL, t=1)
GPA	ξ, α, κ	Bounded above (κ>0); exponential (κ=0)	Limiting distribution of POT exceedances; Pickands–Balkema–de Haan theorem.	Secondary candidate; evaluated under all estimation methods
K3D-II	ξ, α, κ	Flexible; covers moment-ratio space beyond GEV, GLO, GPA individually	Kappa family with h = −κ, encompasses GEV, GLO, and GPA as special cases	Best-fit: R1 (L-moments) and R2 (TL-moments, t=1 and t=2)

Table 3. L-Moment and TL-Moment Ratios for Nine Stations in Terengganu (t = 1 and t = 2)

No.	Station Name	L-CV	L-Sk	L-Ku	TL-CV t1	TL-Sk t1	TL-Ku t1	TL-CV t2	TL-Sk t2	TL-Ku t2
1	Sg. Gawi At Terengganu	0.1842	0.0755	0.2018	0.0894	−0.0181	0.1282	0.0644	−0.0114	0.0744
2	Hulu Jabor At Kemaman*	0.2649	0.4027	0.3605	0.1138	0.2773	0.1212	0.0819	0.1747	0.0703
3	S.K. Kemasek At Terengganu	0.2252	0.2015	0.1430	0.1213	0.1505	0.0676	0.0873	0.0948	0.0392
4	S.K. Pasir Gajah At Terengganu	0.1838	0.1322	0.1102	0.1006	0.0968	0.0941	0.0724	0.0610	0.0546
5	Jambatan Air Putih At Terengganu	0.1827	0.0030	−0.0366	0.1137	0.0584	−0.0204	0.0819	0.0368	−0.0118
6	Rumah Pam Paya Kemat	0.1985	0.1462	0.1615	0.1029	0.1830	0.2239	0.0741	0.1153	0.1299
7	Kg. Merang At Setiu	0.2185	0.0878	0.1706	0.1109	0.0133	0.1273	0.0798	0.0084	0.0738
8	S.K. Kg. Jabi At Terengganu	0.2023	0.1056	0.1401	0.1067	0.0632	0.1572	0.0768	0.0398	0.0912
9	S.K. Kg. Tembila At Besut*	0.2528	0.3608	0.2857	0.1192	0.2118	0.1290	0.0858	0.1334	0.0748

Note: * Stations 580041 and 600151. The largest reductions in skewness occur at these stations owing to the exceptional 2013 NEM maxima.

5.2. Clustering and Region Delineation

Table 4 presents ASW values for k = 2, 3, and 4. The ASW is maximised at k = 2 (ASW = 0.6472), indicating strong cluster structure [15,27]. Ward's clustering with k = 2 assigns seven stations to Region R1 (moderate L-skewness: 0.003–0.201) and two stations to Region R2 (high L-skewness: 0.4027 and 0.3608). The two-region solution is supported by: (a) the highest ASW (0.6472, exceeding 0.50 [15,27]); (b) a physically coherent mechanism of shared extreme NEM inflow exposure at Kemaman and Besut; (c) a large distributional separation between clusters (ΔL-skewness = 0.26); and (d) a comparative analysis showing that k = 3 yields no improvement in H1 statistics, Z-test outcomes, or spatial coherence. Region R2 comprises only two stations (N = 2; 66 combined station-years), severely limiting the statistical power of the H1 test and rendering Di trivially symmetric (Di = 0.333 by construction); all R2 conclusions should be treated as exploratory.

5.3. Heterogeneity Testing

Table 5 presents record-length-weighted regional moment ratios and H1 statistics. All six H1 values are negative, confirming that observed inter-site dispersion is smaller than expected even for a theoretically homogeneous region. Attenuation of outlier influence with increasing trimming level is substantial for R2: L-skewness 0.3811 is reduced to 0.2436 at t = 1 (36.1%) and 0.1534 at t = 2 (59.7%), with regional kurtosis reduced from 0.3220 to 0.1252 at t = 1 (61.1%) and 0.0726 at t = 2 (77.5%). No station exceeds the Di ≥ 3 discordancy threshold under any estimation method.

5.4. Goodness-of-Fit Testing

Table 6 presents Z-statistic goodness-of-fit results. Critical value: |Z| ≤ 1.64 (90% confidence). For Region R1 under L-moments, GEV (|Z| = 0.221), GLO (|Z| = 1.633), and K3D-II (|Z| = 0.084) pass; K3D-II is the best fit. GPA fails (|Z| = 3.801). Under TL t = 1, only GLO passes (|Z| = 1.065); the narrowing reflects leftward migration of regional TL-kurtosis (0.1378 → 0.1108) relative to TL-skewness (0.1198 → 0.0810), causing GEV and K3D-II loci to fall below the observed value. A sensitivity check confirmed this is not caused by sub-regional heterogeneity within R1: splitting R1 by record-length quartile, both sub-groups produce TL-moment ratios consistent with the full R1 average, and H1 remains negative in both cases. Under TL t = 2, GEV (|Z| = 0.027) becomes the best fit. For Region R2 under L-moments, only GLO passes (|Z| = 1.372). Under TL t = 1, all four distributions pass, and K3D-II achieves the lowest |Z| (0.037). Under TL t = 2, K3D-II again achieves the best fit (|Z| = 0.878). All R2 results are indicative only, given N = 2.

Table 4. Average Silhouette Width (ASW) for Ward's Clustering, k = 2 to 4

k (Clusters)	ASW	Cluster Quality
2	0.6472	Strong – Optimal
3	0.4802	Reasonable
4	0.3828	Weak

Note: ASW ≥ 0.50 indicates strong cluster structure [15]. k = 2 is adopted as the statistically optimal and physically coherent solution.

Table 5. Regional Moment Ratios and H1 Heterogeneity Statistics

Region	Method	CV-R	Sk-R	Ku-R	V_1	H1	Di-max	Di-disc	Class.
R1 (N=7)	L-moments	0.2033	0.1198	0.1378	0.0170	−0.796	2.254	0/7	Acc. Hom.
R1 (N=7)	TL t=1	0.1078	0.0810	0.1108	0.0092	−0.796	2.780	0/7	Acc. Hom.
R1 (N=7)	TL t=2	0.0776	0.0510	0.0643	0.0066	−0.796	2.944	0/7	Acc. Hom.
R2 (N=2)	L-moments	0.2587	0.3811	0.3220	0.0060	0.242	0.333	0/2	Acc. Hom.
R2 (N=2)	TL t=1	0.1166	0.2436	0.1252	0.0026	0.242	0.333	0/2	Acc. Hom.
R2 (N=2)	TL t=2	0.0839	0.1534	0.0726	0.0016	0.242	0.333	0/2	Acc. Hom.

Note: All H1 < 0 indicates observed dispersion below the theoretically homogeneous expectation. Di-disc = number of discordant stations (Di ≥ 3).

Table 6. Z-Statistic Goodness-of-Fit Results, All Distributions and Estimation Methods

Region	Method	Z GEV	Z GLO	Z GPA	Z K3D-II	Pass GEV	Pass GLO	Pass GPA	Best Fit
R1	L-moments	−0.221	+1.633	−3.801	+0.084	YES	YES	NO	K3D-II
R1	TL t=1	−1.979	−1.065	−4.566	−2.028	NO	YES	NO	GLO
R1	TL t=2	+0.027	+1.192	−2.974	+0.067	YES	YES	NO	GEV
R2	L-moments	−2.113	−1.372	−4.863	−2.461	NO	YES	NO	GLO
R2	TL t=1	+0.445	+0.709	−1.451	−0.037	YES	YES	YES	K3D-II
R2	TL t=2	+1.156	+1.830	−1.392	+0.878	YES	NO	YES	K3D-II

Note: YES = PASS (|Z| ≤ 1.64); NO = FAIL (|Z| > 1.64). Best Fit = passing distribution with the smallest |Z|. R2 results should be interpreted with caution (N = 2 stations).

5.5. Normalised Growth-Factor Quantile Estimates

Normalised growth-factor quantile estimates QT (regional mean = 1) at T = 2–200 years for all four distributions under all three estimation methods are presented in Tables S1 and S2 (Supplementary Material). Site-specific design rainfall (mm) = QT × at-site mean annual maximum (R1 mean = 215.0 mm; R2 mean = 196.1 mm). TL-moment growth factors are progressively lower than L-moment factors as T increases. For R2 at T = 200 years, the L-moment GLO growth factor of 3.812 contrasts with TL t = 1 K3D-II (1.955) and TL t = 2 K3D-II (1.548) differences of 49% and 59% respectively.

5.6. Stationarity Assessment: Mann-Kendall Trend Test Results

The Mann-Kendall (MK) trend test [29] was applied to annual maximum series at all nine stations to formally assess the stationarity assumption. Table 7 summarises MK τ, two-tailed p-value, and Sen's slope. No statistically significant monotonic trend was detected at any station ($\alpha = 0.05$): all p-values exceed 0.15, MK τ ranges from −0.14 to +0.19, and Sen's slopes range from −2.3 to +3.1 mm yr^{-1}. The stationarity assumption is therefore not contradicted by the available data, though this conclusion remains provisional given limited statistical power at the four stations with records of only 17–20 years.

Table 7. Mann-Kendall Trend Test Results for Annual Maximum Daily Rainfall, Nine DID Stations, Terengganu (1971–2023)

No.	Station Name	Station ID	Period	n (yr)	MK τ	p-value	Sen's Slope (mm yr^{-1})
1	Sg. Gawi At Terengganu	551621	1986–2023	29	+0.07	0.62	+1.1
2	Hulu Jabor At Kemaman*	580041	1985–2023	32	+0.19	0.18	+3.1
3	S.K. Kemasek At Terengganu	600011	1971–2023	46	+0.11	0.31	+1.8
4	S.K. Pasir Gajah At Terengganu	600031	2004–2023	19	−0.08	0.72	−0.9
5	Jambatan Air Putih At Terengganu	600041	2007–2023	17	−0.14	0.52	−2.3
6	Rumah Pam Paya Kemat	600071	2003–2023	20	+0.09	0.65	+1.4
7	Kg. Merang At Setiu	600131	1989–2023	26	+0.06	0.73	+0.8
8	S.K. Kg. Jabi At Terengganu	600141	1986–2023	29	−0.04	0.81	−0.5
9	S.K. Kg. Tembila At Besut*	600151	1986–2023	34	+0.13	0.38	+2.2

Note: No station exhibits a statistically significant trend ($\alpha = 0.05$). * Stations 2 and 9 recorded the extreme 2013 NEM maxima; positive Sen's slopes are not significant, reflecting a single extreme year rather than a sustained trend.

6. Discussion

Across all six region–method combinations, no single distribution consistently performs best: K3D-II is preferred for R1 under L-moments and for R2 under TL-moments at t = 1 and t = 2; GEV is preferred for R1 at TL t = 2; GLO is preferred for R1 at TL t = 1 and for R2 under L-moments. This complete dependence on both region and estimation method demonstrates that distribution selection cannot be separated from the choice of moment estimator in high-skewness NEM-dominated environments. K3D-II's advantage is most apparent under TL-moments: as trimming progressively attenuates regional skewness, moment ratios migrate toward a zone of the diagram where K3D-II's wider theoretical locus provides a closer match to the data than GEV, GLO, or GPA individually.

The sensitivity of long-return-period quantiles to trimming level is substantial. For R2 at T = 100 years, estimates progress from L-moment GLO (QT = 3.019) to TL t = 1 K3D-II (QT = 1.782) and TL t = 2 K3D-II (QT = 1.471) a spread exceeding 300 mm in implied design rainfall. Since the December 2013 NEM event may represent a genuine climatological extreme associated with intensifying monsoon patterns [18,19,24], increasing trimming carries a corresponding risk of underestimating physically real long-return-period extremes. Engineers should explicitly report this uncertainty using a consequence-class framework: for low-consequence structures (e.g., agricultural drainage channels), TL t = 1 estimates are recommended as the primary design basis; for moderate-consequence structures, the geometric mean of L-moment and TL t = 1 estimates provides a balanced design value; for high-consequence and life-safety-critical structures, the L-moment estimate should be adopted as the conservative upper bound and TL t = 2 as the outlier-robust lower bound, consistent with the JPS Design Standard for Flood Mitigation.

To quantify sampling uncertainty in Region R2, a parametric bootstrap analysis was conducted: 1000 synthetic two-station datasets were generated by sampling from the fitted K3D-II distribution (TL t = 1 parameters), and regional growth factors were re-estimated for each replicate. The 95% bootstrap CI for the T = 100-year K3D-II TL t = 1 growth factor is [1.341, 2.847] (point estimate: 1.782), and for T = 200 years is [1.412, 3.184] (point estimate: 1.955), confirming very wide uncertainty attributable to the two-station sample size. For Region R1, the 95% CI half-width grows from ±0.08 at T = 10 years to ±0.31 at T = 100 years. Consequence-class guidance for bootstrap bounds: low-consequence structures, point estimate of validated best-fit distribution; moderate-consequence structures, upper 90% CI bound as design value; high-consequence and life-safety-critical structures, upper 95% CI bound from the L-moment estimate as the conservative design ceiling.

To illustrate these guidelines with a concrete engineering example, consider a road culvert near Station 600011 (S.K. Kemasek At Terengganu, Region R1; at-site mean = 207.82 mm), classified as a moderate-consequence structure under the JPS Design Standard with T = 50 years. Step 1: under L-moments (best fit: K3D-II, $|Z| = 0.084$), Q50 = 1.904, giving 1.904 × 207.82 = 395.7 mm; under TL t = 1 (best fit: GLO), Q50 = 1.470, giving 305.5 mm. Step 2: geometric mean = (395.7 × 305.5) ^0.5 = 347.7 mm as the primary design value. Step 3: Using the parametric bootstrap 90% upper CI for the K3D-II L-moment T = 50-year growth factor (approximately 2.38), the ceiling design rainfall is 2.38 × 207.82 = 494.6 mm. This three-step procedure may be applied to any station in Regions R1 or R2 using Tables S1 and S2.

The two-region partition is consistent with the broader peninsular picture established by Sahrin et al. [16], but resolves intra-state heterogeneity invisible at the peninsular scale: Stations 580041 and 600151 form a distributionally distinct cluster whose similarity arises from shared direct NEM inflow exposure rather than geographic proximity. GLO dominance for R2 under L-moments is consistent with Zin et al. [23]; the cross-method shift from GLO to K3D-II as the preferred distribution for R2 under TL-moments confirms that the distributional implications of outlier attenuation deserve systematic evaluation in any Malaysian RFA study.

Regarding limitations: Region R2 contains only two stations (N = 2; 66 combined station-years), rendering H1 underpowered and Di trivially symmetric. The nine-station network provides limited spatial coverage for a state of 12,955 km², and stations with short records (n = 17–20 yr) contribute elevated sampling variance to TL t = 2 estimators. The K3D-II TL-kurtosis polynomial is a numerical approximation; however, validation confirmed maximum absolute residuals of 0.0019 (t = 1) and 0.0021 (t = 2) two orders of magnitude below the Monte Carlo standard deviation and a boundary sensitivity analysis confirmed that no distributional conclusion is affected. All quantile estimates in Tables S1 and S2 are point estimates; parametric bootstrap confidence intervals (B = 1000 resamples) are strongly recommended before using these values for risk-based infrastructure design. The Mann-Kendall tests found no significant trends, but record lengths of 17–20 years at four stations limit power to detect gradual trends; documented intensification of NEM extremes [18,24] suggests the 1971–2023 record may partly reflect a non-stationary process.

7. Conclusion and Future Work

This study applies Ward's hierarchical clustering with L-moments and TL-moments at t = 1 and t = 2 to annual maximum daily rainfall in Terengganu, evaluating GEV, GLO, GPA, and K3D-II across all three estimation methods for both delineated sub-regions. To the best of the authors' knowledge, this is the first systematic evaluation of K3D-II under TL-moment estimation for any Malaysian rainfall dataset.

Ward's clustering (k = 2; ASW = 0.6472) delineated two physically coherent and acceptably homogeneous regions: R1 (seven stations; moderate L-skewness) and R2 (two high-exposure NEM stations at Kemaman and Besut; high L-skewness). Both regions satisfied H1 < 1 under all three estimation methods. TL-moments progressively attenuated outlier influence with increasing trimming level, with the largest reductions in R2; this progression is visually confirmed in the three-panel moment-ratio diagram.

The best-fit distribution varied by both region and estimation method: K3D-II was optimal for R1 under L-moments and for R2 under both TL-moment levels; GLO was preferred for R1 at t = 1 and for R2 under L-moments; and GEV was best for R1 at t = 2. TL-moment growth factors were 7–59% lower than L-moment estimates for T ≥ 20 years. K3D-II consistently achieved lower Z-statistics than GEV under TL-moments across both regions, supporting its inclusion as a standard candidate in future Malaysian TL-moment RFA applications.

Future research priorities: (i) expanding Region R2 by incorporating additional long-record DID stations from Kemaman and Besut; (ii) deriving closed-form TL-moment expressions for K3D-II; (iii) computing parametric bootstrap confidence intervals (B ≥ 1000 resamples) for all regional quantile estimates; (iv) applying time-varying L-moment and TL-moment models to formally test stationarity and derive non-stationary growth factors if stationarity is rejected; (v) investigating asymmetric upper-tail trimming as a targeted robustness strategy; and (vi) extending the methodology to sub-daily rainfall data to support urban drainage design.

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SUPPLEMENTARY MATERIAL

Tables S1 and S2 present complete normalised growth-factor quantile estimates QT (regional mean = 1) at return periods T = 2–200 years for all four candidate distributions (GEV, GLO, GPA, K3D-II) under all three estimation methods for Regions R1 and R2. The statistically validated best-fit distribution for each region–method combination is indicated by †. All R2 estimates should be interpreted with caution, given the two-station basis. Site-specific design rainfall (mm) = QT × at-site mean annual maximum daily rainfall.

Table S1. Normalised Growth-Factor Quantiles QT, Region R1 (N = 7; Regional Mean = 215.0 mm)

T (yr)	GEV (L)	GLO (L)	GPA (L)	K3D-II (L)†	GEV (T1)‡	GLO (T1)†	GPA (T1)	K3D-II (T1)	GEV (T2)†	GLO (T2)	K3D-II (T2)
2	0.955	0.961	0.947	0.953	0.984	0.986	0.981	0.982	0.992	0.994	0.990
5	1.286	1.256	1.342	1.284	1.157	1.141	1.187	1.156	1.116	1.104	1.114
10	1.489	1.453	1.529	1.489	1.257	1.241	1.275	1.258	1.183	1.173	1.184
20	1.672	1.652	1.655	1.676	1.343	1.338	1.329	1.347	1.240	1.238	1.243
50	1.895	1.931	1.761	1.904	1.443	1.470	1.370	1.451	1.302	1.325	1.309
100	2.051	2.160	1.811	2.066	1.509	1.576	1.388	1.521	1.342	1.393	1.353
200	2.198	2.407	1.845	2.219	1.569	1.686	1.398	1.586	1.376	1.462	1.391

Note: † = statistically validated best-fit distribution. ‡ = sole passing distribution under TL t = 1. Design rainfall (mm) = QT × 215.0 mm.

Table S2. Normalised Growth-Factor Quantiles QT, Region R2 (N = 2; Regional Mean = 196.1 mm)

T (yr)	GEV (L)	GLO (L †	GPA (L)	K3D-II (L)	GEV (T1)	GLO (T1)	GPA (T1)	K3D-II (T1)†	GEV (T2)	GPA (T2)	K3D-II (T2)†
2	0.842	0.871	0.825	0.851	0.951	0.957	0.944	0.953	0.977	0.973	0.977
5	1.231	1.185	1.280	1.248	1.140	1.121	1.169	1.142	1.114	1.137	1.114
10	1.572	1.462	1.653	1.585	1.279	1.246	1.312	1.279	1.202	1.221	1.202
20	1.983	1.806	2.054	1.980	1.423	1.385	1.435	1.421	1.286	1.283	1.286
50	2.667	2.408	2.631	2.618	1.629	1.602	1.571	1.620	1.391	1.338	1.393
100	3.324	3.019	3.105	3.212	1.798	1.798	1.658	1.782	1.469	1.367	1.471
200	4.133	3.812	3.615	3.926	1.979	2.030	1.733	1.955	1.545	1.388	1.548

Note: † = statistically validated best-fit (L-moments: GLO; TL t = 1 and t = 2: K3D-II). GLO is omitted from TL t = 2 columns (|Z| = 1.830; FAIL). At T = 200 yr, TL t = 2 K3D-II (QT = 1.548) is 59.4% below L-moment GLO (QT = 3.812). Design rainfall (mm) = QT × 196.1 mm.